Efficient Fault Tolerant Topology Control for Three-Dimensional Wireless Networks

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***Abstract*— Fault tolerant topology control in wireless networks has been studied recently. In order to achieve both sparseness (i.e., the number of links is linear with the number of nodes) and fault tolerance (i.e., can survive certain level of node/link failures), different geometric topologies were proposed and used as the underlying network topologies for wireless networks. However, most of the existing topology control algorithms can only be applied to 2-dimensional (2D) networks where all nodes are distributed in a 2D plane. In practice, wireless networks may be deployed in 3-dimensional (3D) space, such as under water wireless sensor networks in ocean or ad hoc networks in space. This paper seeks to investigate efficient fault tolerant topology control protocols for 3D wireless networks. Our new protocols not only guarantee the k-connectivity of the network, but also ensure the bounded node degree and constant power stretch factor. All of our proposed protocols are localized algorithms, which only use one-hop neighbor information and constant messages with small time complexity. Our simulation confirms our theoretical proofs for all proposed 3D topologies.**

I. Introduction

Wireless networks (especially, wireless ad hoc and sensor networks) intrigue many challenging research problems because they inherently have some special characteristics and unavoidable limitations compared with traditional fixed infrastructure networks. Energy conservation and network performance are probably the most critical issues because mobile devices are usually powered on batteries and have limited computing capability. By allowing each wireless device to *locally* adjust its transmission range and select certain neighbors for communication, the *topology control* techniques aim to maintain a structure that can support energy efficient routing as well as improve the overall network performance. There exist several topology control techniques, such as localized geometrical structures, dynamic cluster techniques and power management protocols. In this paper, we focus on localized geometrical structure based methods [1]–[4] which utilize the rich geometric properties of wireless network to enable efficient topology control.

In order to be power efficient, traditional topology control algorithms try to reduce the number of links, and thereby, reduce the redundancy available for tolerating node and link failures. Thus, the topology derived with such algorithms is more vulnerable to node failures or link breakages. However, due to constrained power capacity, hostile deployment environment, and other factors, event like individual node failures are more likely to happen, which might cause network partitions and badly degrade the network performance. Therefore, in order to gain certain degree of redundancy and guarantee the overall performance, fault tolerance becomes a critical requirement for the design of wireless networks. As fault tolerance strongly depends on the network connectivity, topology design for such networks needs to consider both power efficiency and fault tolerance. Recently, [5]–[7] extended existing topology control algorithms with fault tolerance.

Most proposed geometrical structure based topology control algorithms [1]–[6] are only applied to 2D networks. However, in practice, wireless networks are often deployed in 3D space, such as notebooks in a multi-floor building and sensor nodes in an *under-water sensor network* (UWSN). Three dimensional UWSN is used to detect and observe phenomena that can not be adequately observed by means of ocean bottom sensor nodes, i.e., to perform cooperative sampling of the 3D ocean environment. In this paper, we study how to efficiently construct topology for 3D networks to maintain fault tolerance, conserve energy and enable energy efficient routing.

The contributions of this paper are as follows: (1) we introduce three new localized geometrical structures, which can be easily constructed using constant messages; (2) we prove all three structures can preserve the k-connectivity of the network; (3) we prove two structures can provide energy efficient routes for routing while one of them can also bound the node degree. To the best of our knowledge, there is no previous research on 3D fault tolerant topology control. Even compared with the existing 3D topology control methods in [7], [8] for keeping 1-connectivity, our proposed structures are much easier to construct.

The rest of the paper is organized as follows. Section II presents our network model and related works on fault tolerant and 3D topology control. In Section III, we describe our new localized fault tolerant 3D topologies and prove their good properties. Section IV presents detailed simulation results and we conclude the paper in Section V.

II. Preliminary

A. Network Model

A 3D wireless network consists of a set V of n wireless nodes distributed in a 3D plane $\mathbb{R}^3$. Each node has the same *maximum transmission range* R. These wireless nodes define a *unit ball graph* (UBG), or called *unit sphere graph*, in which there is an edge uv between two nodes u and v iff (if and only

This work of Yu Wang is supported in part by the US National Science Foundation (NSF) under Grant No. CNS-0721666.

if) the Euclidean distance $\|uv\|$ between u and v in $\mathbb{R}^3$ is at most R. In other words, two nodes can always receive the signal from each other directly if the distance between them is no more than R. We also assume that all wireless nodes have distinctive identities and each node knows its position information either through a low-power GPS receiver or some other ways. By one-hop broadcasting, each node u can gather the location information of all nodes within its transmission range. As in the most common power-attenuation model, the power to support a link uv is assumed to be $\|uv\|^\beta$, where β is a real constant between 2 and 5 depending on the wireless transmission environment. We also assume that UBG is k-connected for some $k > 1$, i.e., given any pair of wireless devices in the network, there are at least k disjoint paths between them. With k-connectivity, the network can survive $k-1$ node/link failures.

B. Preferred Properties

Topology control protocols aim to maintain a structure that can improve the overall network performance. In the literature, the following desirable features of the structure are well-regarded and preferred in wireless networks: (1) To achieve fault tolerance (surviving $k-1$ failures), the constructed topology needs to be k-connected. (2) It is also desirable that node degree in the constructed topology is small and upper-bounded by a constant. A small node degree reduces the MAC-level contention and interference, and may help to mitigate the well-known hidden and exposed terminal problems. (3) A good network topology should be *energy efficient*, i.e., the total power consumption of the least energy cost path between any two nodes in final topology should not exceed a constant factor of the power consumption of the least energy cost path in original network. Given a path $v_1 v_2 \cdots v_h$ connecting two nodes v_1 and v_h, the energy cost of this path is $\sum_{j=1}^{h-1} \|v_j v_{j+1}\|^\beta$. The path with the least energy cost is called the shortest path in a graph. A subgraph H is called a *power spanner* of a graph G if there is a positive real constant ρ such that for any two nodes, the power consumption of the shortest path in H is at most ρ times of the power consumption of the shortest path in G. The constant ρ is called the *power stretch factor*. A power spanner of the communication graph (e.g., UBG) is usually energy efficient for routing. (4) Due to limited resources and high mobility of wireless nodes, it is preferred that the topology can be constructed locally. Here, a topology is *localized*, i.e., can be constructed locally, if every node u can decide all edges incident on itself in the topology by only using the information of nodes within a constant hops of u. Actually, all of our topologies presented here only use 1-hop neighbor information.

C. Related Works

With the objective of achieving energy efficiency and maintaining network connectivity, several geometrical structures have been proposed for topology control in 2D networks, such as *local minimum spanning tree* (LMST) [9], *relative neighborhood graph* (RNG) [10], [11], *Gabriel graph* (GG) [10], [12], *Yao graph* (YG) [2], [3], *cone-based topology control* (CBTC) [1], [4] and so on. In order to construct such topology structures, the nodes' transmission power are minimized, as a result, the number of links in the network is reduced as compare to the *unit disk graph* (UDG), which contains an edge between two nodes iff their distance is at most one. On the other side, lacking of redundancy makes the topology more susceptible to node failures or link breakages. In order to achieve routing redundancy and construct k-connected topology, recent research work extended existing topology control algorithms to incorporate fault tolerance.

Li and Hou [5] proposed a fault tolerant topology control algorithm to construct the k-connected topology, called fault-tolerant local spanning subgraph (FLSS_k). During the topology construction phase, a node builds a spanning subgraph to preserve k-connectivity using a simple greedy algorithm. However, for each step in the greedy algorithm, they need to check the k-connectivity of current local graph, which is very time consuming. Besides the k-connectivity, the authors also proved that FLSS_k can maintain bi-directional links and reduce the power consumption.

In [7], Bahramgiri *et al.* presented a variation of the CBTC algorithm to preserve the k-connectivity. The algorithm increases the transmission power until it reaches the maximum value or the angle between any two consecutive neighbors of the resulted topology is at most $\frac{2\pi}{3k}$. Even though the topology is proved to be k-connected, it does not bound the node degree.

Zhou *et al.* [6] modified the RNG structure as follow to construct k-RNG, which is k-connected. In k-RNG, an edge exists between nodes u and v iff there are at most $k-1$ nodes w that satisfy $\|uw\| < \|uv\|$ and $\|wv\| < \|uv\|$. Zhou *et al.* proved that k-RNG can guarantee the k-connectivity if the original communication graph is k-connected.

Li *et al.* [13] generalized the Yao structure to $YG_{p,k}$, which is defined as follows: at each node u, any equally separated p rays originating from u define p cones, where $p > 6$. In each cone, u chooses the k closest nodes, if there is any, and add directed links from u to these nodes. Li *et al.* proved $YG_{p,k}$ can preserve the k-connectivity. Besides, $YG_{p,k}$ is also a length/power spanner with bounded node degree.

All the algorithms and structures discussed above are only designed for 2D networks. Recently, there are also research work to apply topology control algorithms to 3D networks.

Wang *et al.* [14] studied 3D topology control by extending RNG and GG to 3D case and proposing new 3D Yao-based topologies, which can be constructed locally and efficiently. They proved several properties of these new 3D structures, e.g., bounded node degree and constant power stretch factor.

Bahramgiri *et al.* [7] also generalized CBTC algorithm from 2D to 3D, to preserve the connectivity. Basically, each node u increases its transmission power until there is no empty 3D-cone with angle degree α, i.e., there exists at least a node in each 3D-cone of degree α centered at u, if $\alpha \leqslant \frac{2\pi}{3}$. This algorithm can be extended to ensure k-connectivity with $\alpha \leqslant \frac{2\pi}{3k}$. Even though this approach can guarantee connectivity, the gap detection algorithm applied to check the existence of the empty 3D-cone of degree α is very complicated. The time complexity of the gap detection algorithm at a node u is $O(d^3 \log d)$, where d is the node degree of u. Moreover, their method can not bound the node degree, as shown by [13].

Similarly, Ghosh *et al.* [8] also presented two CBTC-based approaches for 3D wireless networks. Though the first approach, a heuristic based on 2D orthographic projections, can provide excellent performance in practice, it can not guarantee connectivity for sure. In the second approach, a spherical Delaunay triangulation (SDT) is built to determine the existence of empty 3D cones. Though the second approach can guarantee the connectivity of the network, the expense to construct the SDT is very high. As a result, the second approach is not as efficient as our new proposed methods in this paper.

Actually, to solely achieve the connectivity of a 3D network, LMST may be the best choice, since it is very sparse and can be easily constructed even for 3D networks, and can preserve the connectivity. However, it does not preserve the k-connectivity and can have very large power stretch factor.

III. 3D Fault Tolerant Topology Control

Most of the existing 3D topology control methods either can not support fault tolerance [7], [8], [14] or rely on some complex construction method [7]. In this section, we propose several simple localized topology control methods which guarantee to preserve the k-connectivity.

A. 3D k-RNG and 3D k-GG

The first two localized structures are based on 3D RNG and 3D GG. The definitions of 3D k-RNG and 3D k-GG are as follows: an edge $uv \in RNG^k_{3D}$ iff the intersection of two balls centered at u and v with radius $\|uv\|$ contains less than k nodes from the set V; an edge $uv \in GG^k_{3D}$ iff the ball with edge uv as a diameter contains less than k nodes of V. See Figure 1 for illustrations. Based on their definitions, 3D k-RNG and 3D k-GG can be easily constructed using 1-hop neighbors' position information only.

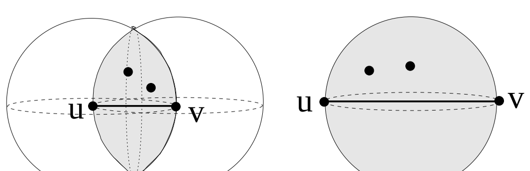


Fig. 1. 3D k-RNG (left) and 3D k-GG (right). An edge uv is kept iff the shaded area has less than k nodes. Here assume $k = 3$, thus uv is kept.

Now we prove both 3D k-RNG and 3D k-GG can preserve the k-connectivity .

Theorem 1: The structures RNG^k_{3D} and GG^k_{3D} are k-connected if the UBG G is k-connected, i.e., both RNG^k_{3D} and GG^k_{3D} can sustain $k-1$ node faults.

Proof: We first prove the theorem for 3D k-RNG. Given a set S of $k-1$ nodes, $S \subset V$, due to the k-connectivity of G, we know that $G-S$ is still connected. To prove RNG^k_{3D} is k-connected, we prove that $RNG^k_{3D} - S$ is connected by contradiction.

Assume that graph $RNG^k_{3D} - S$ is not connected, then, there must exist at least a pair of nodes such that there is no path between them. Let the nodes u, v be the pair with the smallest distance to each other , i.e., $\|uv\| \leq \|u'v'\|$ for any pair of nodes u', v' that are not connected. Since $uv \notin RNG^k_{3D} - S$, $uv \notin RNG^k_{3D}$. According to the definition of RNG^k_{3D}, there should be at least k neighbors w of node u that satisfy the condition $\|uw\| < \|uv\|$ and $\|wv\| < \|uv\|$. Assume the removed $k-1$ nodes are neighbors of node u, then there is at least one neighbor w left in $RNG^k_{3D} - S$ with $\|wv\| < \|uv\|$. As nodes u, v is the pair with the smallest distance among those disconnected pairs in $RNG^k_{3D} - S$, nodes w, v must be connected. Therefore, nodes u, v are also connected via w, which is a contradiction to the assumption. Thus, $RNG^k_{3D} - S$ is connected, and RNG^k_{3D} is k-connected. Note that for the case where removed nodes are not all neighbors of u, the proof also holds.

The above proof can be easily adopted for 3D k-GG. Due to space limit, we ignore the detail here. ■

We then prove the power efficiency for 3D k-GG.

Theorem 2: The structure GG^k_{3D} is a power spanner with spanning ratio bounded by one.

Proof: According to [14], GG^1_{3D} is a power spanner with the power stretch factor of one. From the construction of GG^k_{3D}, it is easy to know that $GG^1_{3D} \subseteq GG^k_{3D}$, with $k > 1$, as a result, GG^k_{3D} is also a power spanner with spanning ratio bounded by one. ■

This theorem shows that all links in the least energy cost paths are kept in 3D k-GG. In other words, 3D k-GG provides energy efficient routes for routing algorithms.

B. 3D k-YG

Since both 3D k-RNG and 3D k-GG can not bound node degree (a node with large number of nodes on the surface of its transmission ball), we are also interested in constructing fault tolerant Yao graph for 3D networks.

In 2D networks, Yao graph is defined as follows: at each node u, any p equally-separated rays originating at u define p cones. In each cone, only keep the shortest edge uv among all edges emanated from u, if there is any. The node out-degree of Yao graph is bounded by constant p. However, Yao graph can not be directly extended to 3D, while RNG and GG can. It is hard to define a fixed partition boundary of Yao structure of a node in 3D. Notice that a disk in 2D can be easily divided into p equal cones which do not intersect with each other, but in 3D case, it is impossible to divide a ball into p equal 3D cones without intersections among each other. Therefore, here we use equal size 3D cones which are defined by neighbors' position and can intersect to each other. Fortunately, we can prove that the number of such cones used in our protocol is still bounded by a constant.

Algorithm 1 illustrates our localized algorithm for a node u to build the 3D k-YG. We use identical cones with a top angle θ to partition the transmission ball at node u, and where to define the cones depends on the locations of its neighbors. Here θ is an adjustable parameter that could be any angle smaller than $\pi/3$. Notice that these cones are identical with the same size/shape and can intersect with each other. The algorithm first orders all links uv_i in term of link length. Then it processes link uv_i from the shortest link and follows an ascending order. When it processes uv_i, it defines cone C_{uv_i}, adds the link uv_i as well as other $k-1$ shortest links in C_{uv_i} and marks all other links in C_{uv_i} as *processed*. Figure 2(a) shows an example of the algorithm when $k = 3$.

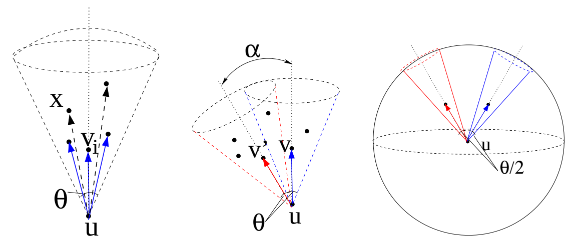


Fig. 2. Illustrations of Fault Tolerant 3D Yao Graph: (a) k shortest links kept in one cone; (b) and (c) bounded node degree by a volume argument.

Algorithm 1 Construct Fault Tolerant 3D Yao Structure YG^k_{3D} for Node u.

1: u collects the positions of its neighbors $N_1(u)$ in UBG.
2: Sort all neighbors $v_i \in N_1(u)$ by its length such that $\|uv_i\| \leq \|uv_{i+1}\|$, where $i = 1$ to d. Here d is the number of neighbors.
3: Set PROCESSED$(v_i) = 0$ for all neighbor $v_i \in N_1(u)$.
4: **for** $i = 1$ to d **do**
5:   **if** PROCESSED$(v_i) = 0$ **then**
6:     In the cone C_{uv_i}, add k shortest edges in YG^k_{3D}. As an example shown in Figure 2(a), with $k = 3$, uv_i and the other two short edges are added in the cone C_{uv_i}.
7:     Set PROCESSED$(x) = 1$ for all neighbor $x \in C_{uv_i}$.

The message complexity of the above algorithm is $O(n)$, since each node only sends one message in Line 1 including its location information. The time complexity of the algorithm is $O(d \log d)$, since sorting in Line 2 can be done in $O(d \log d)$ and the "for" loop (Lines 4-7) takes $O(d)$. Significantly, this is more efficient than the method in [7], whose time complexity is $O(d^3 \log d)$.

Now we prove some properties of YG^k_{3D} in the following three theorems.

Theorem 3: The structure YG^k_{3D} is k-connected if the original UBG G is k-connected, i.e., YG^k_{3D} can sustain $k-1$ node faults.

Proof: For simplicity, assume that all $k-1$ fault nodes v_1, v_2, ..., v_{k-1} are neighbors of a node u. We show that the remaining graph of YG^k_{3D} after removing the $k-1$ nodes is still connected.

Notice that G is k-connected, thus, the degree of each node is at least k. Additionally, with the $k-1$ fault nodes removed, there is still a path in G to connect any pair of remaining nodes. Assume that the path uses node u and have a link uw, we will prove by induction that there is a path in the remaining graph to connect u and w.

If uw has the smallest distance among all pairs of nodes, according to Algorithm 1 uw must be in YG^k_{3D}. Assume the statement is true for node pair whose distance is the rth shortest. Consider uw with the $(r+1)$th shortest length.

If w is one of the k closest nodes to u in some cone, the link uw remains in the remaining graph. Otherwise, for the cone in which node w resides, there must be other k nodes which are closer to u than w and they are connected with u in YG^k_{3D}. Since we only have $k-1$ failure nodes, at least one of the links of YG^k_{3D} in that cone will survive, say link ux. As $\angle xuw < \theta < \frac{\pi}{3}$, xw in triangle xuw is not the longest edge. Thus, $\|xw\| < \|uw\|$, and nodes x and w are connected. Then link uw can be replaced by link ux and a path from x to w by induction. This finishes the proof.

Note that for the case where the nodes removed are not all neighbors of the same node, the induction proof also holds. Induction is based on all pair of nodes. ■

Theorem 4: The node out-degree of YG^k_{3D} is bounded by $\frac{2k}{1-cos(\frac{\theta}{4})}$.

Proof: After node u processes C_{uv}, it marks all links inside C_{uv} as processed and those links will never be processed, again. And each processed cone adds at most k outgoing links in the final structure. Therefore, we only need to prove the number of processed cones is bounded by a constant c, then the node out-degree will be bounded by kc.

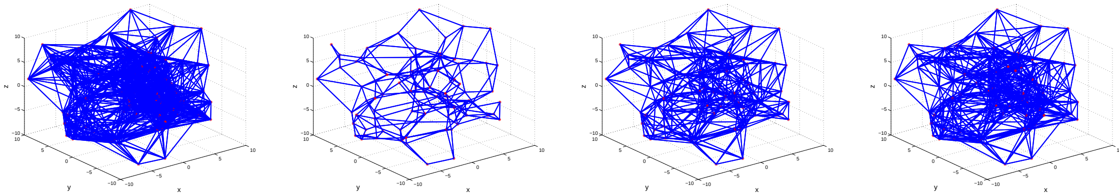
It's easy to prove that for any two processed cones, the angle α between their axes satisfies $\alpha \geq \frac{\theta}{2}$. See Figure 2(b). Assume there exists any two processed cones C_{uv} and $C_{uv'}$, the angle between their axes $\angle vuv' = \alpha < \frac{\theta}{2}$. Then, v' is inside C_{uv} and v is inside $C_{uv'}$. One of v and v' will be processed first. Let us assume it is v. Then, after adding the k shortest links in Algorithm 1, u will mark all nodes in C_{uv} as processed, including v'. Therefore, v' will never be processed, which is a contradiction.

Then we show that the number of processed cones is bounded by a constant. Since $\alpha \geq \frac{\theta}{2}$, the cones with uv and uv' as axes and with $\frac{\theta}{2}$ as top angle can not intersect with each other. Therefore, the total number of processed cones is bounded by how many such $\frac{\theta}{2}$ cones can be put into a unit ball so that they do not intersect with each other. See Figure 2(c). By using a volume argument, this number is bounded by $\frac{4\pi/3}{2\pi(1-cos(\theta/4))/3} = \frac{2}{1-cos(\frac{\theta}{4})}$.

Therefore, the out-degree of YG^k_{3D} is bounded by $\frac{2k}{1-cos(\frac{\theta}{4})}$. ■

Notice that YG^k_{3D} can only bound the out-degree at each node, however, we can apply Algorithm 1 again for all incoming links in YG^k_{3D} to bound node in-degree, same as the sparse Yao graph [3] in 2D case.

Finally, we prove a theorem which guarantees that for any pair of nodes there is at least an energy efficient route in YG^k_{3D}. Here an energy efficient route means the path uses



(a) UBG (b) 3D k-RNG (b) 3D k-GG (c) 3D k-YG

Fig. 3. 3D localized topologies which are constructed from the same UBG. Here, $n = 100$, $k = 2$ and $P^{max} = R^2 = 81$.

at most constant time of energy than the original least energy cost path in UBG. Our proof uses the following lemma from [2], [3].

Lemma 5: [2], [3] The power stretch factor of the Yao-based graph is at most $\frac{1}{1-(2\sin\frac{\delta}{2})^{\beta}}$, if for every link uv that is not in the final graph, there exists a shorter link uw in the graph and $\angle vuw < \delta$, where δ is a constant smaller than $\pi/3$.

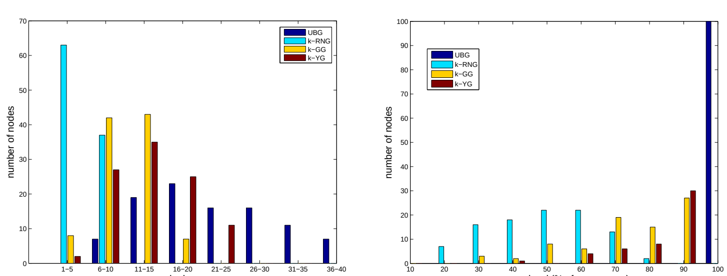
Theorem 6: YG^k_{3D} is a power spanner with spanning ratios bounded by a constant, $\frac{1}{1-(2sin(\frac{\theta}{2}))^{\beta}}$.

Proof: For a link $ux \notin YG^k_{3D}$, there must exist k shorter links in the resulting graph. See Figure 2(a), when $k = 3$, three edges are shorter than the removed link ux. The angle between the removed link ux and any of the k-shorter links is less than θ. Since, $\theta < \pi/3$, by Lemma 5, the power spanning ratio of YG^k_{3D} is $\frac{1}{1-(2sin(\frac{\theta}{2}))^{\beta}}$. ∎

IV. Simulations

In order to evaluate the performance of our 3D topology control protocols, we conduct simulations by generating random networks in 3D. In our experiments, we randomly generate a set V of n wireless nodes and the UBG, then test the connectivity of UBG. If it is k-connected, we construct different localized topologies proposed in this paper on it, and measure the node degree as well as power efficiency of these topologies.

Figure 3 shows a set of topologies generated for a UBG with 100 wireless nodes. In the experimental results presented here, we generate n random wireless nodes in a $20 \times 20 \times 20$ cube; the parameter $\theta = \pi/4$ for 3D k-YG; the fault tolerance requirement $k = 2$; the maximum transmission range R is set to 9 and the power constant $\beta = 2$, thus the maximum transmission power $P^{max} = R^2 = 81$.



(a) node degree (b) transmission power

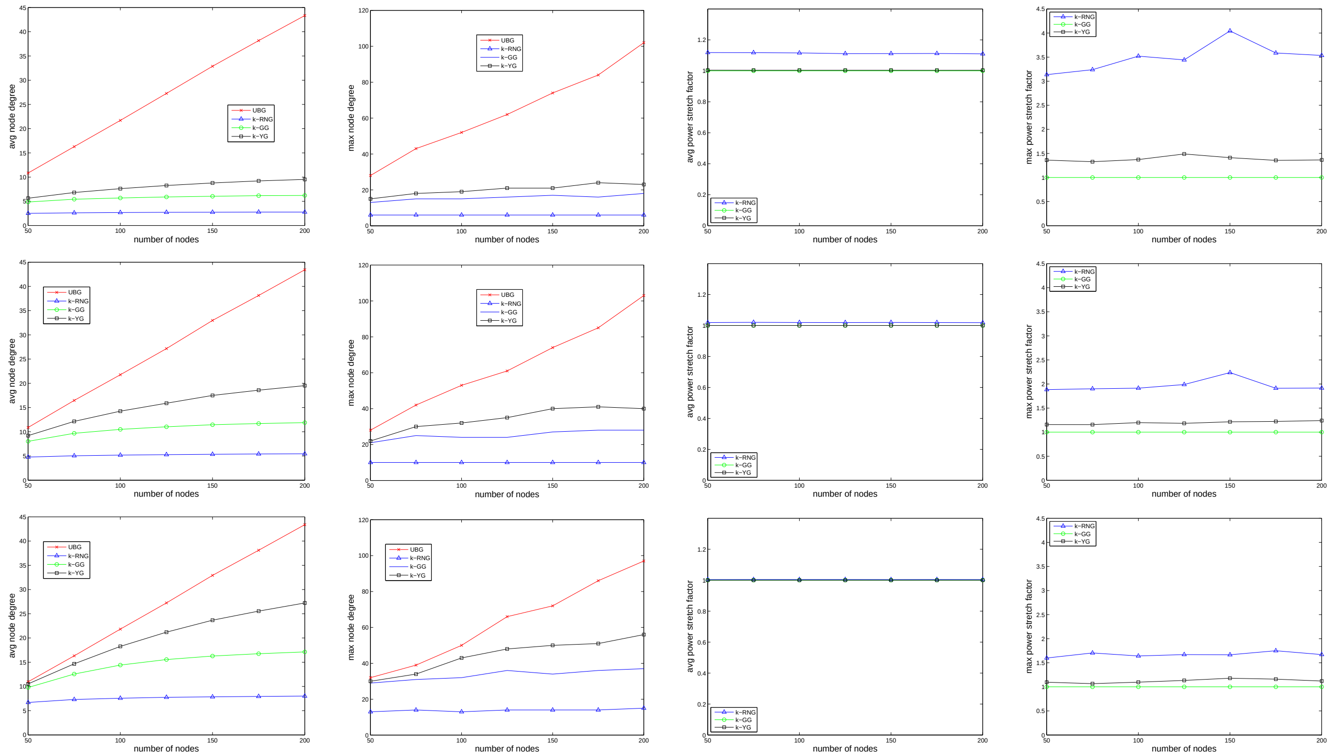
Fig. 4. (a) Node degrees of the UBG and other topologies; (b) Final assigned transmission power levels of nodes. Here, $n = 100$, $k = 2$ and $P^{max} = 81$.

For the same instance, we plotted the node degrees of different topologies in Figure 4(a). It is clear that UBG has more nodes with high node degree. While k-RNG, k-GG and k-YG can drastically reduce the node degrees. Figure 4(b) shows the assigned minimal power levels for all the nodes. Here we assume that each node can shrink its power level to support the longest link in the generated topology. Clearly, no node needs to transmit at its maximum power level anymore in these sparse topologies. k-RNG uses the smallest power level, since it is the sparest graph. Both k-GG and k-YG can also save a lot of energy.

We then vary the number of nodes n in the network from 50 to 200, where 100 vertex sets are generated for each case. The average and the maximum are computed over all these 100 vertex sets. We consider various fault tolerance requirements: $k = 1, 2, 3$. All experimental results are plotted in Figure 5. Three rows in Figure 5 are for $k = 1, 2, 3$ respectively.

The average node degree of wireless networks should not be too large to avoid interference, collision, and overhead. It should not be too small either: a low node degree usually implies that the network has a low fault tolerance and tends to increase the overall network power consumption as longer paths may have to be taken. The first two columns in Figure 5 show that, all localized topologies have lower degrees compared with UBG and keep small degrees when the UBG becomes denser and denser (i.e., the number of nodes in the network increases). k-RNG is sparser than k-GG, and k-GG is sparser than k-YG. These can also be verified by Figure 3. Notice that k-RNG and k-GG do not have large degrees in this experiment, the reason is that the nodes are distributed randomly in the area. In real life, the network may not be distributed randomly, it is possible that k-RNG and k-GG have large degrees. Such examples and simulation results in 2D networks can be found in [3]. We also observe that higher fault tolerance requirement k will cause larger node degree in all topologies.

Besides connectivity, the most important design metric of wireless networks is perhaps the power efficiency, because it directly affects both the nodes and the network lifetime. The last two columns in Figure 5 show that, all proposed structures have small power stretch factors even when the network is very dense. Notice that k-RNG has higher maximum power stretch factor than k-GG and k-YG, however, it is still smaller than 5. The reason again is that the nodes are distributed randomly in the area and the number of nodes is not too large. As we



(a) avg degree (b) max degree (c) avg stretch factor (d) max stretch factor

Fig. 5. Average/maximum node degree and average/maximum power stretch factor for $k = 1, 2, 3$ (from upper row to bottom row, respectively).

expected, k-GG has a power stretch factor of one and the power stretch factor of k-YG is smaller than the theoretical bound $\frac{1}{1-(2sin(\frac{\theta}{2}))^{\beta}} = \frac{1}{1-(2sin(\frac{\pi}{8}))^{2}} \approx 2.4$.

We also conduct simulations when the maximum transmission power P^{max} from 40 to 100. Due to space limit, we can not include those results in this paper. However, the basic conclusions from the simulations are coherent with previous simulations.

V. Conclusion

Topology control for wireless networks has been extensively studied recently and different geometric topologies were proposed to achieve the sparseness and fault tolerance. However, most of them are only applied to 2D networks. In this paper, we introduce three 3D geometric topologies (3D k-RNG, 3D k-GG, and 3D k-YG) which can be constructed locally and efficiently. We formally prove all these topologies can preserve the k-connectivity of the network. In addition, 3D k-GG has power stretch factor one, while 3D k-YG not only has constant power stretch factor but also has constant bounded node out-degree. Our simulation results confirm the good performance of these 3D topologies. This paper is the first step towards research for 3D fault tolerant topology control, and there are still a number of challenging questions left. E.g., if position information is not accurate, how to achieve fault tolerance and power efficiency of these geometric topologies.

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