Contact States: Representation and Recognizability in the Presence of Uncertainties*

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Abstract

Recognition of contact states is often necessary or preferred in planning and executing contact-based assembly motions in the presence of uncertainties. However, due to inevitable uncertainties, not all contact states can be identified at all contact configurations. A crucial question is what kinds of contact states at what kinds of configurations are impossible to be distinguished or identified. In this paper, we address the question by analyzing the recognizability of contact states between convex polyhedral objects with both position/orientation and force/moment sensing in the presence of sensing uncertainties, i.e., by characterizing configurations where contact states are intrinsically indistinguishable with those sensing means. To reflect contact recognizability in contact representation, we propose a fuzzy definition of contact states. We further introduce a practical approach towards real-time automatic contact state identification based on the recognizability analysis.

1 Introduction

Contact motions are necessary for manipulation tasks and tight-clearance assembly tasks. In order to plan and execute contact motions, knowledge of contact states are required, and in particular, reasoning about sensory feedback to detect and recognize contact states are required to deal with uncertainties and inaccuracies.

There are two major issues about contact states: *representation* and *recognition*. While there exists considerable research addressing either issues[10, 5, 2, 1, 6, 14, 7, 8, 11, 15, 13], little attention is paid on how the two issues are related. However, as recognition is inevitably subject to uncertainties, one has to ask, from a practical perspective, how a representation scheme (1) facilitates the detection of different contact states by minimizing cases where two states are distinct by definition but indistinguishable by sensing in practice and (2) acknowledges the reality that there are always cases where contact states are intrinsically indistinguishable regardless of sensing means. As more than one contact *configuration*[1] usually share the same *contact characteristics*, a "contact state" is normally viewed as of *higher* level than that of the contact configurations. Hence, another question is how contact configurations are related to contact states in representation as well as in recognition, which is subject to uncertainties. In essence, the question is about how to characterize contact configurations which lead to indistinguishable or unrecognizable contact states.

As a first step, this paper addresses these issues and questions with a focus on contact states between two convex polyhedra. In Section 2, we discuss how to relate contact representation to recognition in the presence of uncertainties through reviewing different representation schemes in order to gain insight on proper contact state primitives. In Section 3, we discuss contact configurations vs. contact states, analyze configurations where contact states are unrecognizable with respect to position/orientation and/or force/moment sensing in recognition, and propose a fuzzy representation of contact states as the result. In Section 4, we introduce a practical approach to automatic contact state recognition based on position/orientation and force/moment sensing. We summarize the paper and discuss future work in Section 5.

2 Existing Schemes of Representation

Among existing representation schemes for contact states, topological representation is most common, where a contact state is described in terms of the topological surface elements which are in contact. The most widely used topological representation was the point-contact notion for polyhedra introduced by Lozano-Pérez [10] and Donald [5] in their attempt to construct

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[1] A contact configuration is meant the relative configuration or location between the objects in contact.

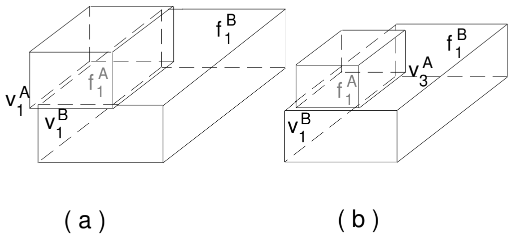

Figure 1: Indistinguishable states due to uncertainties

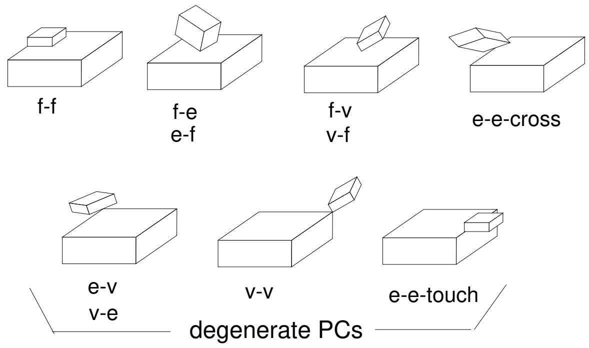

Figure 2: Principal contacts (PCs)

C-obstacles. Here point contacts, for example, vertex-edge contacts in 2D polygons and vertex-face, edge-edge (which intersect at a point) contacts in 3D polyhedra, were used as contact primitives, and a contact state was described as a set of such primitives. Another topological representation was introduced by Desai *et al.* [4] where they used the single contacts between a pair of topological surface elements of two polyhedra as primitives, called *elemental contacts* (ECs), and described a contact state (between the two polyhedra) in terms of the set of ECs formed. However, both representations result in states which are different by definition but indistinguishable in recognition due to uncertainties. Fig. 1 shows an example.

To minimize such problem, Xiao [16, 17] introduced the concept of *principal contacts* (PCs) as primitives to describe contact states. Denoting the boundary elements of a face as the edges and vertices bounding it, and the boundary elements of an edge as the vertices bounding the edge, a PC can be defined as the following: *a PC denotes the contact between a pair of surface elements (i.e., faces, edges, or vertices) which are not the boundary elements of other contacting surface elements (more than one pair are in contact).* Clearly, a PC is an EC (introduced earlier), but not every EC is a PC. With the notion of PCs, the contact states shown in Fig. 1 become a single contact state, described by the single PC f_1^A-f_1^B. For polyhedral objects, there are six types of non-degenerate PCs and four types of degenerate ones as shown in Fig. 2. The non-degenerate ones are sufficient to characterize a realistic contact situation [17].

PCs are of higher-level primitives than both the point-contact primitives and ECs with several advantages. One convenience is that every contact between two convex polyhedra forms a single PC; if a PC is viewed as a set of contact configurations, then all possible PCs form a partition of the set of all possible contact configurations between two convex objects. On the other hand, if more than one PC is formed in a contact, one of the contacting object has to be non-convex. The presence of more than one PC indicates concavity in a contact situation. More importantly, using PCs as contact state primitives simply eliminates the otherwise indistinguishable contact states due to *planar position/orientation uncertainty* along a contact plane[2] in that they become the same contact state.

However, even in terms of PCs, not all contact states are distinguishable in all contact configurations as long as there is uncertainty, which is inevitable. This prompts us to focus on the characterization of contact configurations where contact states are indistinguishable and, as the result, propose a fuzzy approach to represent contact states, which, from the perspective of recognition, seems much more adequate than conventional crisp representations. We report our work in the next section.

3 Principal Contacts (PCs) as Fuzzy Sets

Recall that in the crisp notion of contact states based on PCs, a contact state between two convex polyhedral objects A and B is represented by a single PC of one of the six non-degenerate types shown in Fig. 2. Furthermore, each PC consists of a crisp set of contact configurations. However, as there exist certain contact configurations where to recognize the PC with absolute certainty is impossible, it becomes necessary to blur the boundaries of a contact state. In this section, we analyze the configurations where PCs cannot be uniquely identified with respect to position/orientation and force/moment sensing, and based on the analysis, characterize the memebership functions of single-PC contact states as fuzzy sets.

[2] Such a contact plane is determined by a contacting face or the two contacting edges in an e-e-cross PC.

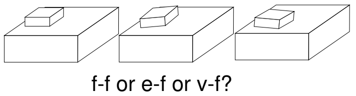

Figure 3: Examples of indistinguishable configurations which lead to indistinguishable PCs due to configuration uncertainty

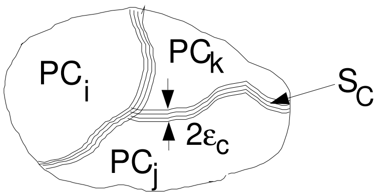

Figure 4: Region S_C of configurations where PCs are indistinguishable by position/orientation sensing

3.1 Configurations of Indistinguishable PCs

Without losing generality, let us assume object B is fixed and describe contact configurations between A and B in the configuration space of A. With configuration uncertainty, i.e., position/orientation uncertainties, there are obviously indistinguishable contact configurations, which, if in a neighborhood of boundary contact configurations between two PCs or among more PCs, are the contact configurations where PCs become indistinguishable *from position/orientation sensing only.* Fig. 3 shows some examples of indistinguishable configurations which lead to indistinguishable PCs. If the configuration uncertainty is bounded by ϵ_C, which defines the radius of the maximum 6-D neighborhood of any configuration $\mathbf{C}$ such that the contact configurations in the neighborhood are indistinguishable from $\mathbf{C}$ and each other, then the set S_C of configurations of indistinguishable PCs form the "belt" regions of width $2\epsilon C$ covering all between-PC boundaries in the configuration space of A, as illustrated in Fig. 4.

If force/moment sensing is used, the PC of certain configuration in S_C, where it is unrecognizable from sensed position/orientation data, can now become recognizable from the sensed force/moment data. Assume that a force/moment sensor is attached to the gripper, the contact force has a uniform distribution, and there is no gravity. Fig. 5a shows a 2-D example where the two PCs can be distinguished by the different directions of the sensed (contact) forces as long as such directional difference is greater than that dictated by the force sensing uncertainty. In Fig. 5b, based on the force/moment analysis, there is no moment exerted to the gripper in

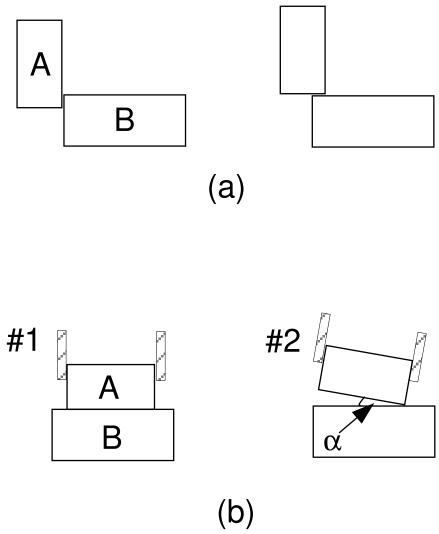

Figure 5: Configurations where PCs are distinguishable by force/moment sensing

the configuration #1, but there is a considerable moment applied to the gripper in the configuration #2, as the result of the non-zero contact force[3] and the sudden increase of the distance (to the axis of rotation). This is true *regardless how small the angle α is so long as it is not zero.* Such a moment discontinuity can be used to distinguish the two PCs *as long as the change in the moment is greater than the moment uncertainty.*

On the other hand, as force/moment uncertainties are inevitable, there are contact configurations where the PCs are indistinguishable by force/moment sensing. Let ϵ_F be the bound on force uncertainty such that two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are indistinguishable if $||\mathbf{F}_1 - \mathbf{F}_2|| \leq \epsilon_F$. Let ϵ_M be the bound on moment uncertainty[4] such that two moments $\mathbf{M}_1$ and $\mathbf{M}_2$ are indistinguishable if $||\mathbf{M}_1 - \mathbf{M}_2|| \leq \epsilon_M$. It is easy to show that from any two contact configurations of different PCs with sensed forces and moments $\mathbf{F}_1$, $\mathbf{M}_1$ and $\mathbf{F}_2$, $\mathbf{M}_2$ respectively, if

$$\angle(\mathbf{F}_1, \mathbf{F}_2) \leq \arctan \frac{\epsilon_F}{\min(||\mathbf{F}_1||, ||\mathbf{F}_2||)} \tag{1}$$

then the PCs cannot be distinguished from force sensing, and if

$$||\mathbf{M}_1 - \mathbf{M}_2|| \leq \epsilon_M \tag{2}$$

then the PCs cannot be distinguished from moment sensing. If both inequalities hold, then the corresponding two configurations belong to the configuration set $S_{F/M}$ of indistinguishable PCs by force/moment sensing (Fig. 6a and b show example pairs of configurations where their PCs are indistinguishable).

[3] Although in theory, a contact force can be infinitesimally small while still keeping the contact, in practice, the contact force is always finite as the result of compliant motion or force guard.

[4] It can be related to the force uncertainty.

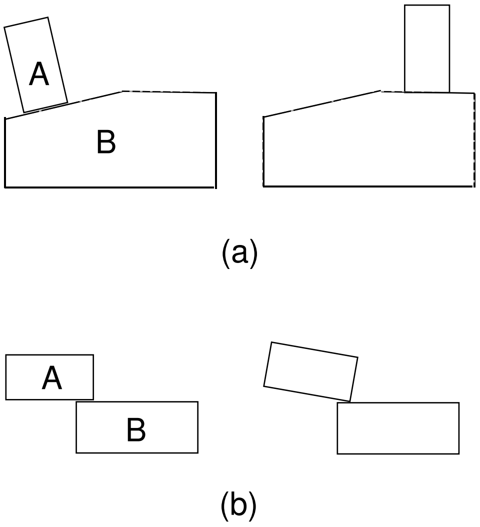

Figure 6: Configurations where PCs are indistinguishable by force/moment sensing

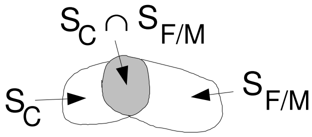

Figure 7: S_C and $S_{F/M}$

One may ask how $S_{F/M}$ is related to S_C. While there are certain configurations in S_C that are not in $S_{F/M}$ (e.g., Fig. 5), there are also configurations in $S_{F/M}$ but not in S_C (e.g., Fig. 6a, where the top two faces of B are almost parallel), and in addition, there are configurations that are in both S_C and $S_{F/M}$ (e.g., Fig. 6b). Thus, $S_C \bigcap S_{F/M} \neq \emptyset$ and is a subest *smaller* than both S_C and $S_{F/M}$ (Fig. 7), which shows why *both* position/orientation and force/moment sensing are preferred in contact recognition but also shows that there are inevitably configurations $S_C \bigcap S_{F/M}$ where PCs are indistinguishable no matter how realistically good the sensors are.

3.2 PC Memberships of Contact Configurations

The inevitable existence of configurations of indistinguishable PCs make it necessary to define each PC as a fuzzy set of configurations rather than a crisp set. The membership function for each PC$_i$ is a function of many independent variables – including configuration variables and contact force variables and can be generally viewed as having a flat "top" with value 1. This "top" corresponds to contact configurations where PC$_i$ can be uniquely identified so that these configurations have zero membership values of PCs other than PC$_i$. For those configurations in $S_C \bigcap S_{F/M}$ which are near or on the boundary between PC$_i$ and other PCs, their PC$_i$ membership values are less than 1. In general, if $n(>1)$ PCs are indistinguishable (or unrecognizable) at $\mathbf{C} \in S_C \bigcap S_{F/M}$, then the PC membership values of these PCs at $\mathbf{C}$ are all greater than zero and less than 1 respectively, and the sum of the n values equals to 1. The specific boundary of domain and the characteristics of a PC membership function depends on the specific geometry of the parts and the PC, the way of sensing and characteristics of uncertainties (in terms of uncertainty bounds and/or distributions), and the way the movable part is grasped (which affects the force/moment sensed), among others. Note that the membership function of the fuzzy intersection of any two PCs (i.e., their fuzzy sets) is simply zero, characterizing the fact that any contact configuration $\mathbf{C}$ cannot form two PCs at the same time. This result can be obtained by applying either *drastic intersection* or *bounded difference* [9].

As a PC membership function is highly multi-variable and depends on so many task-specific factors, it is difficult and perhaps impossible to describe it in an explicit, analytical form. On the other hand, a procedural or algorithmic description seems highly suitable to capture both the general characteristics and the case-dependent characteristics provided as input. In the following section, we present a practical approach to determine or identify the PC memberships of a contact configuration automatically.

4 A Practical Approach to PC Recognition

Our approach consists of two phases. The **phase 1** is to identify PC memberships of a contact configuration $\mathbf{C}$ using the sensed position/orientation, taking into account position/orientation uncertainty (i.e. ϵ_C). If $\mathbf{C}$ is *outside* S_C, **phase 1** identifies a unique PC (i.e., its membership value is "1"). Otherwise (i.e., $\mathbf{C} \in S_C$), it identifies the set of PCs for which $\mathbf{C}$ may have nonzero membership values (i.e., they are indistinguishable at $\mathbf{C}$). In such cases, **phase 2** is further applied to $\mathbf{C}$, which uses the sensed force/moment at $\mathbf{C}$ to either uniquely identify a PC if $\mathbf{C}$ is outside $S_{F/M}$ or estimate the membership values of the indistinguishable PCs at $\mathbf{C}$ if $\mathbf{C} \in S_C \bigcap S_{F/M}$. Note that in addition to the sensing inputs, both **Phase 1** and **Phase 2** also input the geometric information of the parts in contact.

For **phase 1**, there exists a general algorithm developed by Xiao and Zhang [18] which identify possible PCs from the intersection between the grown regions of the two objects by their position/orientation uncertainties, where the objects can be arbitrary polyhedra. In addition, we have recently developed a more

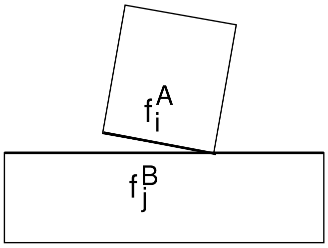

Figure 8: The surface elements possibly involved in contact are f_i^A, f_j^B and their boundary edges and vertices.

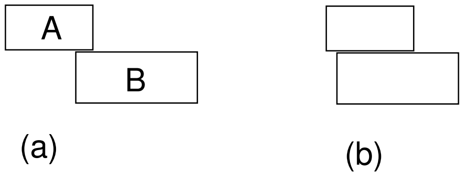

Figure 9: An f-f PC in (a) an e-e-touch look-alike set and (b) a f-f look alike set

efficient algorithm for contacting convex polyhedra that utilizes the distance algorithm by Ong [12] to obtain a contact point, which is then used to locate the possibly contacting surface elements so that the scope of search for possible PCs is much *narrowed* (Fig. 8 shows an example). Ong's algorithm is able to compute both separation distance (for two objects apart) and penetration distance (for two objects intersecting) and is thus a convenient tool for computation in the presence of position/orientation uncertainties. Our new algorithm has a time complexity linear to the total number of vertices in the two convex polyhedra, comparable to that of Ong's distance algorithm it employs.

The output of **phase 1** based on the sensed contact configuration $\mathbf{C}$ is either (1) a unique PC of e-e-cross, v-f, or e-f type; or (2) a set of PCs characterized as the "look-alike" PCs of one PC_0, where the PC_0 is an f-f, an e-e-touch, a v-e (or e-v), or a v-v PC. Such a "look-alike" set provides us both the topological information of possible PCs for configuration $\mathbf{C}$ *and* the geometrical information that those PCs look like PC_0 at the configuration $\mathbf{C}$. In other words, $\mathbf{C}$ is closest to the center of the region of configurations for PC_0 than to those for other PCs. For example, if PC_0 is an e-e-touch PC, it indicates that an f-f PC in its look-alike set has a close-to-line contact region (Fig. 9a) rather than a vast-area one (Fig. 9b); if PC_0 is an f-f PC, then an e-f PC in its look-alike set has a close-to-zero angle between a face of the edge e and the face f (Fig. 5b). This kind of information is extremely valuable for **phase 2** of recognition based on the sensed force/moment data.

In **phase 2**, at those configurations where **phase 1** is unable to yield a unique PC, the sensed force/moment data are used to further differentiate the PCs identified by **phase 1**. First, the direction of sensed forces can be used to differentiate PCs associated with different contact planes, i.e., the PCs obtained from **phase 1** can be easily divided into different subsets such that each subset of PCs has the same contact plane. Then, if there is only one subset whose contact plane normal is roughly along the direction of the sensed force with uncertainties taken into account, that subset is identified as holding the possible PCs, and the other subsets can be discarded. Next, to differentiate PCs within such a subset, the sensed moment can be added in the reasoning. In Section 3.1., we described how an f-f and an f-e PC in an "f-f look-alike" set (e.g., Fig. 5b) can be distinguished by $d = ||\mathbf{M}||/||\mathbf{F}||$. Generally, if $\mathbf{F}$ is approximately normal to the contact plane, d can be viewed approximately as a distance from the projection of the rotation axis of $\mathbf{M}$ on the contact plane to a vertex, an edge, or an internal face point in contact, depending on whether the PC is of f-v, f-e, or f-f type (with the same contact plane). Thus, d can have very distinct values for different types of PCs sharing the same contact plane, as determined by the contact geometry, even with sensing uncertainties. Such distinct values can be used to identify PCs.

If the output of **phase 2** is a unique PC, it means that the contact configuration $\mathbf{C}$ is outside $S_C \bigcap S_{F/M}$ and has the membership value of that PC being 1; otherwise, if the output is a set of $n(> 1)$ still indistinguishable PCs $G = \{\mathrm{PC}_1, ..., \mathrm{PC}_n\}$, we need to further determine their membership function values at $\mathbf{C}$. Let $N_G \subset S_C \bigcap S_{F/M}$ denotes the set of configurations in which PC_1, ..., PC_n are indistinguishable. The membership function of each PC_i at $\mathbf{C}$, $i = 1, ..., n$, can be defined by the conditional probability $P(\mathrm{PC}_i|N_G)$ of occurance of PC_i for configurations in N_G. $P(\mathrm{PC}_i|N_G)$ is related to specific system/task characteristics and uncertainty characteristics and can be estimated empirically. If all PC_1, ..., PC_n are equally likely to occur for N_G, then for any $\mathbf{C} \in N_G$, its membership function value for PC_i, $i = 1, ..., n$, is $1/n$.

5 Discussion and Future Work

We have proposed a fuzzy representation of contact states in which PCs (as contact primitives) are represented as fuzzy sets of contact configurations to take into account the fact that there always exist contact configurations where it is impossible to identify the corresponding contact state uniquely due to limited sensing accuracy.

We have outlined a practical approach to identify automatically the PC memberships of an unknown contact configuration between two convex polyhedra through combining known information of the objects and sensed information about both the contact configuration and force/moment, taking into account sensing uncertainties. Our approach has a more comprehensive nature than existing ones. On one hand, it uses all types of explicit information which characterizes a contact, including geometrical and topological descriptions of objects and sensed locational and physical information of the contact, as *input* to a process of explicit reasoning independent of specific objects/tasks, and hence it is systematic and general. On the other hand, it allows the incorporation of processes utilizing implicit knowledge gained through learning to handle the effects of hard-to-model factors, such as uncertainties. We are currently implementing the **phase 2** part of identification with a combination of symbolic reasoning and neural network learning in hope to best utilize both the general, explicit theory and the specific, implicit knowledge.

For cases where there involve non-convex objects, a contact state usually consists of more than one PC, i.e., two or more PCs co-exist, which impose more constraints on contact configurations than a single PC does and thus can reduce the ambiguity in recognition. In many cases, a contact state corresponds to a unique configuration or only a few finite ones [19]. This suggests that a PC's recognizability can be increased when it co-exists with other PCs. One crucial future task is to understand the fuzziness of a multi-PC contact state and that of each PC in it for the sake of general contact recognition.

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