TEST II: Sample Problems

**Problem 1**

Assume that two-dimensional space MxN contains 8 objects listed in Table 1. Apply k-means (k=2) algorithm to cluster Y.

|  |  |  |
| --- | --- | --- |
| Y | M | N |
| y1 | 1 | 2 |
| y2 | 2 | 4 |
| y3 | 6 | 2 |
| y4 | 1 | 8 |
| y5 | 6 | 6 |
| y6 | 1 | 4  |
| y7 | 4 | 2 |
| y8 | 5 | 3 |

Table 1

**Solution**

Let’s take K=2 and assume that Y3, Y6 are the seeds.

We need to build clusters C(Y3), C(Y6).

Take Y1, d(Y1,Y3)=5, d(Y1,Y6)=2.

So, C(Y3)={Y3}, C(Y6)={Y6,Y1}

Take Y2, d(Y2,Y3)=6, d(Y2,Y6)=1.

So, C(Y3)={Y3}, C(Y6)={Y6,Y1,Y2}

Take Y4, d(Y4,Y3)=11, d(Y4,Y6)=4.

So, C(Y3)={Y3}, C(Y6)={Y6,Y1,Y2,Y4}

Take Y5, d(Y5,Y3)=4, d(Y5,Y6)=7.

So, C(Y3)={Y3,Y5}, C(Y6)={Y6,Y1,Y2,Y4}

Take Y7, d(Y7,Y3)=2, d(Y7,Y6)=5.

So, C(Y3)={Y3,Y5,Y7}, C(Y6)={Y6,Y1,Y2,Y4}

Take Y8, d(Y8,Y3)=2, d(Y8,Y6)=5.

So, C(Y3)={Y3,Y5,Y7,Y8}, C(Y6)={Y6,Y1,Y2,Y4}

Now we have to find center for C(Y3) and C(Y6)

C(Y3)

|  |  |  |
| --- | --- | --- |
| Y | M | N |
| y3 | 6 | 2 |
| y5 | 6 | 6 |
| y7 | 4 | 2 |
| y8 | 5 | 3 |

 21/4 13/4 -> Seed1 = (5.25, 3.25)

C(Y6)

|  |  |  |
| --- | --- | --- |
| Y | M | N |
| y1 | 1 | 2 |
| y2 | 2 | 4 |
| y4 | 1 | 8 |
| y6 | 1 | 4  |

 5/4 18/4 -> Seed2= (1.25, 4.5)

Now, we have to build clusters around these two seeds.

**Problem 2**

Assume that two-dimensional space MxN contains 8 objects listed in Table 1 (see Problem 1). Apply agglomerative strategy (bottom-up) to cluster objects in Y. Use Manhattan distance

/ d(yi, yj) = |Mi – Mj | + |Ni – Nj | / for objects yi, yjin Yand the distance d(R,Q)= 1/2**⋅**d(A,Q) + 1/2**⋅**d(B,Q) - 1/2**⋅**d(A,B) between clusters R and Q, if R is formed by merging clusters A and B in Y.

Distance matrix for objects in Y

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 | Y7 | Y8 |
| Y1 | x |  |  |  |  |  |  |  |
| Y2 | 3 | x |  |  |  |  |  |  |
| Y3 | 5 | 6 | x |  |  |  |  |  |
| Y4 | 6 | 5 | 11 | x |  |  |  |  |
| Y5 | 11 | 6 | 4 | 7 | x |  |  |  |
| Y6 | 2 | 1 | 7 | 4 | 7 | x |  |  |
| Y7 | 3 | 4 | 2 | 9 | 6 | 5 | x |  |
| Y8 | 5 | 4 | 2 | 9 | 4 | 5 | 2 | x |

**Solution**

d(yi, yj) = |Mi – Mj | + |Ni – Nj | for objects yi, yjin Y

d(R,Q)= 1/2**⋅**d(A,Q) + 1/2**⋅**d(B,Q) - 1/2**⋅**d(A,B) where cluster R is formed by merging clusters A and B.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 | Y7 | Y8 |
| Y1 | x |  |  |  |  |  |  |  |
| Y2 | 3 | x |  |  |  |  |  |  |
| Y3 | 5 | 6 | x |  |  |  |  |  |
| Y4 | 6 | 5 | 11 | x |  |  |  |  |
| Y5 | 11 | 6 | 4 | 7 | x |  |  |  |
| Y6 | 2 | 1 | 7 | 4 | 7 | x |  |  |
| Y7 | 3 | 4 | 2 | 9 | 6 | 5 | x |  |
| Y8 | 5 | 4 | 2 | 9 | 4 | 5 | 2 | x |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | {Y1} | {Y2, Y6} | {Y3} | {Y4} | {Y5} | {Y7} | {Y8} |
| {Y1} | x |  |  |  |  |  |  |
| {Y2, Y6} | 3 | x |  |  |  |  |  |
| {Y3} | 5 | ? | x |  |  |  |  |
| {Y4} | 6 | ? | 11 | x |  |  |  |
| {Y5} | 11 | ? | 4 | 7 | x |  |  |
| {Y7} | 3 | ? | 2 | 9 | 6 | x |  |
| {Y8} | 5 | ? | 2 | 9 | 4 | 2 | x |

d(R,Q)= 1/2**⋅**d(A,Q) + 1/2**⋅**d(B,Q) - 1/2**⋅**d(A,B), where R = A∪B

d({Y2,Y6},{Y3})= 1/2⋅d(Y2,Y3)+1/2⋅d(Y6,Y3)-1/2⋅d({Y2},{Y6})=

 (1/2)⋅6 + (1/2)⋅7 – (1/2)⋅1 = 3+3 = 6

d({Y2,Y6},{Y4})= 1/2⋅d(Y2,Y4)+1/2⋅d(Y6,Y4)-1/2⋅d({Y2},{Y6})=

 (1/2)⋅5 + (1/2)⋅4 – (1/2)⋅1 = 2+2 = 4

d({Y2,Y6},{Y5})= 1/2⋅d(Y2,Y5)+1/2⋅d(Y6,Y5)-1/2⋅d({Y2},{Y6})=

 (1/2)⋅6 + (1/2)⋅7 – (1/2)⋅1 = 3 + 3 = 6

d({Y2,Y6},{Y7})= 1/2⋅d(Y2,Y7)+1/2⋅d(Y6,Y7)-1/2⋅d({Y2},{Y6})=

 (1/2)⋅4 + (1/2)⋅5 – (1/2)⋅1 = 2 + 2 = 4

d({Y2,Y6},{Y8})= 1/2⋅d(Y2,Y8)+1/2⋅d(Y6,Y8)-1/2⋅d({Y2},{Y6})=

 (1/2)⋅4 + (1/2)⋅5 – (1/2)⋅1 = 2 + 2 = 4

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | {Y1} | {Y2, Y6} | {Y3} | {Y4} | {Y5} | {Y7} | {Y8} |
| {Y1} | x |  |  |  |  |  |  |
| {Y2, Y6} | 3 | x |  |  |  |  |  |
| {Y3} | 5 | 6 | x |  |  |  |  |
| {Y4} | 6 | 4 | 11 | x |  |  |  |
| {Y5} | 11 | 6 | 4 | 7 | x |  |  |
| {Y7} | 3 | 4 | 2 | 9 | 6 | x |  |
| {Y8} | 5 | 4 | 2 | 9 | 4 | 2 | x |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | {Y1} | {Y2, Y6} | {Y3, Y7} | {Y4} | {Y5} | {Y8} |
| {Y1} | x |  |  |  |  |  |
| {Y2, Y6} | 3 | x |  |  |  |  |
| {Y3, Y7} | ? | ? | x |  |  |  |
| {Y4} | 6 | 4 | ? | x |  |  |
| {Y5} | 11 | 6 | ? | 7 | x |  |
| {Y8} | 5 | 4 | ? | 9 | 4 | x |

d(R,Q)= 1/2**⋅**d(A,Q) + 1/2**⋅**d(B,Q) - 1/2**⋅**d(A,B), where R = A∪B

d({Y1},{Y3,Y7}) = 1/2⋅d({Y1},{Y3}) + 1/2⋅d({Y1},{Y7}) – 1/2⋅d({Y3,Y7}) =

 = (1/2)⋅5 + (1/2)⋅3 – (1/2)⋅2 = 4 – 1 = 3

d({Y2,Y6},{Y3,Y7}) =

1/2⋅d({Y2,Y6},{Y3}) + 1/2⋅d({Y2,Y6},{Y7}) – 1/2⋅d({Y3,Y7}) =

 (1/2)⋅6 + (1/2)⋅4 – (1/2)⋅2 = 5 – 1 = 4

d({Y4},{Y3,Y7}) = 1/2⋅d({Y4},{Y3}) + 1/2⋅d({Y4},{Y7}) – 1/2⋅d({Y3,Y7}) =

 (1/2)⋅11 + (1/2)⋅9 - (1/2)⋅2 = 10 – 1 = 9

d({Y5},{Y3,Y7}) = 1/2⋅d({Y5},{Y3}) + 1/2⋅d({Y5},{Y7}) – 1/2⋅d({Y3,Y7}) =

 (1/2)⋅4 + (1/2)⋅6 - (1/2)⋅2 = 5 – 1 = 4

d({Y8},{Y3,Y7}) = 1/2⋅d({Y8},{Y3}) + 1/2⋅d({Y8},{Y7}) – 1/2⋅d({Y3,Y7}) =

 (1/2)⋅2 + (1/2)⋅2 - (1/2)⋅2 = 1

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | {Y1} | {Y2, Y6} | {Y3, Y7} | {Y4} | {Y5} | {Y8} |
| {Y1} | x |  |  |  |  |  |
| {Y2, Y6} | 3 | x |  |  |  |  |
| {Y3, Y7} | 3 | 4 | x |  |  |  |
| {Y4} | 6 | 4 | 9 | x |  |  |
| {Y5} | 11 | 6 | 4 | 7 | x |  |
| {Y8} | 5 | 4 | 1 | 9 | 4 | x |

**Problem 3.** Follow DEAR1 algorithm to extract action rules reclassifying objects from the class D0 to the class D1 hidden in table T. Attributes A, C are stable.

 A B C D

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| X1 |  2 |  2 |  1 |  0 |
| X2 |  1 |  1 |  1 |  1 |
| X3 |  2 |  1 |  2  |  1 |
| X4 |  1 |  3 |  1 |  0 |
| X5 |  1 |  3 |  2 |  1 |
| X6 |  1 |  1 |  2 |  0 |

Table T.

**Solution.**

d0\*={x1,x4,x6}, d1\*={x2,x3,x5}

a1\*={x2,x4,x5,x6}, a2\*={x1,x3}

b1\*={x2,x3,x6}, b2\*={x1}<d0\*, b3\*={x4,x5}

c1\*={x1,x2,x4}, c2\*={x3,x5,x6}

a1.b1\*= {x2,x6} ~~a1.b3\*=b3\*~~, a1.c1\*={x2,x4} a1.c2\*= {x5,x6}

a2.b1\*={x3}<d1\* ~~a2.b3\*=~~  a2.c1\*= {x1}<d0\* a2.c2\*= {x3}<d1\*

b1.c1\*={x2}<d1\* b1.c2\*={x3,x6} b3.c1\*={x4}<d0\* b3.c2\*={x5}<d1\*

a1.b1.c1\*={x2} <d1\*, a1.b1.c2\*={x6}<d0\*

|  |  |  |  |
| --- | --- | --- | --- |
| A | B | C | D |
|  | B2 |  | D0 |
| A2 | B1 |  | D1 |
| A2 |  | C1 | D0 |
| A2 |  | C2 | D1 |
|  | B1 | C1 | D1 |
|  | B3 | C1 | D0 |
|  | B3 | C2 | D1 |
| A1 | B1 | C1 | D1 |
| A1 | B1 | C2 | D0 |

T - table

T(D0)

|  |  |  |
| --- | --- | --- |
| A | B | C |
|  | B2 |  |
| A2 |  | C1 |
|  | B3 | C1 |
| A1 | B1 | C2 |

T(D0,A1)

|  |  |
| --- | --- |
| B | C |
| B1 | C2 |

T(D0,A1,C2)

|  |
| --- |
| B |
| B1 |

T(D0,A2)

|  |  |
| --- | --- |
| B | C |
|  | C1 |

T(D0,A2,C1)

|  |
| --- |
| B |
|  |

T(D0,A?)

|  |  |
| --- | --- |
| B | C |
| B2 |  |
| B3 | C1 |

T(D0,A?,C?)

|  |
| --- |
| B |
| B2 |

T(D0,A?,C1)

|  |
| --- |
| B |
| B3 |

T(D1)

|  |  |  |
| --- | --- | --- |
| A | B | C |
| A2 | B1 |  |
| A2 |  | C2 |
|  | B1 | C1 |
|  | B3 | C2 |
| A1 | B1 | C1 |

T(D1, A1)

|  |  |
| --- | --- |
| B | C |
| B1 | C1 |

T(D1,A1,C1)

|  |
| --- |
| B |
| B1 |

T(D1,A2)

|  |  |
| --- | --- |
| B | C |
| B1 |  |
|  | C2 |

T(D1,A?)

|  |  |
| --- | --- |
| B | C |
| B1 | C1 |
| B3 | C2 |

Creating Action Rules from leaves of two trees:

T(D0,A?,C?)

|  |
| --- |
| B |
| B2 |

T(D0,A?,C1)

|  |
| --- |
| B |
| B3 |

T(D1,A1,C1)

|  |
| --- |
| B |
| B1 |

A1.C1.(B2 ->B1) => (D0->D1)

A1.C1.(B3->B1) => (D0->D1)

**Problem 4.** Let S=(X, {a, b, c, d}) be a decision system, where all attributes are flexible. Attribute d is the decision attribute. Find action rules reclassifying objects from the class d1 to d2 using action reducts.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | a | b | c | d |
| x1 | a3 | b1 | c3 | d1 |
| x2 | a3 | b2 | c1 | d2 |
| x3 | a1 | b1 | c1 | d2 |
| x4 | a2 | b1 | c1 | d1 |
| x5 | a1 | b1 | c3 | d1 |
| x6 | a2 | b2 | c2 | d2 |

System S

**Solution:**

|  |  |  |  |
| --- | --- | --- | --- |
|  | **X2** | **X3** | **X6** |
| **X1** | **b2, c1** | **a1, c1** | **a2,b2,c2** |
| **X4** | **a3, b2** | **a1** | **b2,c2** |
| **X5** | **a3, b2, c1** | **c1** | **a2,b2,c2** |

R(x2)= (b2+c1)(a3+b2)(a3+b2+c1) =

 (b2+c1)(a3+b2)=b2.a3+b2+c1.a3+c1.b2 = b2+c1.a3

R(x3)=(a1+c1).a1.c1 = a1.c1 R(x6)= b2 + c2

{c1,a3}, {b2}, {a1,c1}, {c2} – Reducts

b1.(c, -> c1).(a, -> a3) => (d, d1 -> d2) Dom= {x1,x4,x5} b - can not be used

(c, -> c1).(a, -> a3) => (d, d1 -> d2) Dom= {x1,x4,x5} - does not change

a1.(b,b1 -> b2) => (d, d1->d2) Dom= {x5}

a2.(b, b1-> b2) => (d, d1->d2) Dom= {x4}

a3.(b,b1 -> b2) => (d, d1->d2) Dom= {x1}

c1.(b, b1-> b2) => (d, d1->d2) Dom= {x4}

c3.(b, b1-> b2) => (d, d1->d2) Dom= {x1,x5}

b1.(a, -> a1).(c, ->c1) => (d, d1 -> d2) Dom= {x1,x4,x5} b – can not be used

(a, -> a1).(c, ->c1) => (d, d1 -> d2) Dom= {x1,x4,x5} - does not change

a1.(c, c3->c2) => (d, d1->d2) Dom= {x5}

a2.(c, c1->c2) => (d, d1->d2) Dom={x4}

a3.(c, c3->c2) => (d, d1->d2) Dom={x1}