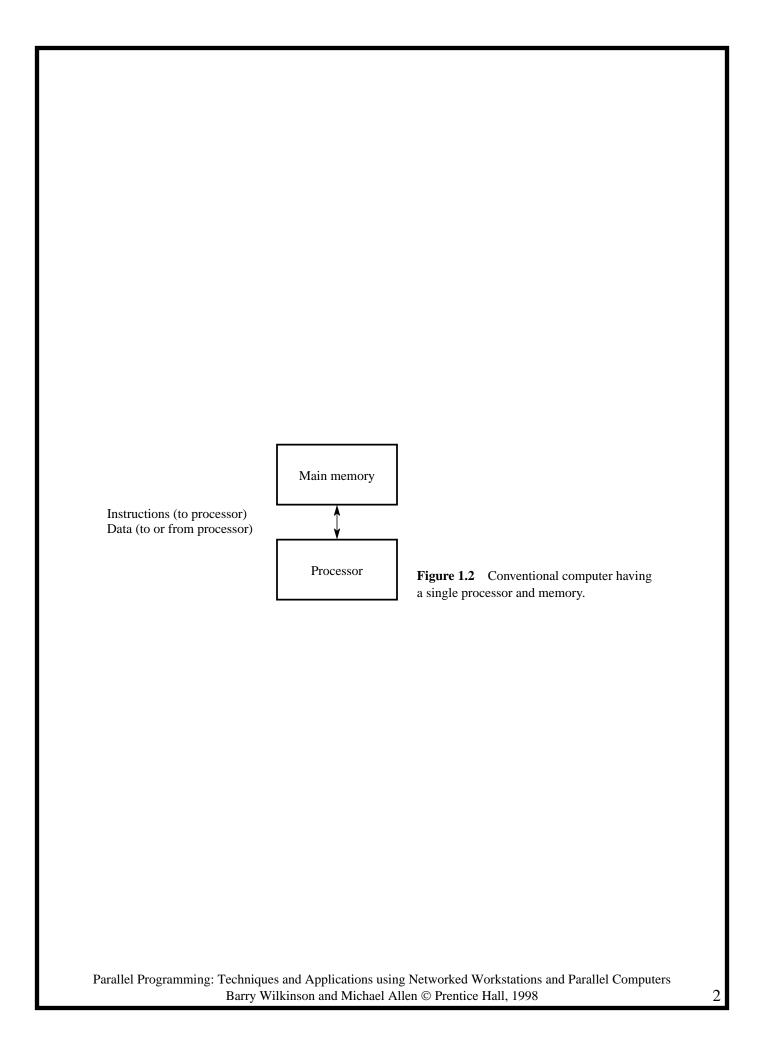
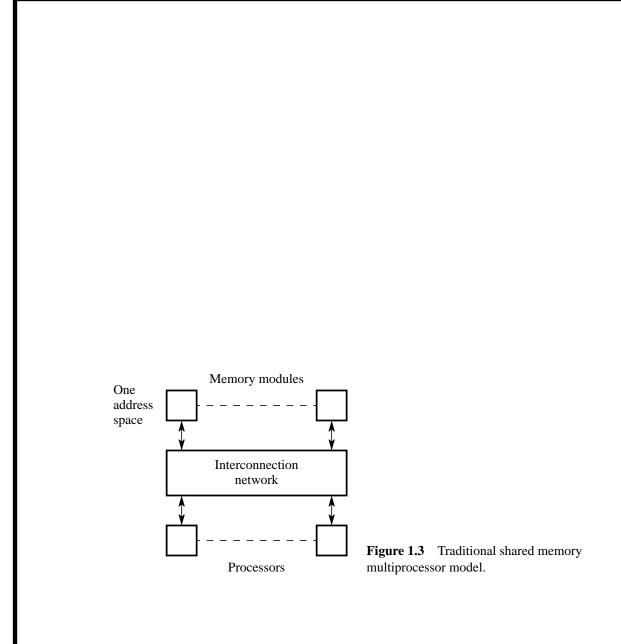
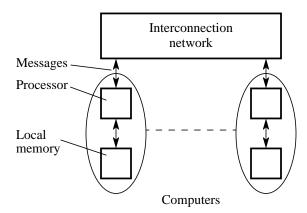


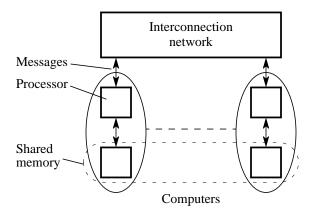
**Figure 1.1** Astrophysical *N*-body simulation by Scott Linssen (undergraduate University of North Carolina at Charlotte [UNCC] student).



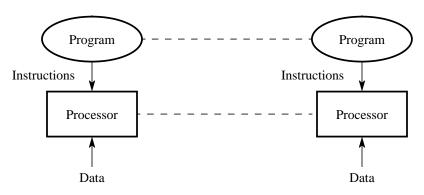




**Figure 1.4** Message-passing multiprocessor model (multicomputer).



**Figure 1.5** Shared memory multiprocessor implementation.



**Figure 1.6** MPMD structure.

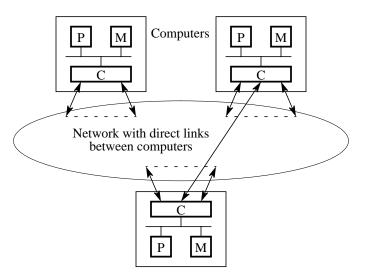
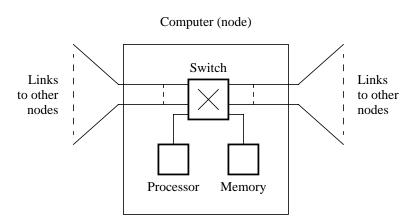
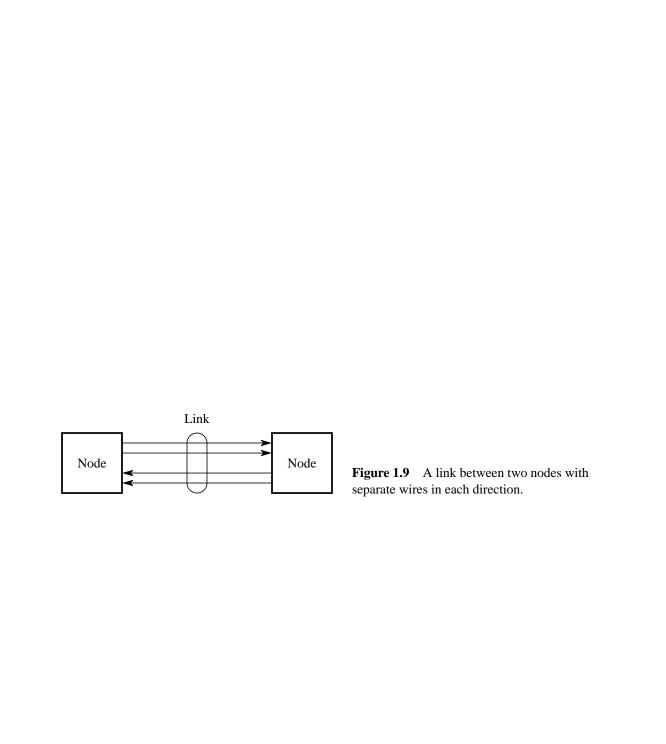
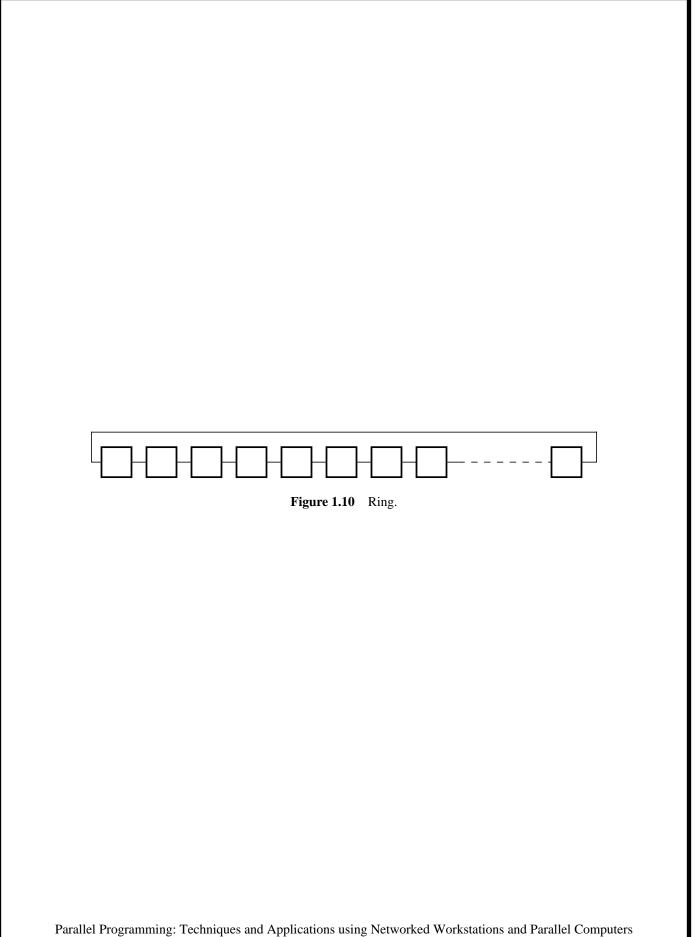


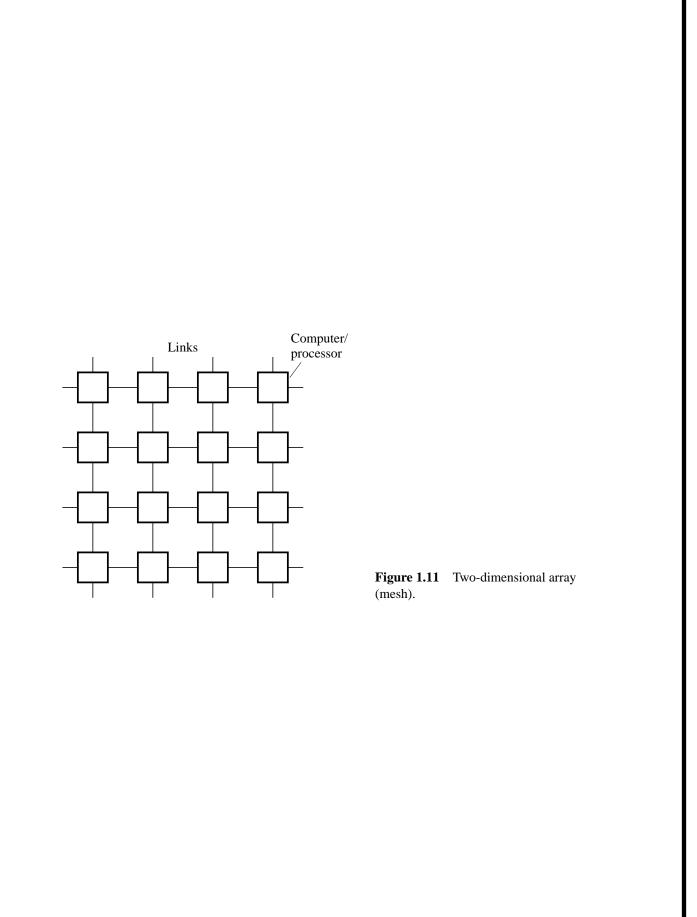
Figure 1.7 Static link multicomputer.



**Figure 1.8** Node with a switch for internode message transfers.







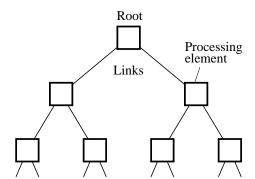
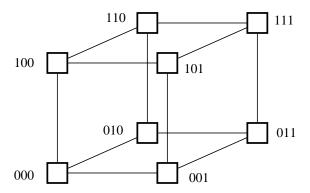
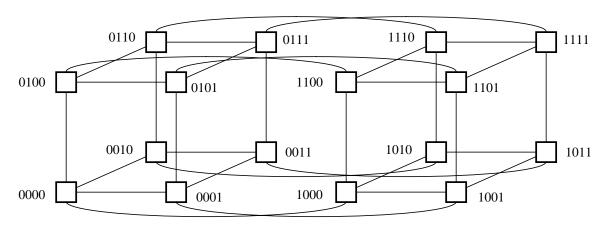


Figure 1.12 Tree structure.



**Figure 1.13** Three-dimensional hypercube.



**Figure 1.14** Four-dimensional hypercube.

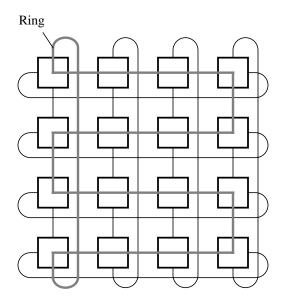
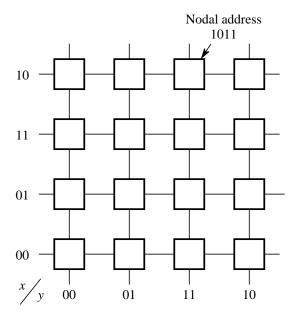
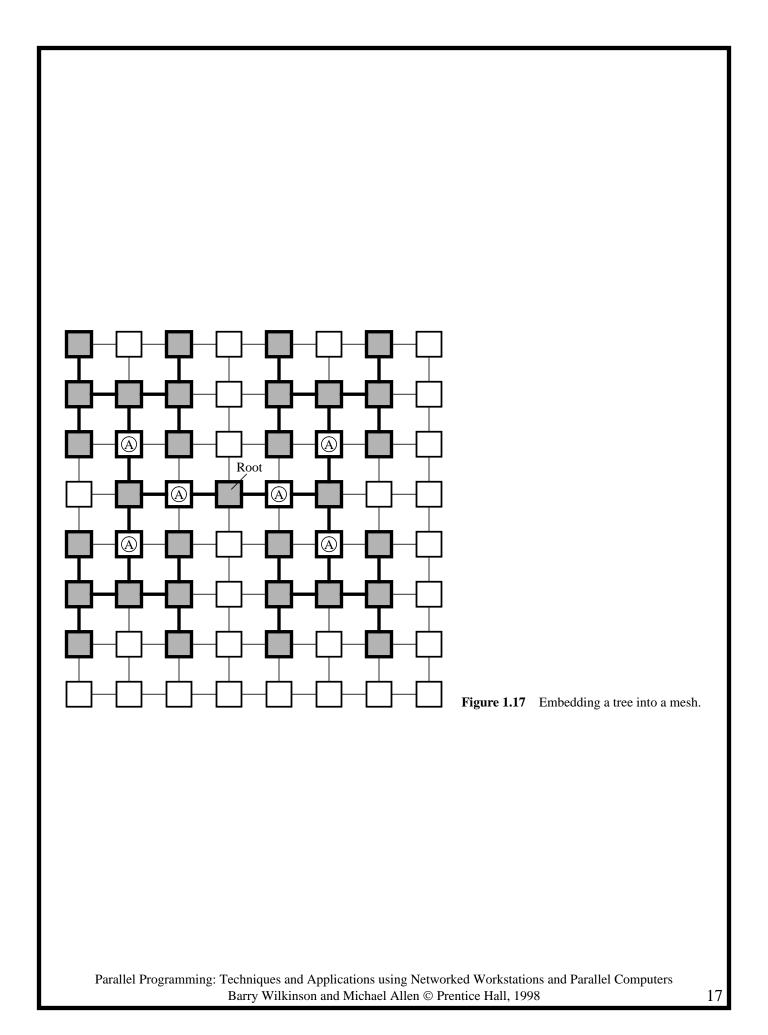


Figure 1.15 Embedding a ring onto a torus.



**Figure 1.16** Embedding a mesh into a hypercube.



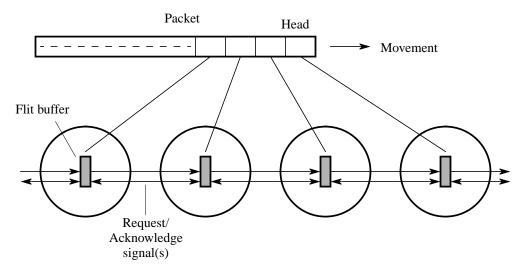
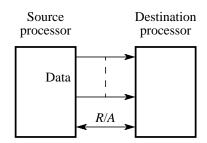


Figure 1.18 Distribution of flits.



**Figure 1.19** A signaling method between processors for wormhole routing (Ni and McKinley, 1993).

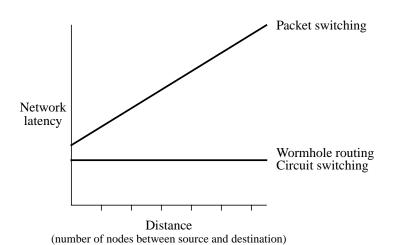
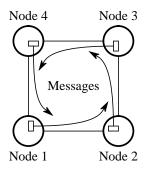


Figure 1.20 Network delay characteristics.



**Figure 1.21** Deadlock in store-and-forward networks.

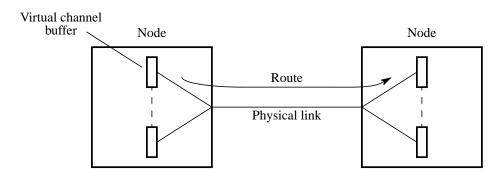
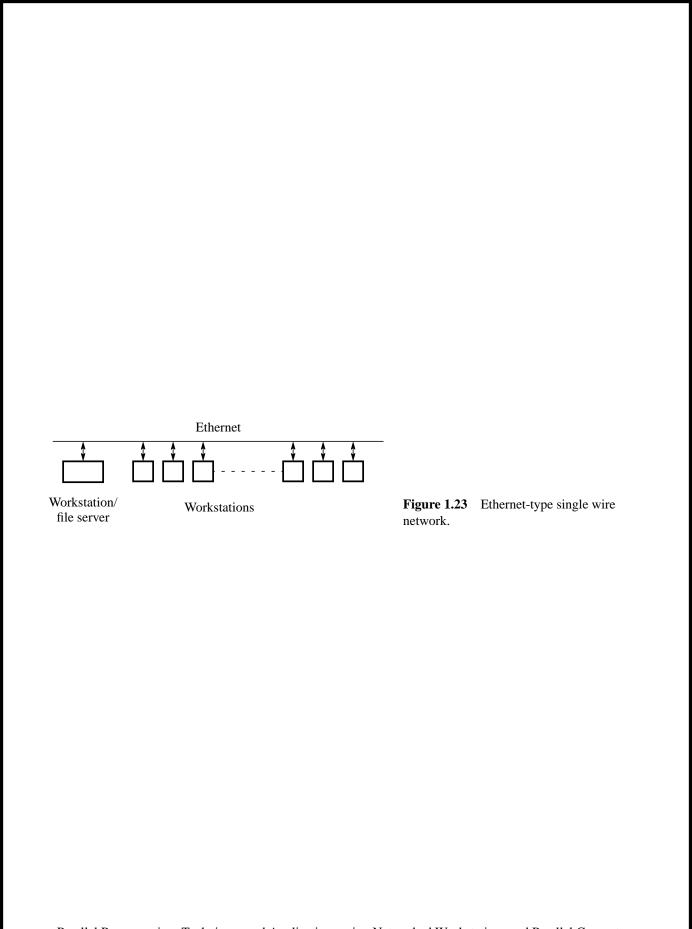
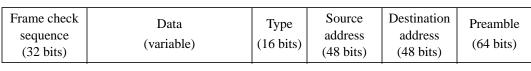


Figure 1.22 Multiple virtual channels mapped onto a single physical channel.





Direction

Figure 1.24 Ethernet frame format.

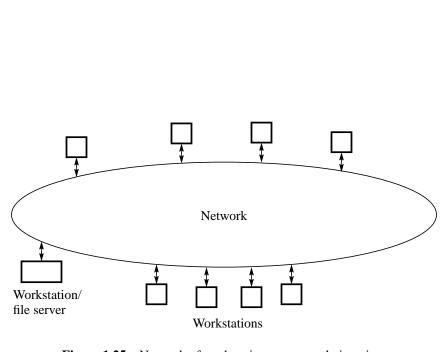
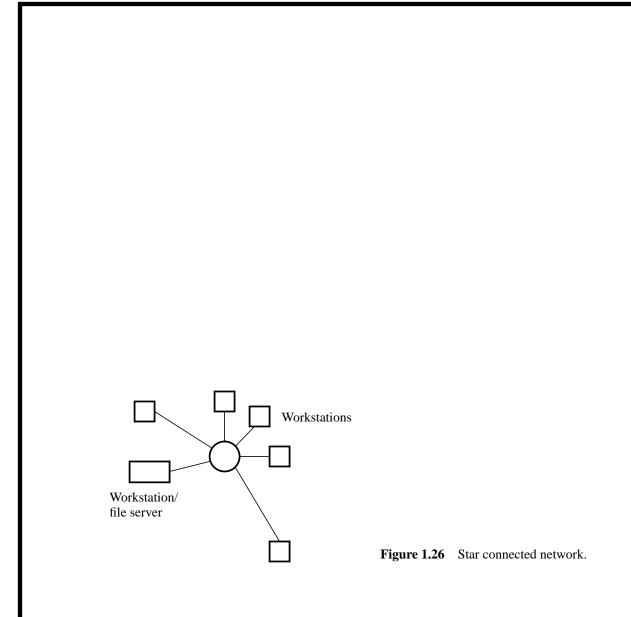
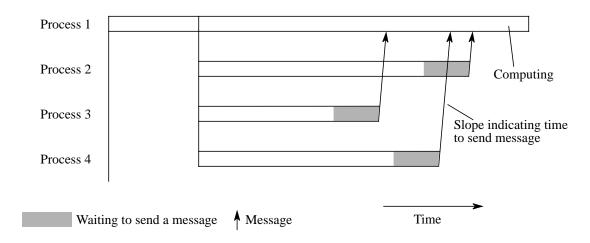


Figure 1.25 Network of workstations connected via a ring.



## Parallel programming cluster (a) Using specially designed adaptors (b) Using separate Ethernet interfaces

Figure 1.27 Overlapping connectivity Ethernets.



**Figure 1.28** Space-time diagram of a message-passing program.

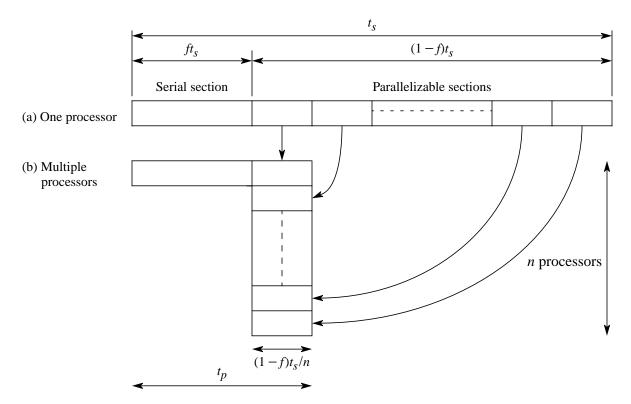
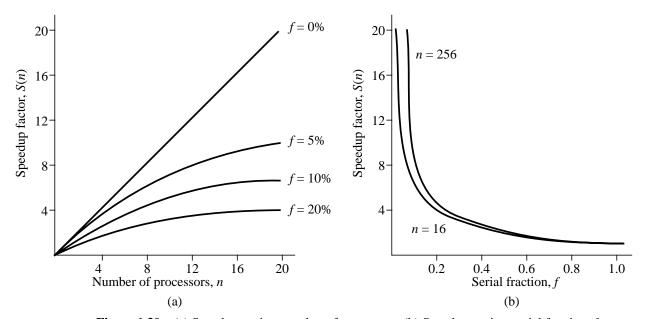
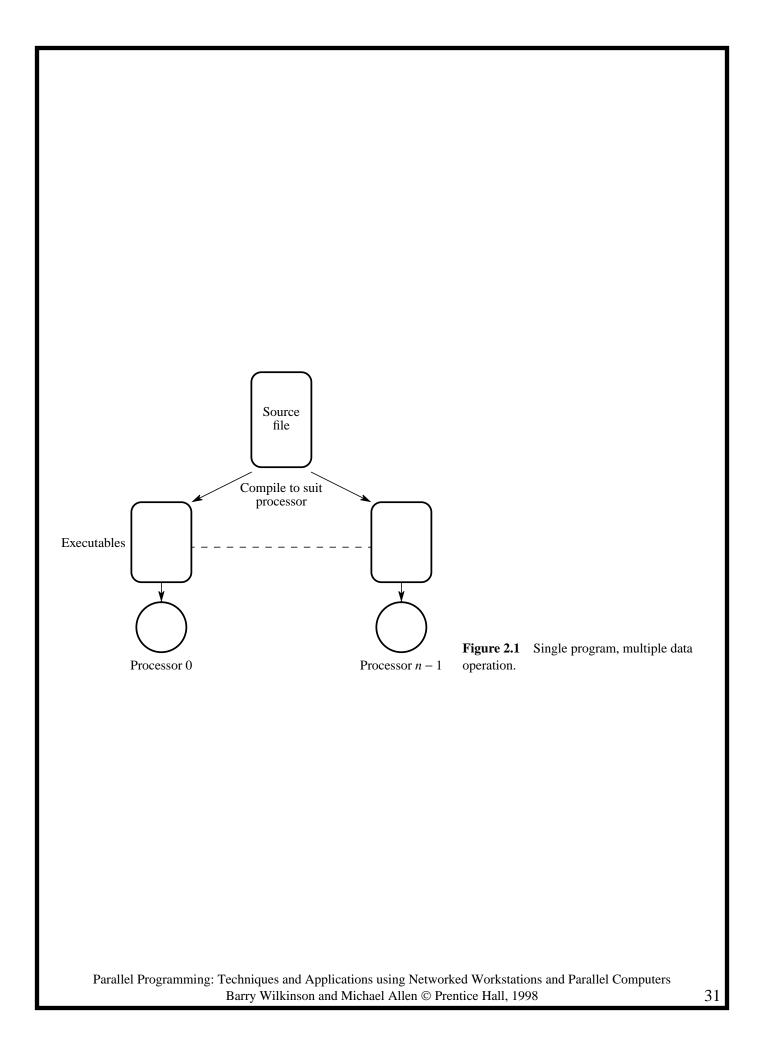
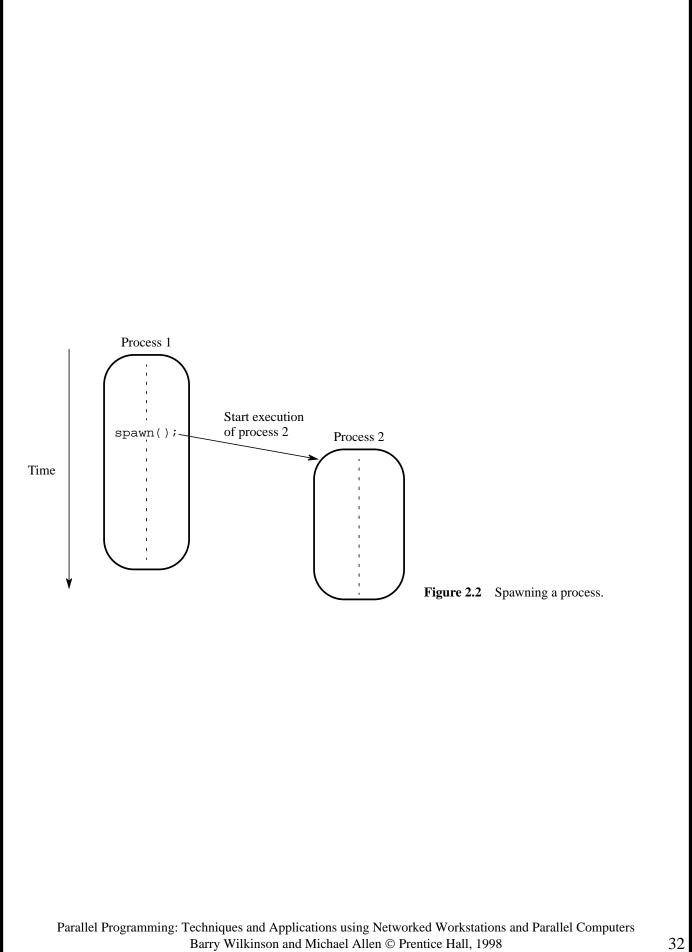


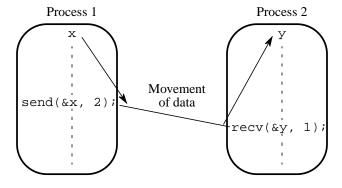
Figure 1.29 Parallelizing sequential problem — Amdahl's law.



**Figure 1.30** (a) Speedup against number of processors. (b) Speedup against serial fraction, f.







**Figure 2.3** Passing a message between processes using send() and recv() library calls.

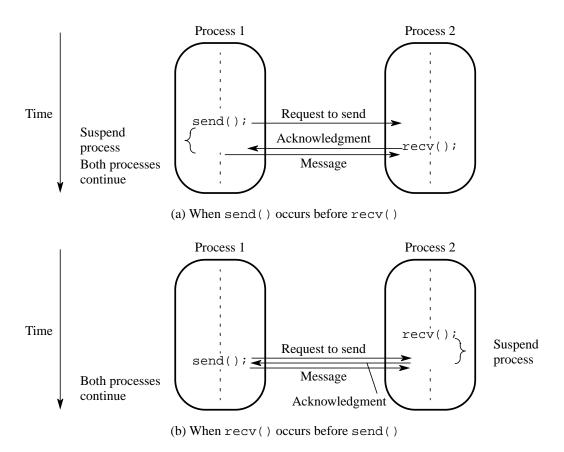


Figure 2.4 Synchronous send() and recv() library calls using a three-way protocol.

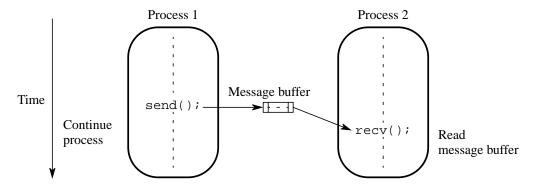
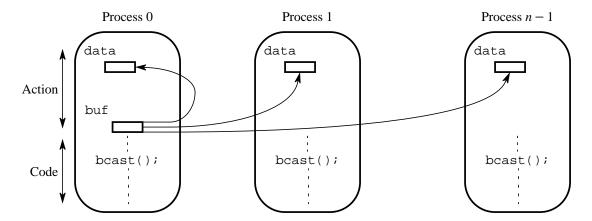


Figure 2.5 Using a message buffer.



**Figure 2.6** Broadcast operation.

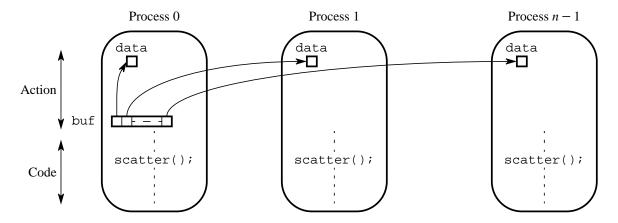
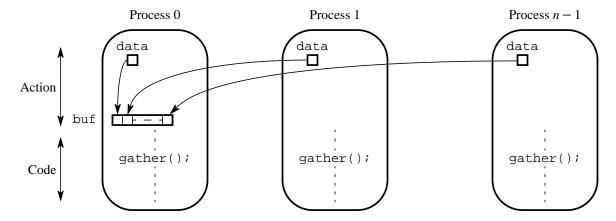
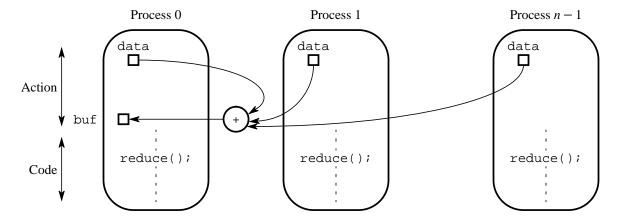


Figure 2.7 Scatter operation.



**Figure 2.8** Gather operation.



**Figure 2.9** Reduce operation (addition).

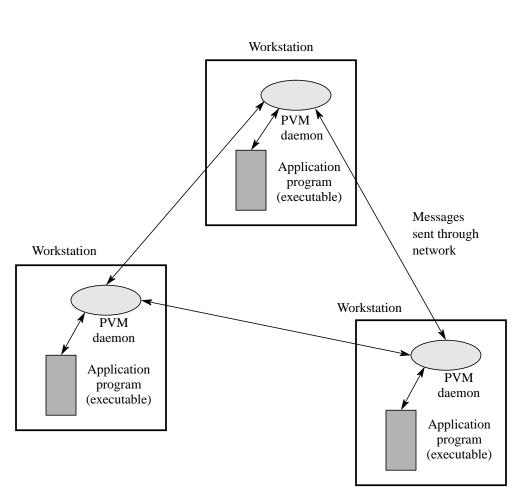


Figure 2.10 Message passing between workstations using PVM.

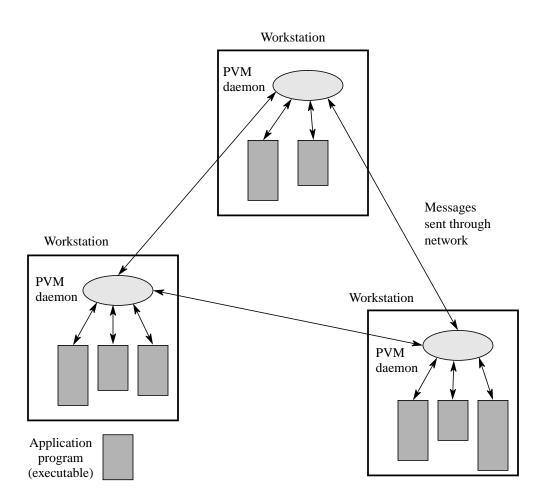


Figure 2.11 Multiple processes allocated to each processor (workstation).

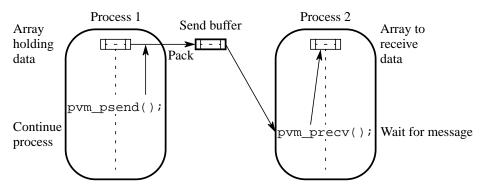


Figure 2.12 pvm\_psend() and pvm\_precv() system calls.

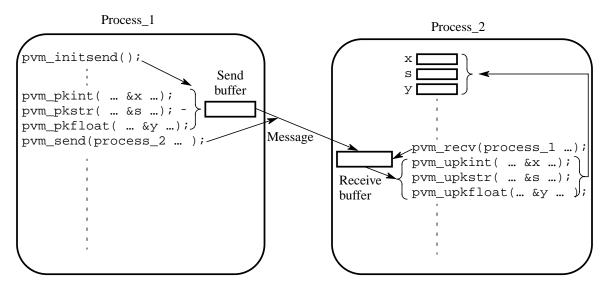


Figure 2.13 PVM packing messages, sending, and unpacking.

```
‡include <stdio.h>
                             Master
‡include <stdlib.h>
‡include <pvm3.h>
‡define SLAVE "spsum"
‡define PROC 10
#define NELEM 1000
nain() {
  int mytid,tids[PROC];
  int n = NELEM, nproc = PROC;
  int no, i, who, msgtype;
                                                                   Slave
  int data[NELEM],result[PROC],tot=0;
                                                      #include <stdio.h>
  char fn[255];
                                                      #include "pvm3.h"
  FILE *fp;
                                                     #define PROC 10
  mytid=pvm_mytid();/*Enroll in PVM */
                                                     #define NELEM 1000
/* Start Slave Tasks */
                                                     main() {
  no=
                                                        int mytid;
   pvm_spawn(SLAVE,(char**)0,0,"",nproc,tids);
                                                        int tids[PROC];
  if (no < nproc) {</pre>
                                                        int n, me, i, msgtype;
    printf("Trouble spawning slaves \n");
                                                        int x, nproc, master;
    for (i=0; i<no; i++) pvm_kill(tids[i]);</pre>
                                                        int data[NELEM], sum;
    pvm_exit(); exit(1);
                                                        mytid = pvm_mytid();
/* Open Input File and Initialize Data */
                                                      /* Receive data from master */
  strcpy(fn,getenv("HOME"));
                                                        msgtype = 0;
  strcat(fn,"/pvm3/src/rand_data.txt");
                                                        pvm_recv(-1, msgtype);
  if ((fp = fopen(fn,"r")) == NULL) {
                                                        pvm_upkint(&nproc, 1, 1);
    printf("Can't open input file %s\n",fn);
                                                        pvm_upkint(tids, nproc, 1);
    exit(1);
                                                        pvm_upkint(&n, 1, 1);
                                                        pvm_upkint(data, n, 1);
  for(i=0;i<n;i++)fscanf(fp,"%d",&data[i]);</pre>
                                                      /* Determine my tid */
/* Broadcast data To slaves*/
                                                        for (i=0; i<nproc; i++)</pre>
  pvm_initsend(PvmDataDefault);
                                                           if(mytid==tids[i])
  msgtype = 0;
                                                              {me = i;break;}
  pvm_pkint(&nproc, 1, 1);
  pvm_pkint(tids, nproc, 1);
                                       Broadcast data
                                                      /* Add my portion Of data */
  pvm_pkint(&n, 1, 1);
                                                        x = n/nproc;
  pvm_pkint(data, n, 1);
                                                        low = me * x;
  pvm_mcast(tids, nproc, msgtag);
                                                        high = low + x;
                                                        for(i = low; i < high; i++)
                                                           sum += data[i];
/* Get results from Slaves*/
  msgtype = 5;
                                                      /* Send result to master */
  for (i=0; i<nproc; i++){
                                                        pvm_initsend(PvmDataDefault);
    pvm_recv(-1, msgtype);
                                      Receive results
                                                        pvm_pkint(&me, 1, 1);
    pvm_upkint(&who, 1, 1);
                                                        pvm_pkint(&sum, 1, 1);
    pvm_upkint(&result[who], 1, 1);
                                                        msgtype = 5;
    printf("%d from %d\n",result[who],who);
                                                        master = pvm_parent();
                                                        pvm_send(master, msgtype);
/* Compute global sum */
                                                      /* Exit PVM */
  for (i=0; i<nproc; i++) tot += result[i];</pre>
                                                        pvm exit();
  printf ("The total is %d.\n\n", tot);
                                                        return(0);
 pvm_exit(); /* Program finished. Exit PVM */
 return(0);
                             Figure 2.14 Sample PVM program.
```

Parallel Programming: Techniques and Applications using Networked Workstations and Parallel Computers

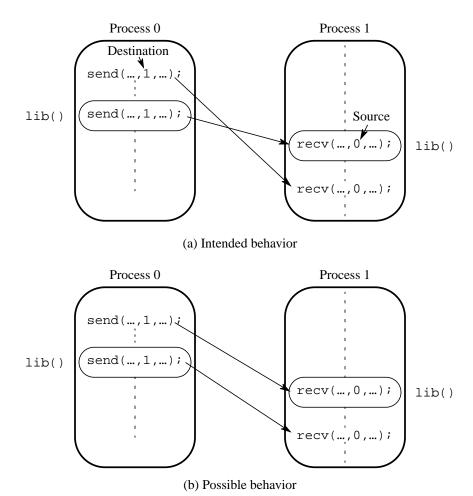
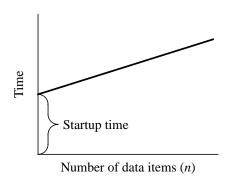


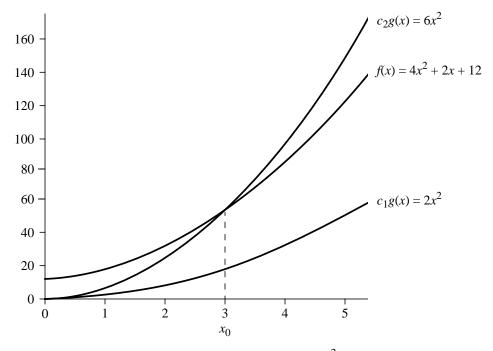
Figure 2.15 Unsafe message passing with libraries.

```
#include "mpi.h"
#include <stdio.h>
#include <math.h>
#define MAXSIZE 1000
void main(int argc, char *argv)
  int myid, numprocs;
  int data[MAXSIZE], i, x, low, high, myresult, result;
  char fn[255];
  char *fp;
  MPI_Init(&argc,&argv);
  MPI_Comm_size(MPI_COMM_WORLD,&numprocs);
  MPI_Comm_rank(MPI_COMM_WORLD,&myid);
  if (myid == 0) {
                               /* Open input file and initialize data */
     strcpy(fn,getenv("HOME"));
     strcat(fn,"/MPI/rand_data.txt");
     if ((fp = fopen(fn,"r")) == NULL) {
       printf("Can't open the input file: %s\n\n", fn);
       exit(1);
     for(i = 0; i < MAXSIZE; i++) fscanf(fp, "%d", &data[i]);</pre>
  /* broadcast data */
  MPI_Bcast(data, MAXSIZE, MPI_INT, 0, MPI_COMM_WORLD);
/* Add my portion Of data */
  x = n/nproc;
  low = myid * x;
  high = low + x;
  for(i = low; i < high; i++)
     myresult += data[i];
  printf("I got %d from %d\n", myresult, myid);
/* Compute global sum */
  MPI_Reduce(&myresult, &result, 1, MPI_INT, MPI_SUM, 0, MPI_COMM_WORLD);
  if (myid == 0) printf("The sum is %d.\n", result);
  MPI_Finalize();
```

Figure 2.16 Sample MPI program.



**Figure 2.17** Theoretical communication time.



**Figure 2.18** Growth of function  $f(x) = 4x^2 + 2x + 12$ .

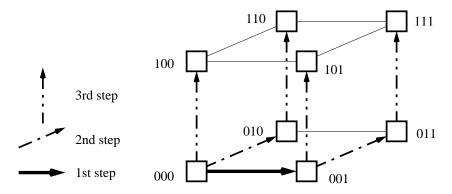


Figure 2.19 Broadcast in a three-dimensional hypercube.

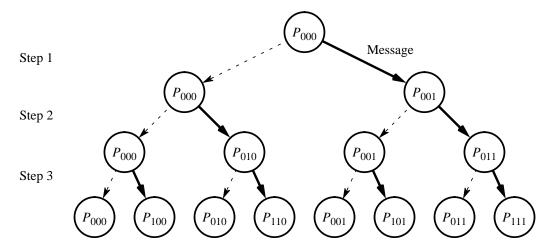
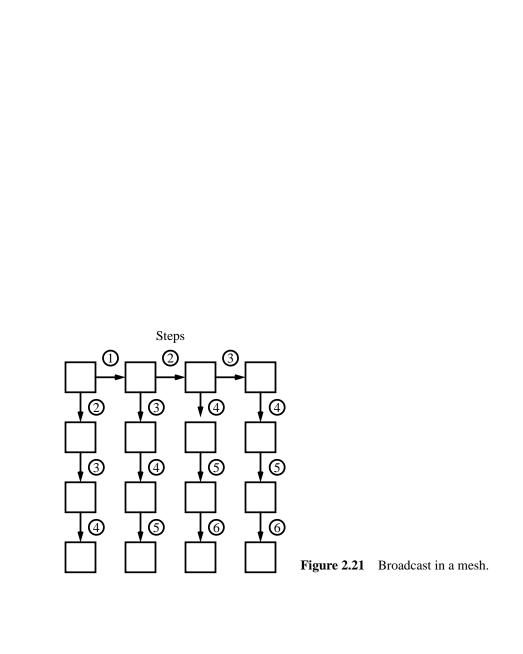
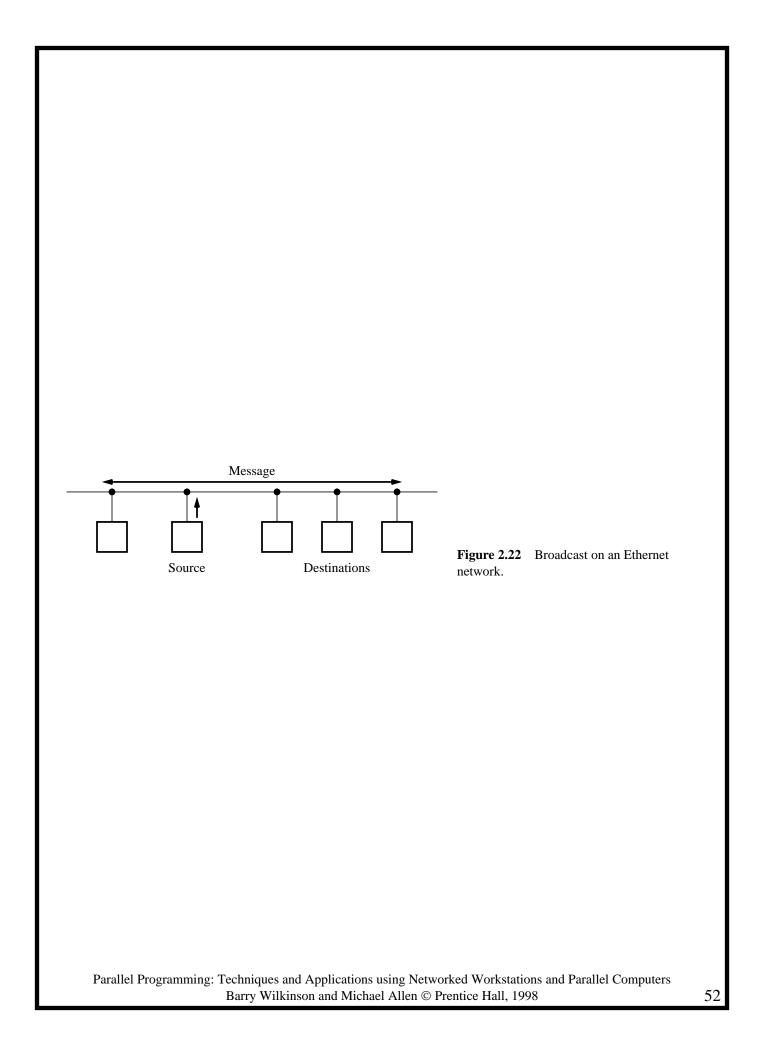
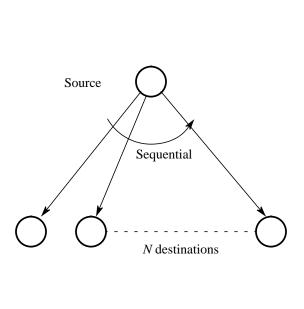


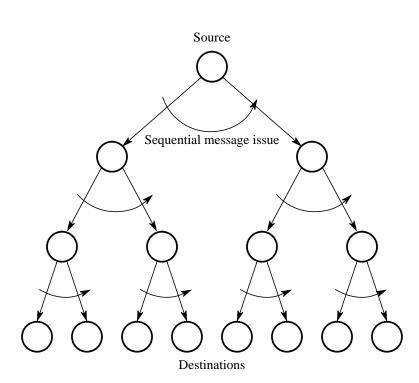
Figure 2.20 Broadcast as a tree construction.



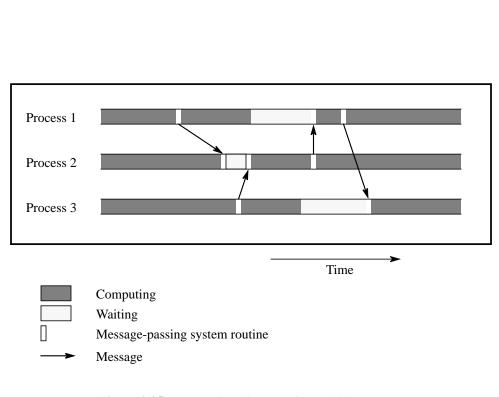




**Figure 2.23** 1-to-*N* fan-out broadcast.



**Figure 2.24** 1-to-*N* fan-out broadcast on a tree structure.



**Figure 2.25** Space-time diagram of a parallel program.

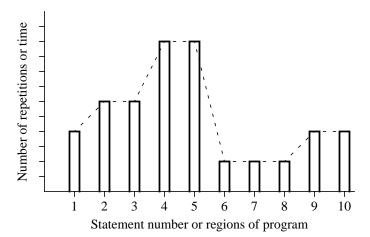
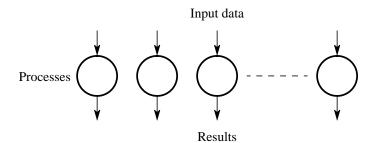
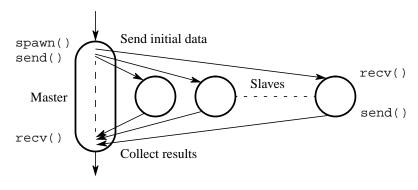


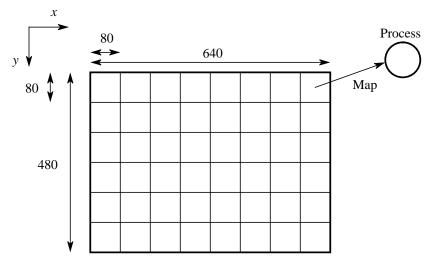
Figure 2.26 Program profile.

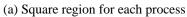


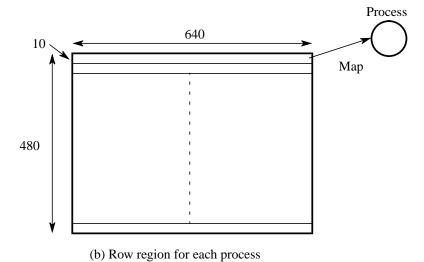
**Figure 3.1** Disconnected computational graph (embarrassingly parallel problem).



**Figure 3.2** Practical embarrassingly parallel computational graph with dynamic process creation and the master-slave approach.







**Figure 3.3** Partitioning into regions for individual processes.

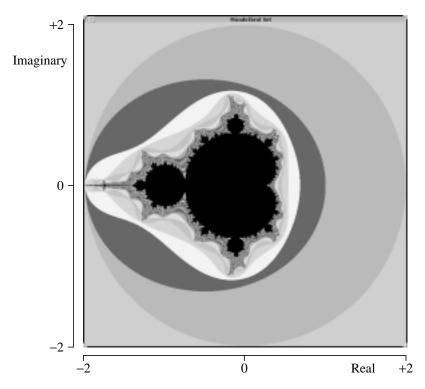


Figure 3.4 Mandelbrot set.

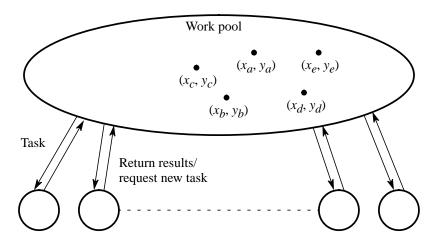
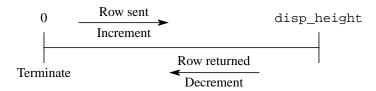
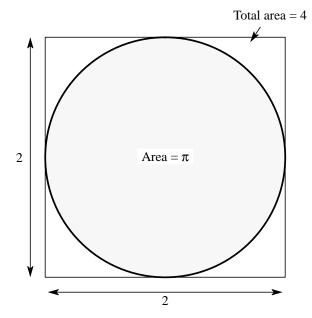


Figure 3.5 Work pool approach.

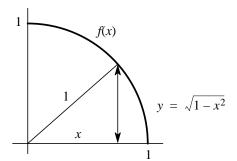
Rows outstanding in slaves (count)



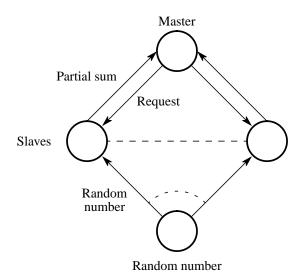
**Figure 3.6** Counter termination.



**Figure 3.7** Computing  $\pi$  by a Monte Carlo method.



**Figure 3.8** Function being integrated in computing  $\pi$  by a Monte Carlo method.



process

**Figure 3.9** Parallel Monte Carlo integration.

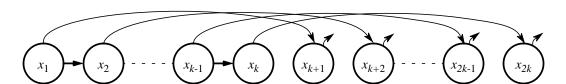


Figure 3.10 Parallel computation of a sequence.

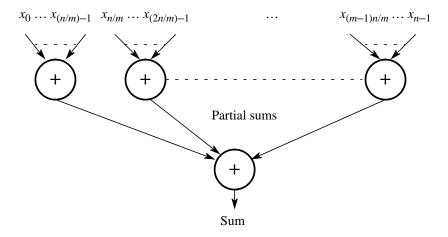


Figure 4.1 Partitioning a sequence of numbers into parts and adding the parts.

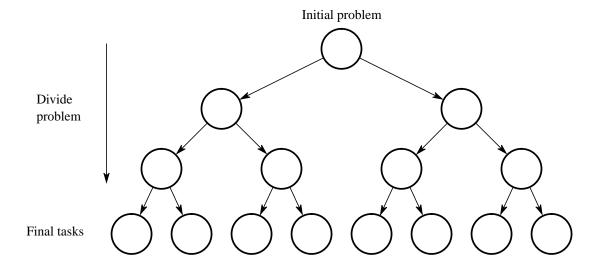
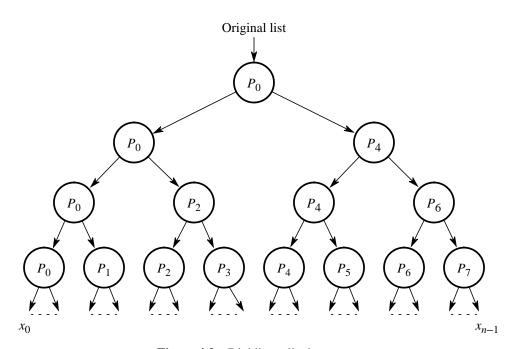


Figure 4.2 Tree construction.



**Figure 4.3** Dividing a list into parts.

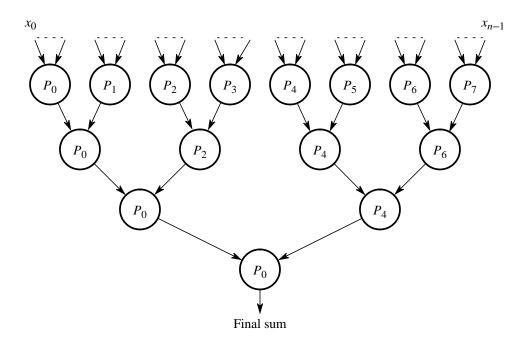
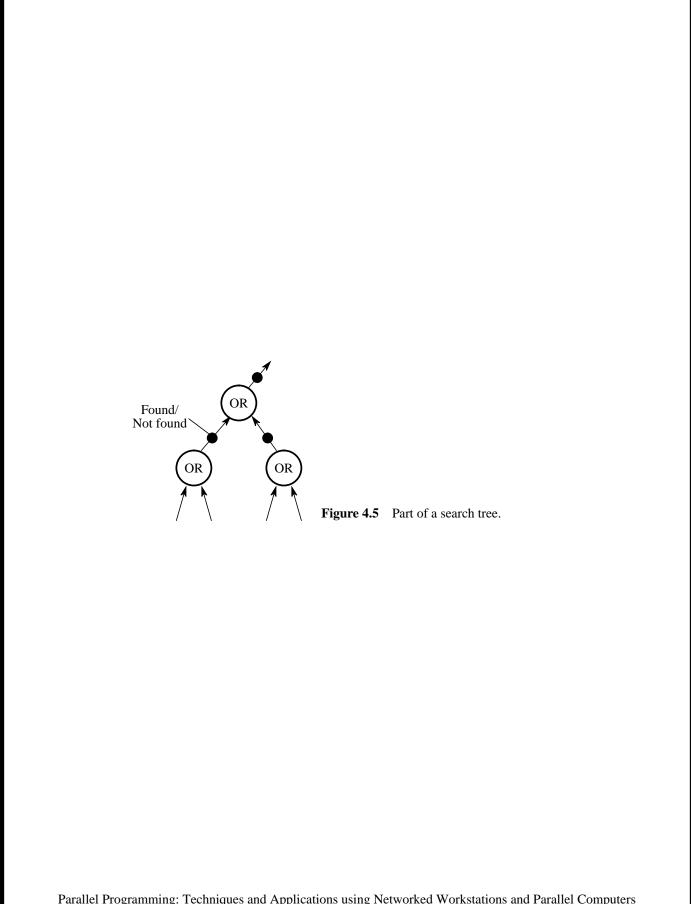


Figure 4.4 Partial summation.



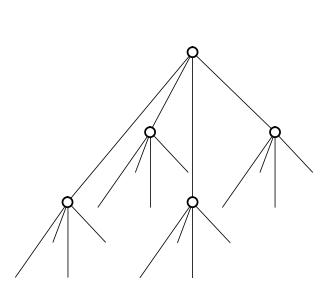
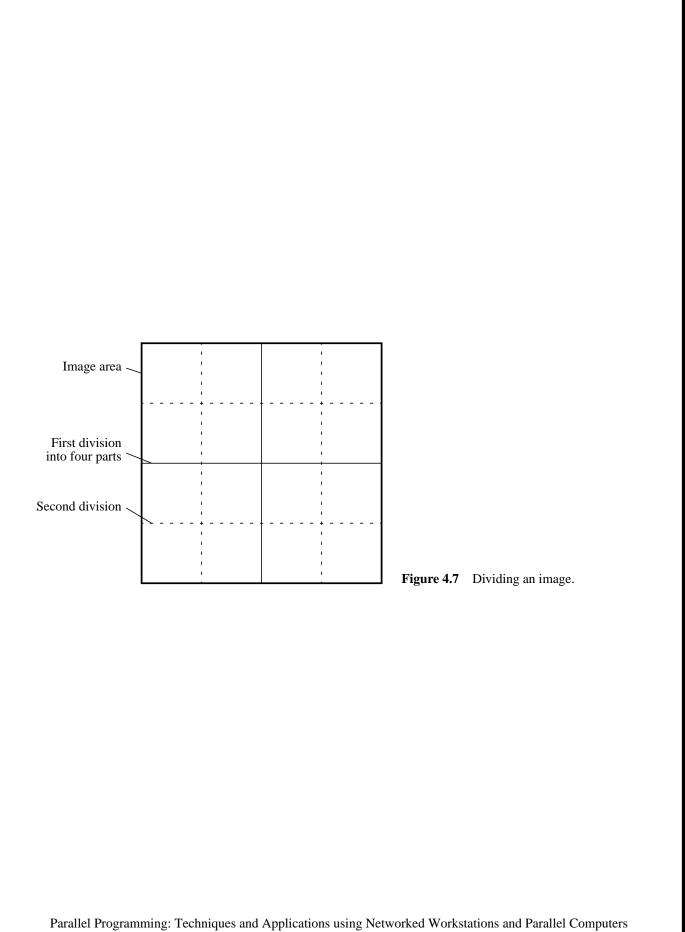


Figure 4.6 Quadtree.



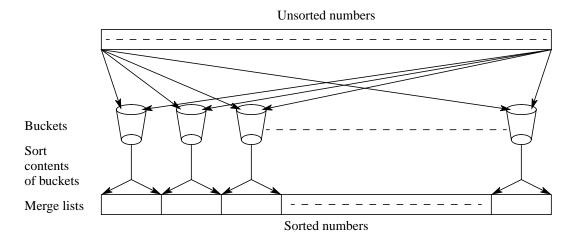
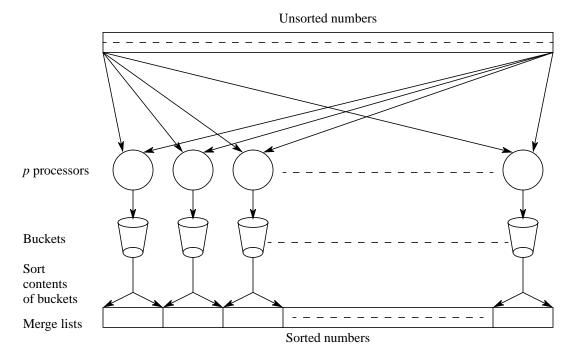


Figure 4.8 Bucket sort.



**Figure 4.9** One parallel version of bucket sort.

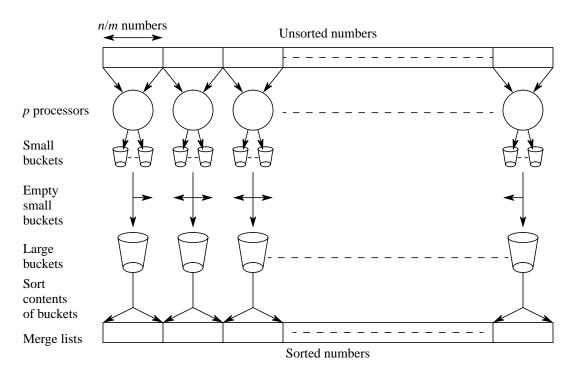


Figure 4.10 Parallel version of bucket sort.

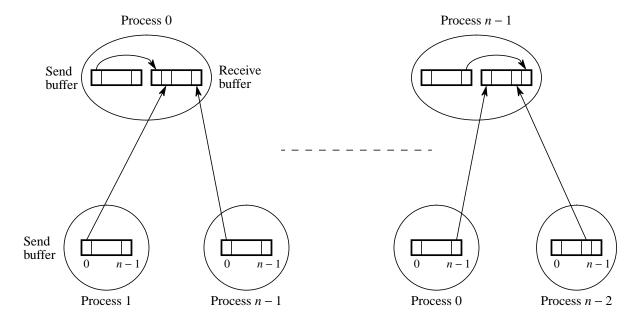
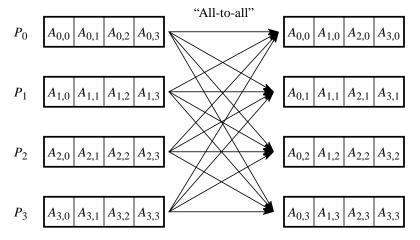
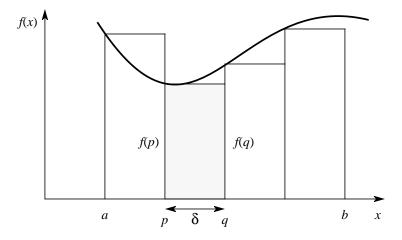


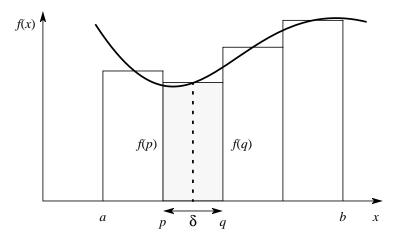
Figure 4.11 "All-to-all" broadcast.



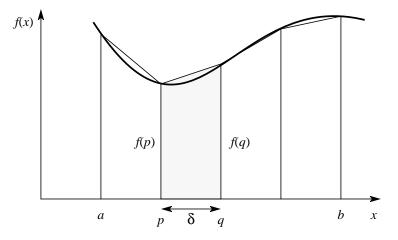
**Figure 4.12** Effect of "all-to-all" on an array.



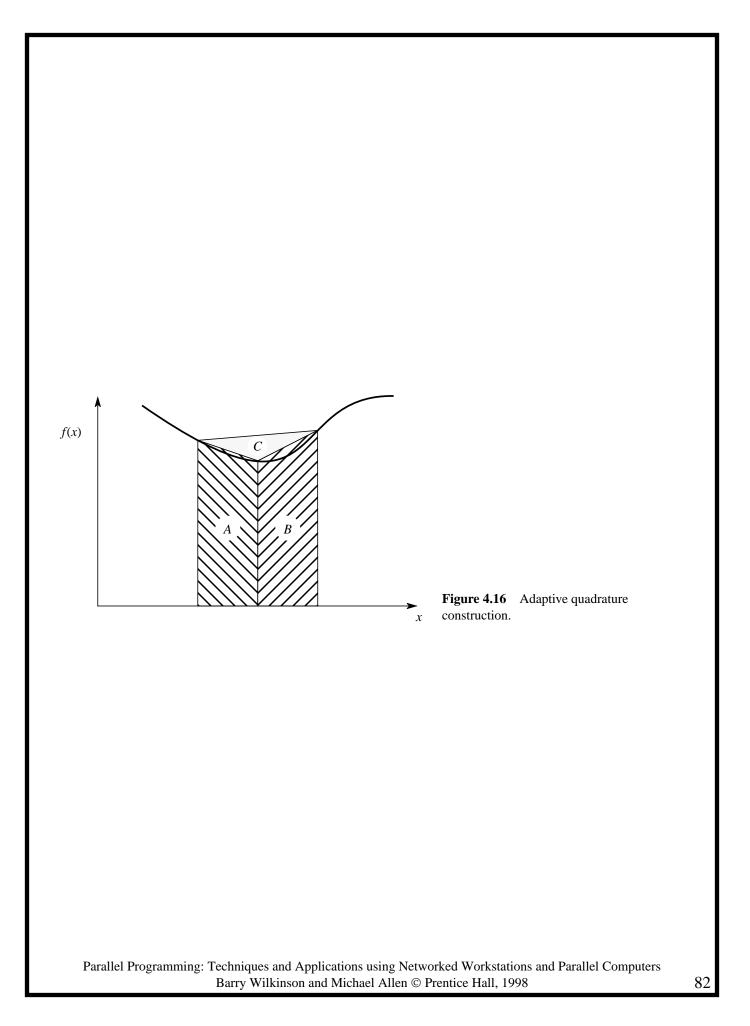
**Figure 4.13** Numerical integration using rectangles.

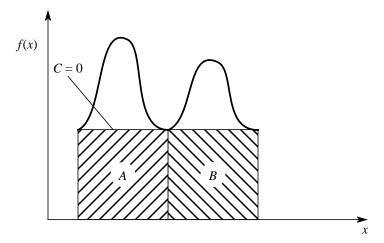


**Figure 4.14** More accurate numerical integration using rectangles.

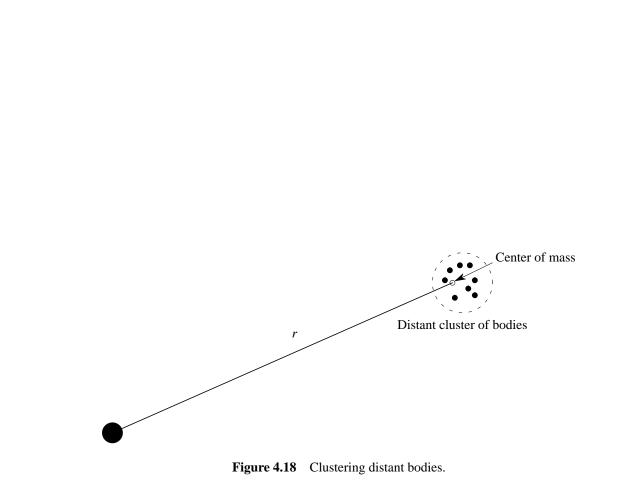


**Figure 4.15** Numerical integration using the trapezoidal method.





**Figure 4.17** Adaptive quadrature with false termination.



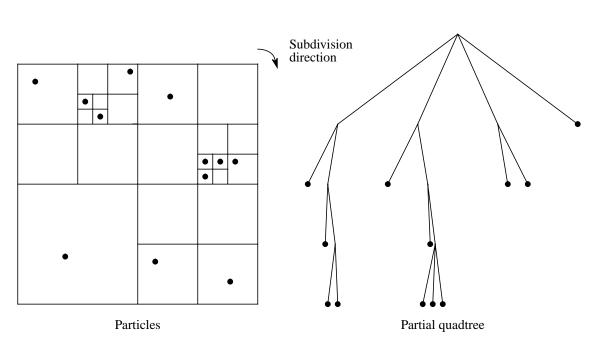
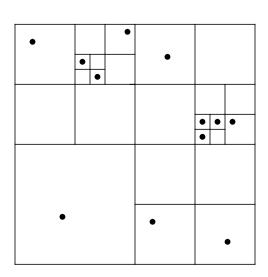
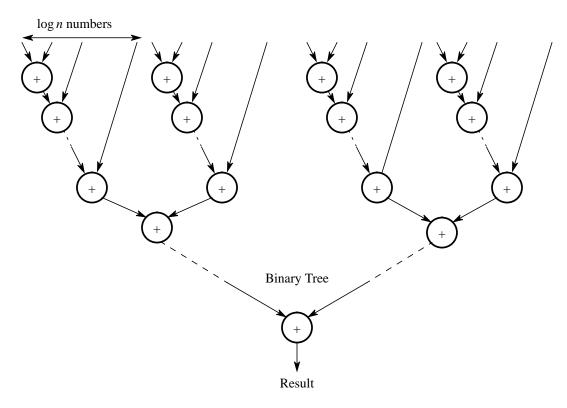


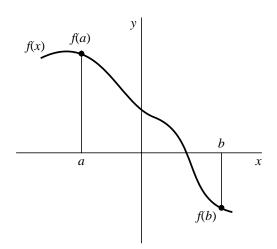
Figure 4.19 Recursive division of two-dimensional space.



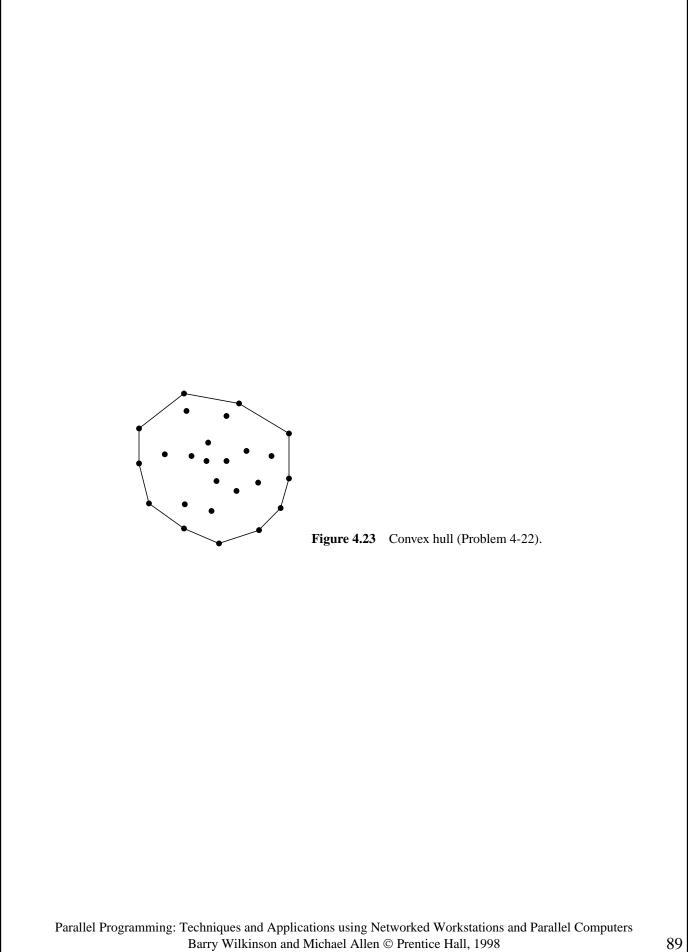
**Figure 4.20** Orthogonal recursive bisection method.



**Figure 4.21** Process diagram for Problem 4-12(b).



**Figure 4.22** Bisection method for finding the zero crossing location of a function.



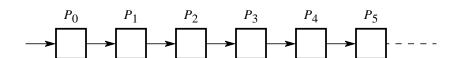


Figure 5.1 Pipelined processes.

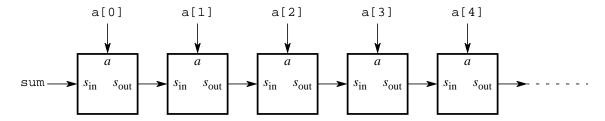
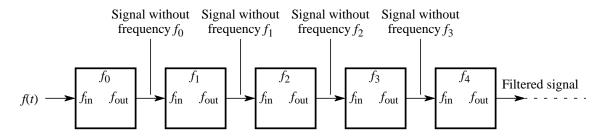


Figure 5.2 Pipeline for an unfolded loop.



**Figure 5.3** Pipeline for a frequency filter.

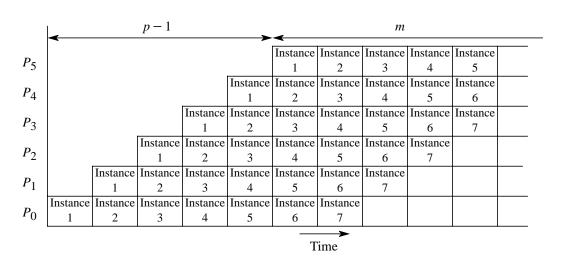
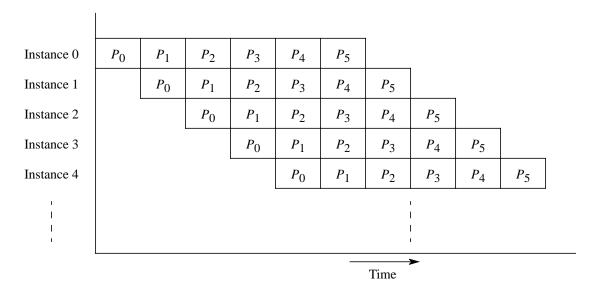


Figure 5.4 Space-time diagram of a pipeline.



**Figure 5.5** Alternative space-time diagram.

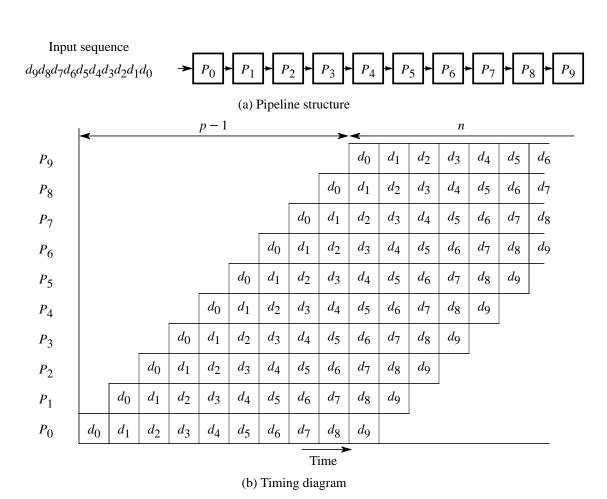
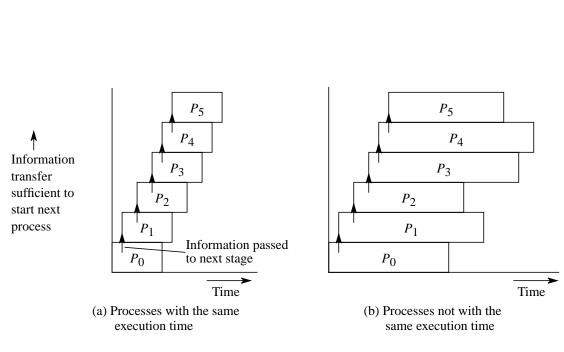
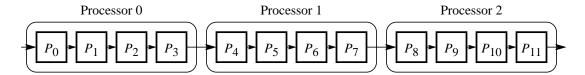


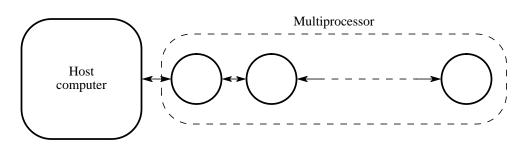
Figure 5.6 Pipeline processing 10 data elements.



**Figure 5.7** Pipeline processing where information passes to next stage before end of process.



**Figure 5.8** Partitioning processes onto processors.



**Figure 5.9** Multiprocessor system with a line configuration.

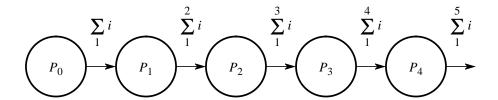


Figure 5.10 Pipelined addition.

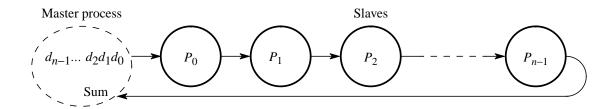
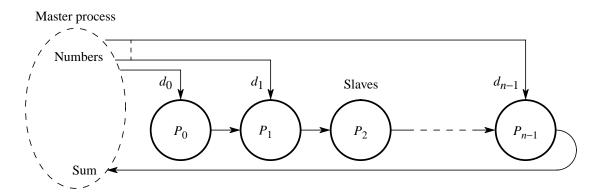


Figure 5.11 Pipelined addition numbers with a master process and ring configuration.



**Figure 5.12** Pipelined addition of numbers with direct access to slave processes.

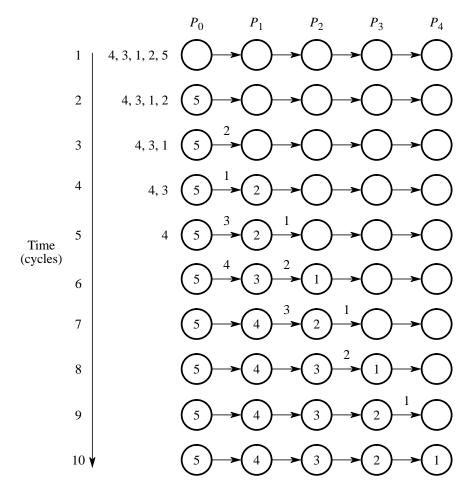
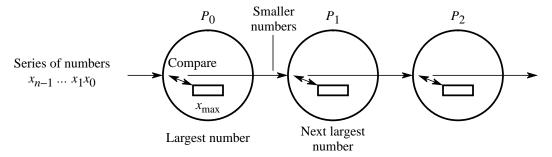


Figure 5.13 Steps in insertion sort with five numbers.



**Figure 5.14** Pipeline for sorting using insertion sort.

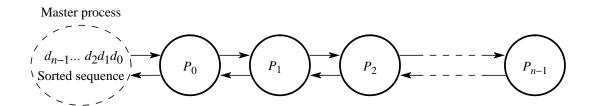


Figure 5.15 Insertion sort with results returned to the master process using a bidirectional line configuration.

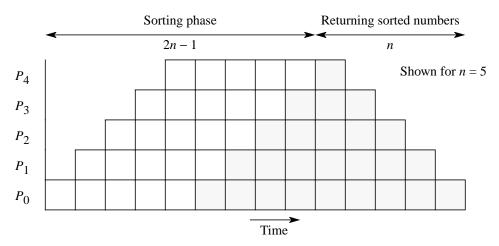


Figure 5.16 Insertion sort with results returned.

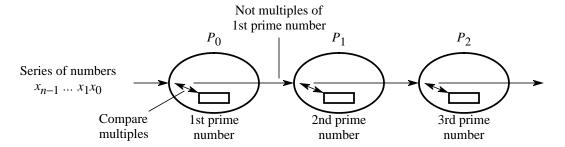


Figure 5.17 Pipeline for sieve of Eratosthenes.

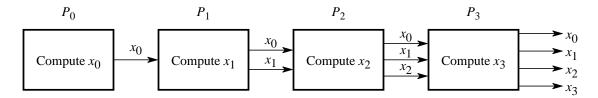
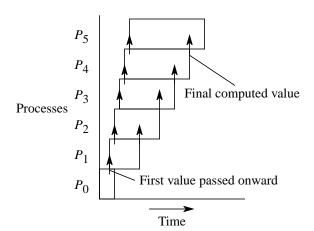
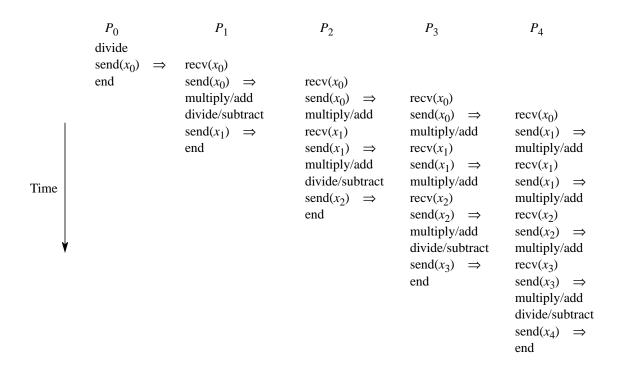


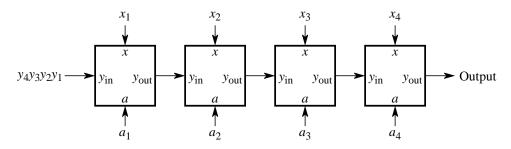
Figure 5.18 Solving an upper triangular set of linear equation using a pipeline.



**Figure 5.19** Pipeline processing using back substitution.



**Figure 5.20** Operations in back substitution pipeline.



**Figure 5.21** Pipeline for Problem 5-9.

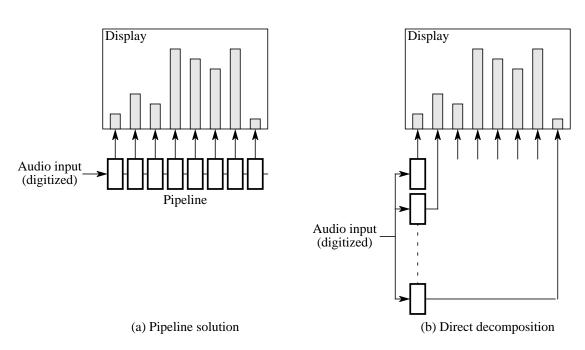
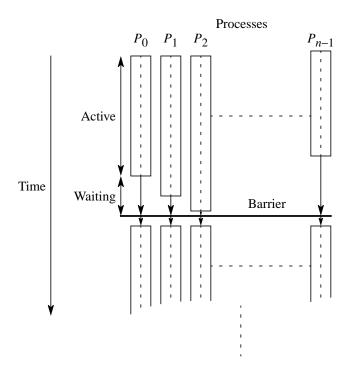


Figure 5.22 Audio histogram display.



**Figure 6.1** Processes reaching the barrier at different times.

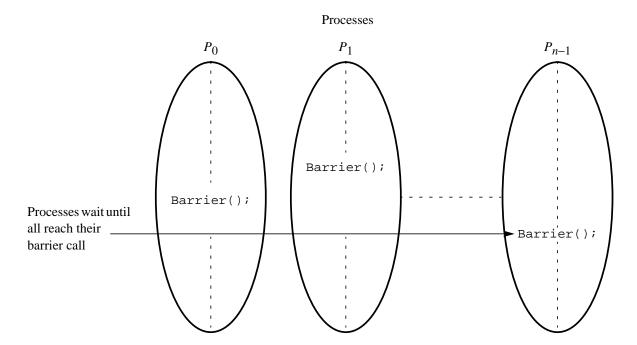


Figure 6.2 Library call barriers.

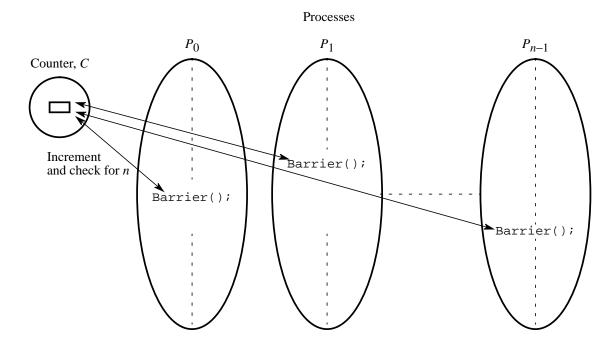


Figure 6.3 Barrier using a centralized counter.

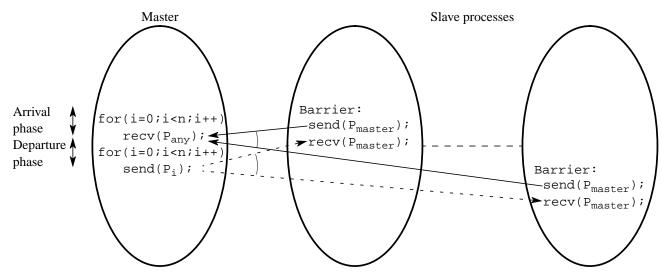


Figure 6.4 Barrier implementation in a message-passing system.

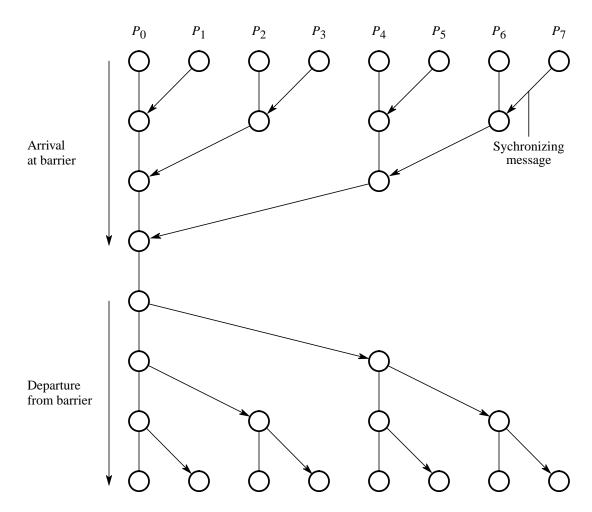


Figure 6.5 Tree barrier.

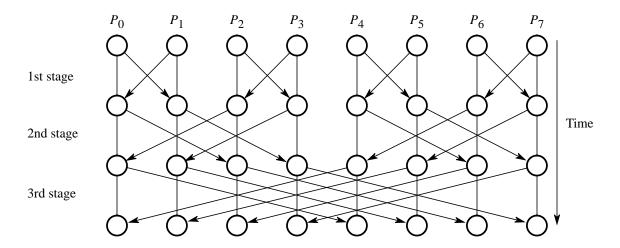


Figure 6.6 Butterfly construction.

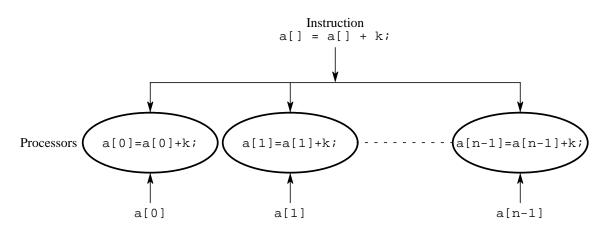
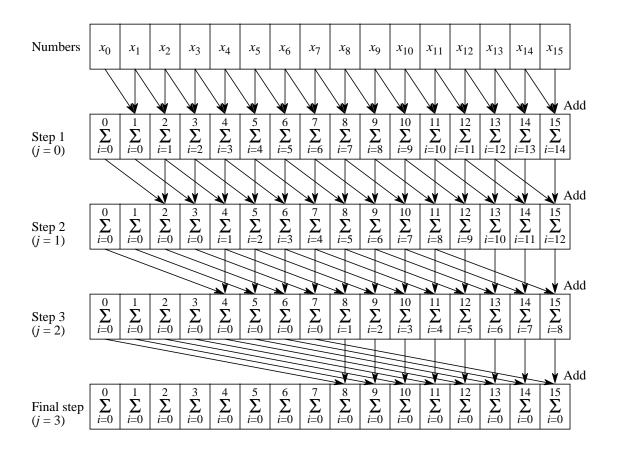
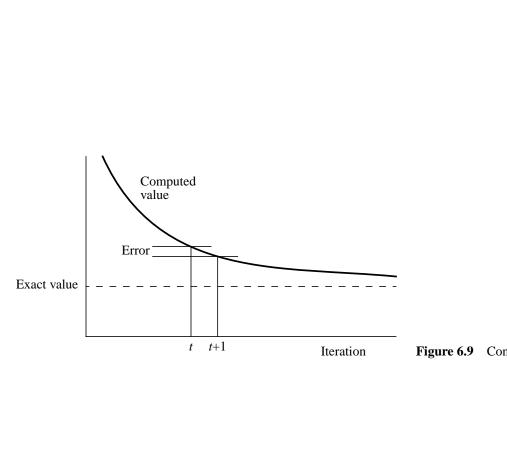


Figure 6.7 Data parallel computation.



**Figure 6.8** Data parallel prefix sum operation.



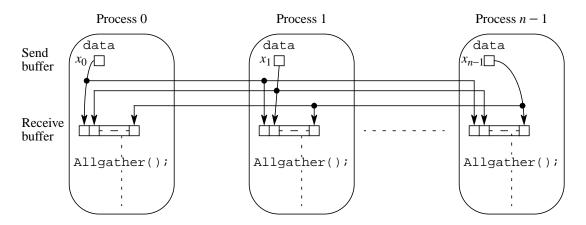


Figure 6.10 Allgather operation.

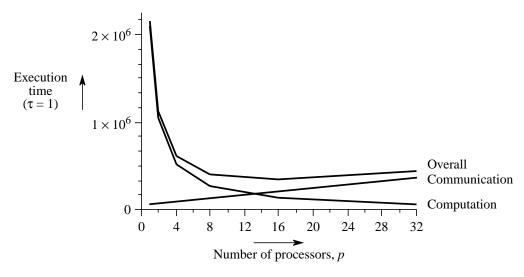


Figure 6.11 Effects of computation and communication in Jacobi iteration.

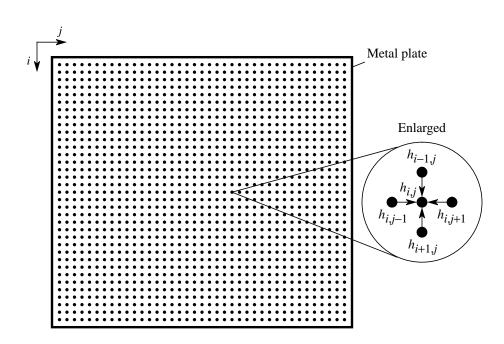
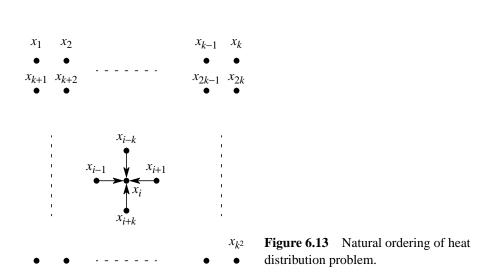
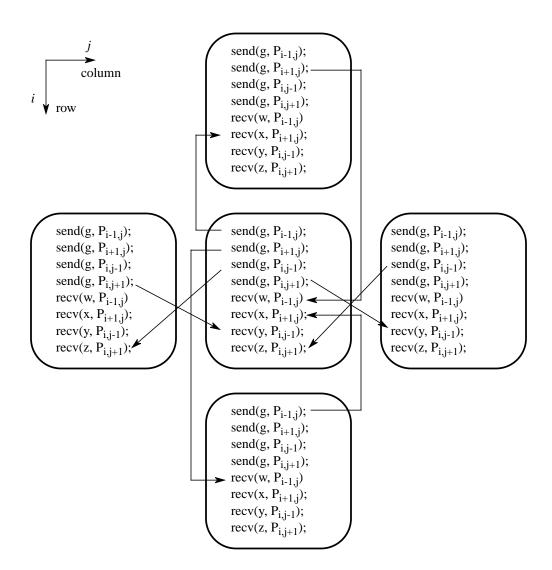
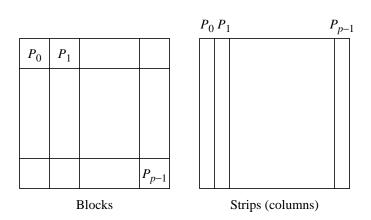


Figure 6.12 Heat distribution problem.





**Figure 6.14** Message passing for heat distribution problem.



**Figure 6.15** Partitioning heat distribution problem.

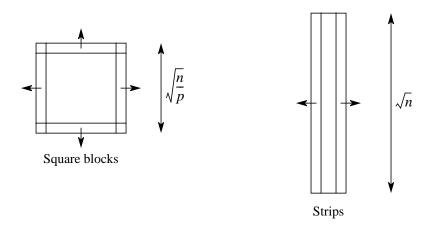
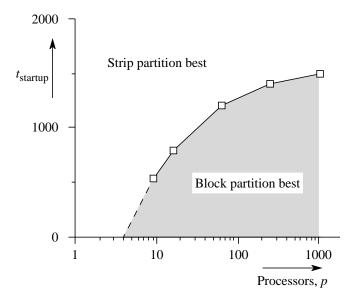


Figure 6.16 Communication consequences of partitioning.



**Figure 6.17** Startup times for block and strip partitions.

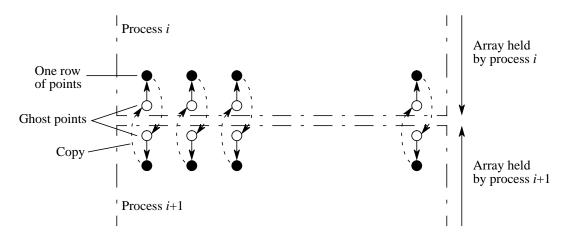
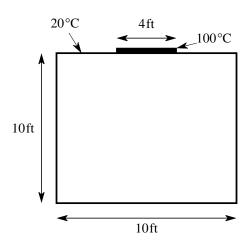
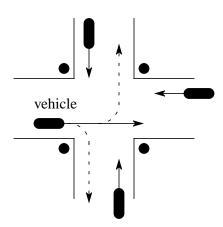


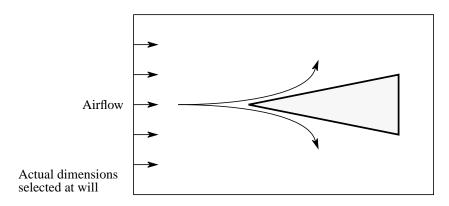
Figure 6.18 Configurating array into contiguous rows for each process, with ghost points.



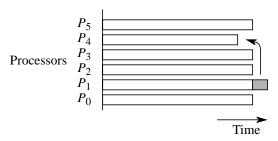
**Figure 6.19** Room for Problem 6-14.



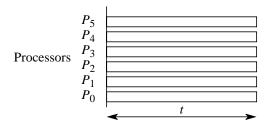
**Figure 6.20** Road junction for Problem 6-16.



**Figure 6.21** Figure for Problem 6-23.

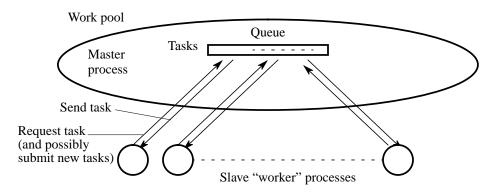


(a) Imperfect load balancing leading to increased execution time

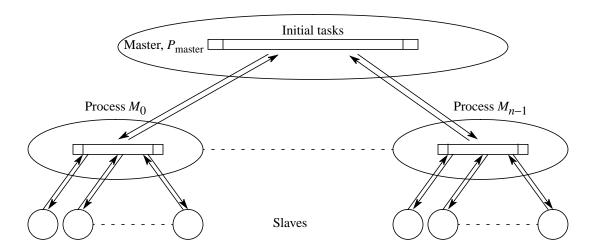


(b) Perfect load balancing

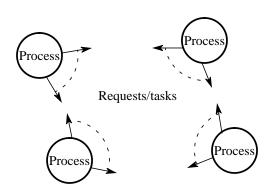
**Figure 7.1** Load balancing.



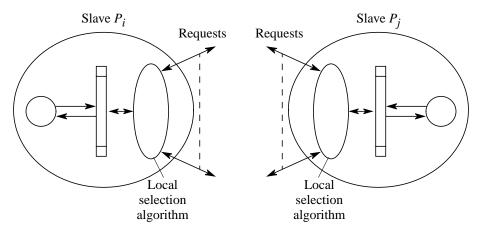
**Figure 7.2** Centralized work pool.



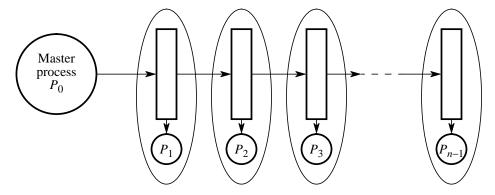
**Figure 7.3** A distributed work pool.



**Figure 7.4** Decentralized work pool.



**Figure 7.5** Decentralized selection algorithm requesting tasks between slaves.



**Figure 7.6** Load balancing using a pipeline structure.

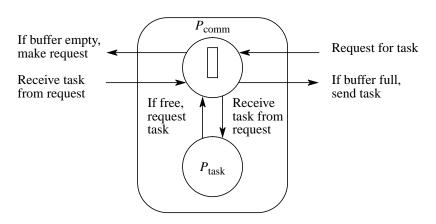
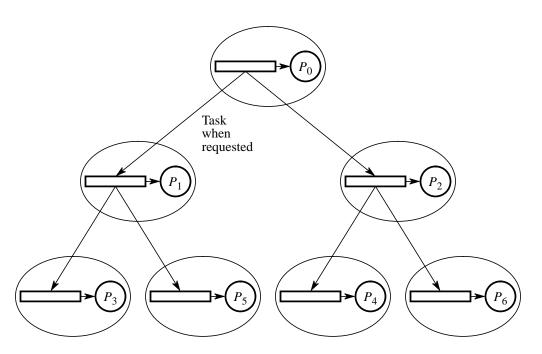
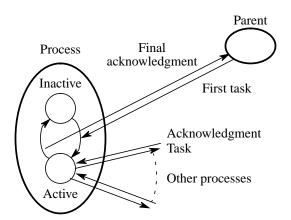


Figure 7.7 Using a communication process in line load balancing.

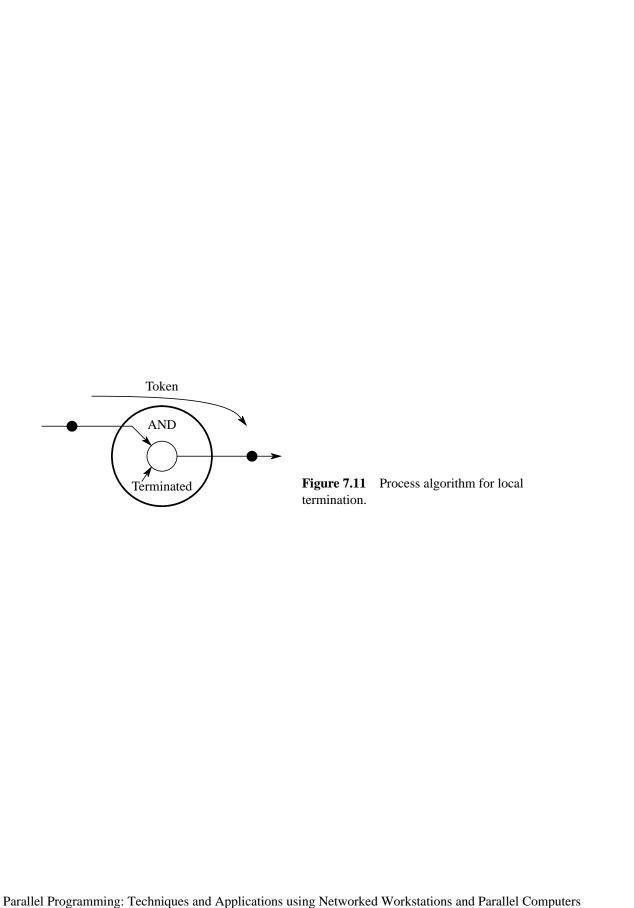


**Figure 7.8** Load balancing using a tree.



**Figure 7.9** Termination using message acknowledgments.

Token passed to next processor when reached local termination condition  $P_0 \qquad P_1 \qquad P_2 \qquad P_{n-1}$ 



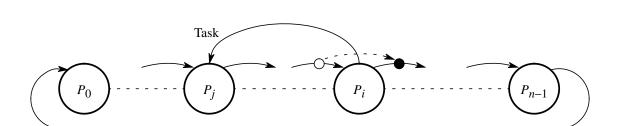
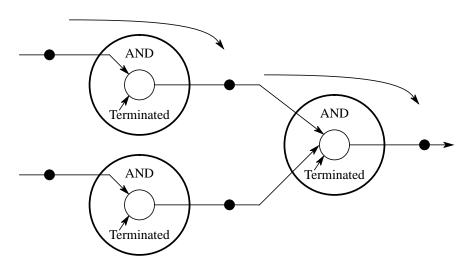


Figure 7.12 Passing task to previous processes.



**Figure 7.13** Tree termination.

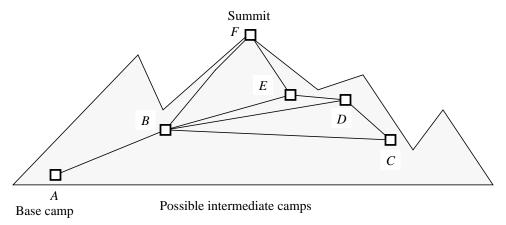
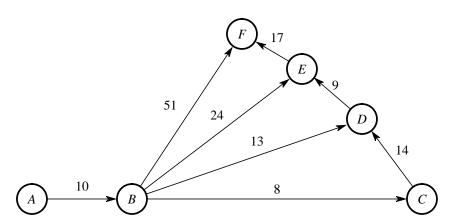
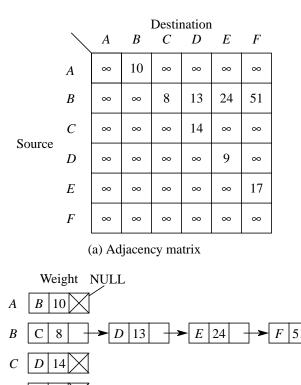


Figure 7.14 Climbing a mountain.



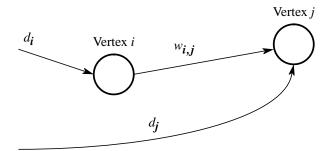
**Figure 7.15** Graph of mountain climb.



(b) Adjacency list

Source

Figure 7.16 Representing a graph.



**Figure 7.17** Moore's shortest-path algorithm.

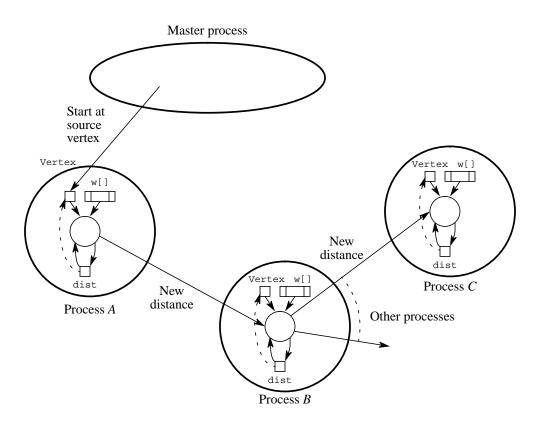
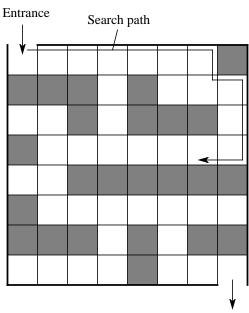
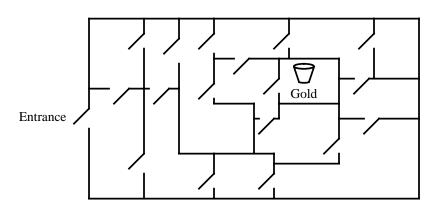


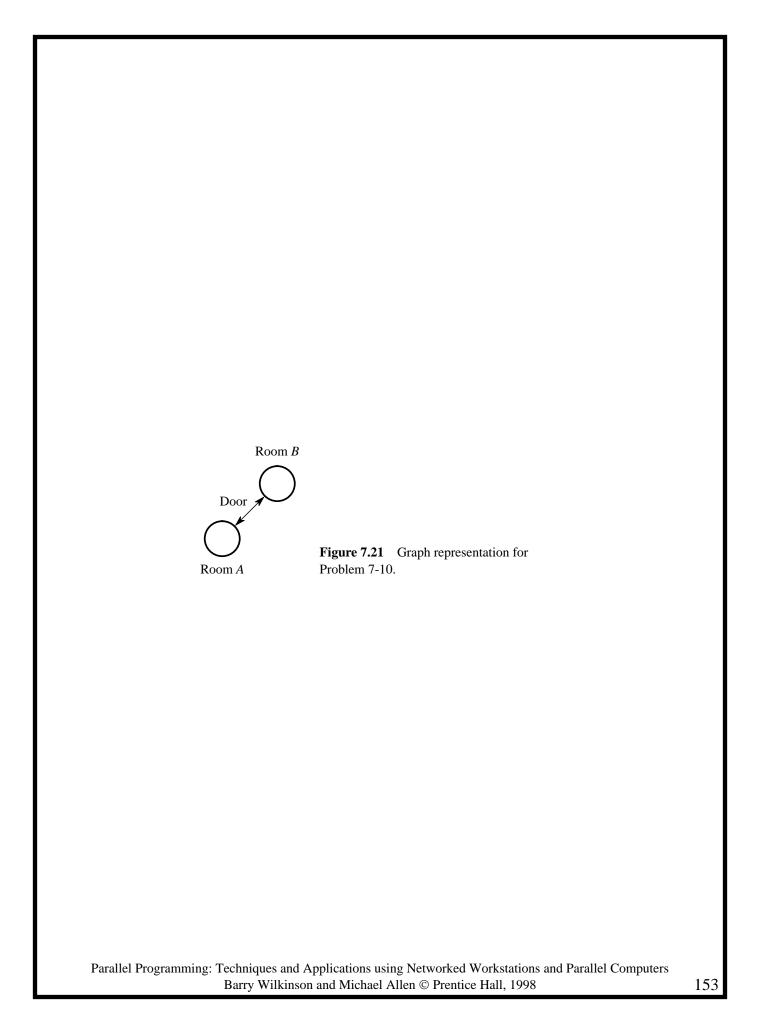
Figure 7.18 Distributed graph search.



Exit **Figure 7.19** Sample maze for Problem 7-9.



**Figure 7.20** Plan of rooms for Problem 7-10.



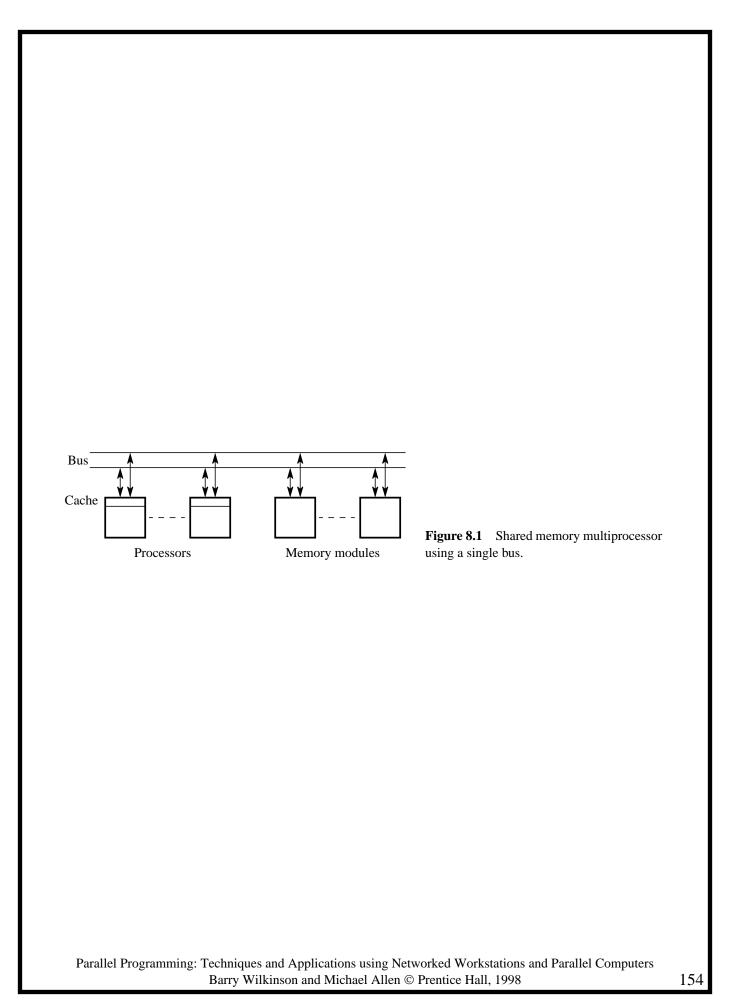


TABLE 8.1 SOME EARLY PARALLEL PROGRAMMING LANGUAGES

Language	Originator/date	Comments	
Concurrent Pascal	Brinch Hansen, 1975 <sup>a</sup>	Extension to Pascal	
Ada	U.S. Dept. of Defense, 1979 <sup>b</sup>	Completely new language	
Modula-P	Bräunl, 1986 <sup>c</sup>	Extension to Modula 2	
C*	Thinking Machines, 1987 <sup>d</sup>	Extension to C for SIMD systems	
Concurrent C	Gehani and Roome, 1989 <sup>e</sup>	Extension to C	
Fortran D	Fox et al., 1990 <sup>f</sup>	Extension to Fortran for data parallel programming	

a. Brinch Hansen, P. (1975), "The Programming Language Concurrent Pascal," *IEEE Trans. Software Eng.*, Vol. 1, No. 2 (June), pp. 199–207.

b. U.S. Department of Defense (1981), "The Programming Language Ada Reference Manual," *Lecture Notes in Computer Science*, No. 106, Springer-Verlag, Berlin.

c. Bräunl, T., R. Norz (1992), *Modula-P User Manual*, Computer Science Report, No. 5/92 (August), Univ. Stuttgart, Germany.

d. Thinking Machines Corp. (1990), *C\* Programming Guide, Version 6*, Thinking Machines System Documentation.

e. Gehani, N., and W. D. Roome (1989), *The Concurrent C Programming Language*, Silicon Press, New Jersey.

f. Fox, G., S. Hiranandani, K. Kennedy, C. Koelbel, U. Kremer, C. Tseng, and M. Wu (1990), *Fortran D Language Specification*, Technical Report TR90-141, Dept. of Computer Science, Rice University.

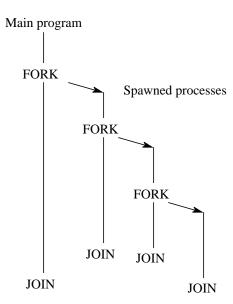
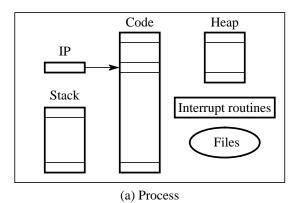
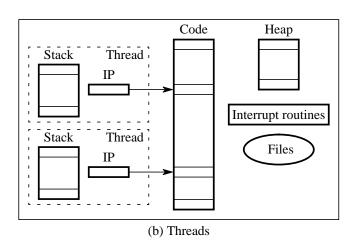


Figure 8.2 FORK-JOIN construct.





**Figure 8.3** Differences between a process and threads.

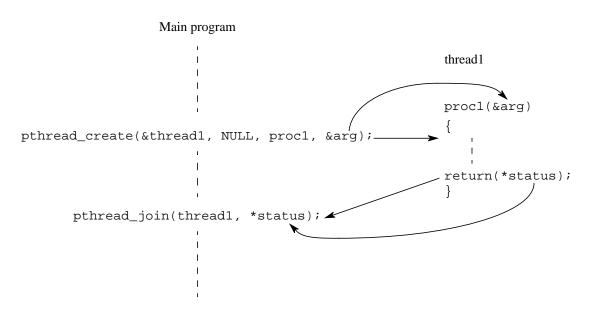


Figure 8.4 pthread\_create() and pthread\_join().

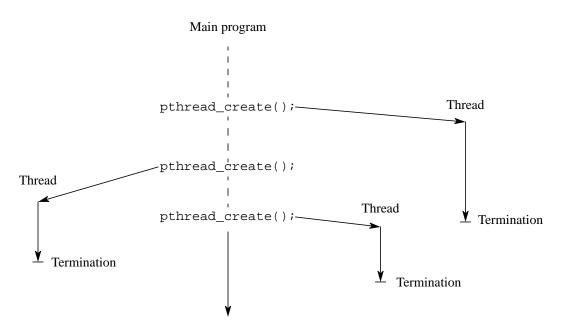
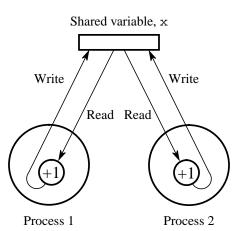
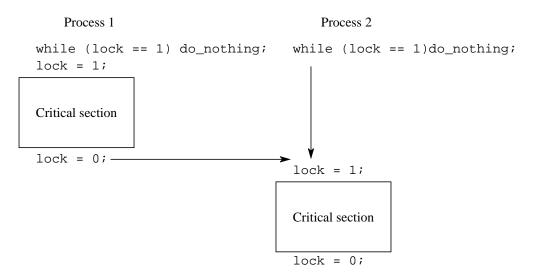


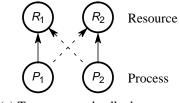
Figure 8.5 Detached threads.



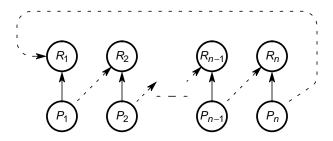
**Figure 8.6** Conflict in accessing shared variable.



**Figure 8.7** Control of critical sections through busy waiting.



(a) Two-process deadlock



(b) *n*-process deadlock

Figure 8.8 Deadlock (deadly embrace).

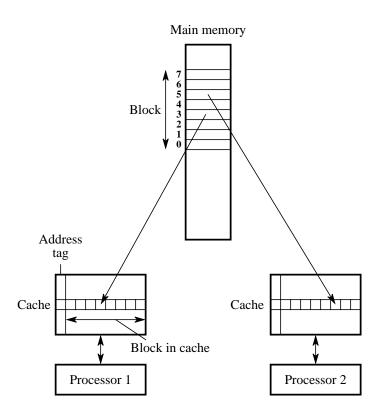
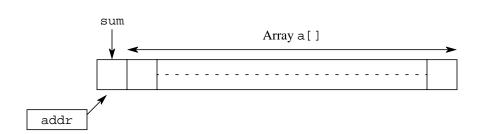
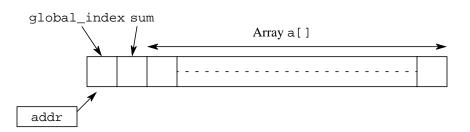


Figure 8.9 False sharing in caches.



**Figure 8.10** Shared memory locations for Section 8.4.1 program example.



**Figure 8.11** Shared memory locations for Section 8.4.2 program example.

TABLE 8.2 LOGIC CIRCUIT DESCRIPTION FOR FIGURE 8.12

Gate	Function	Input 1	Input 2	Output
1	AND	Test1	Test2	Gate1
2	NOT	Gate1		Output1
3	OR	Test3	Gate1	Output2

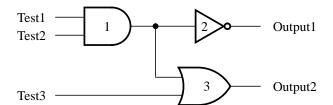
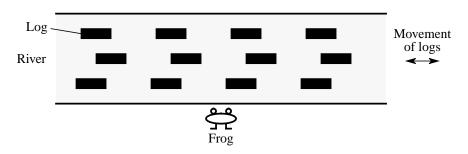
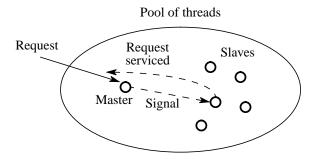


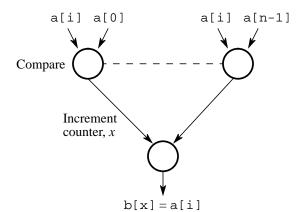
Figure 8.12 Sample logic circuit.



**Figure 8.13** River and frog for Problem 8-23.



**Figure 8.14** Thread pool for Problem 8-24.



**Figure 9.1** Finding the rank in parallel.

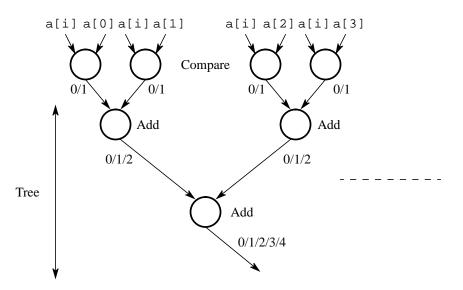
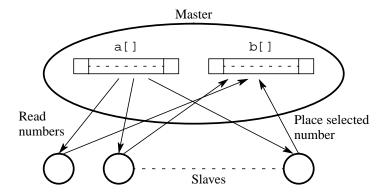
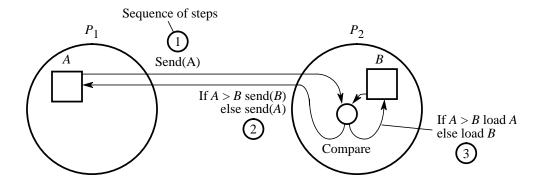


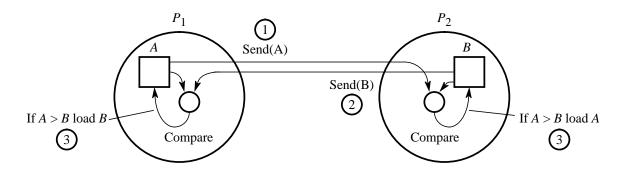
Figure 9.2 Parallelizing the rank computation.



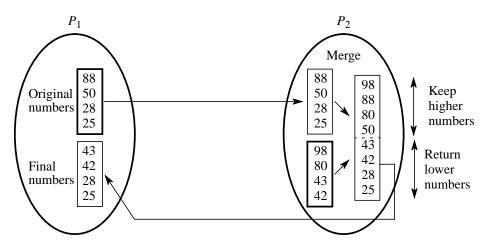
**Figure 9.3** Rank sort using a master and slaves.



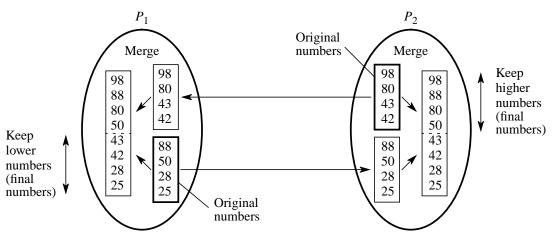
**Figure 9.4** Compare and exchange on a message-passing system — Version 1.



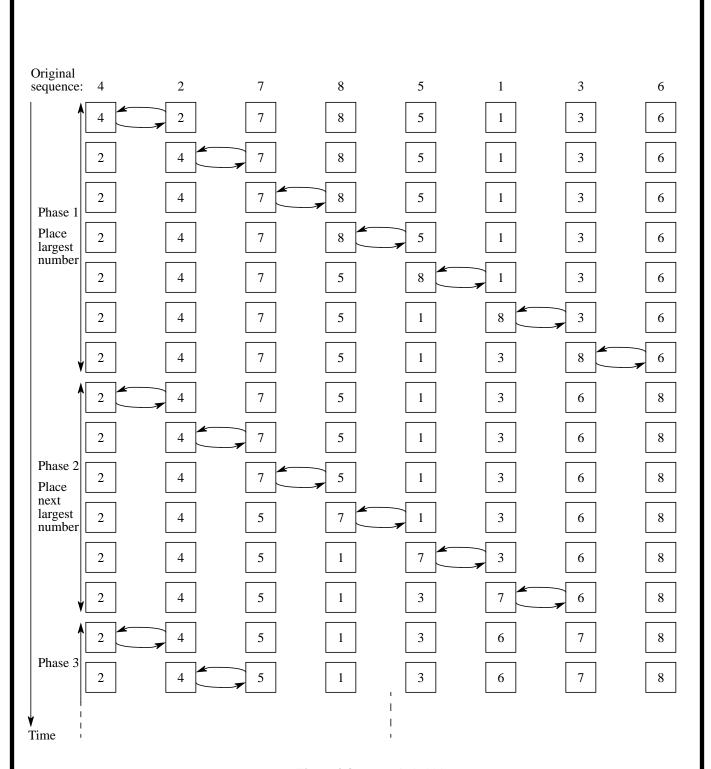
**Figure 9.5** Compare and exchange on a message-passing system — Version 2.



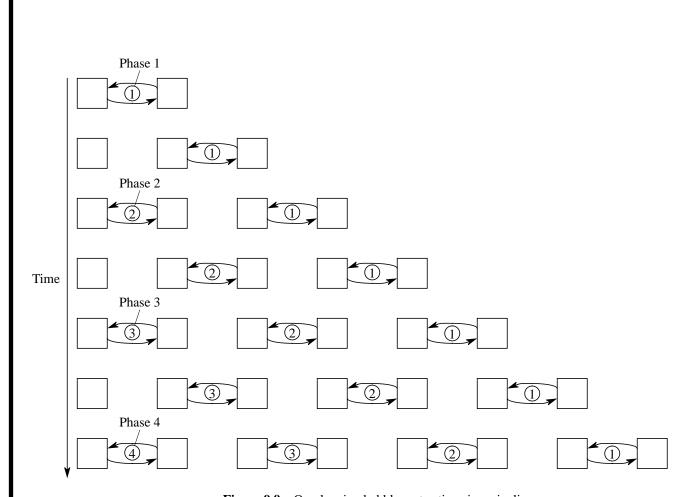
**Figure 9.6** Merging two sublists — Version 1.



**Figure 9.7** Merging two sublists — Version 2.



**Figure 9.8** Steps in bubble sort.



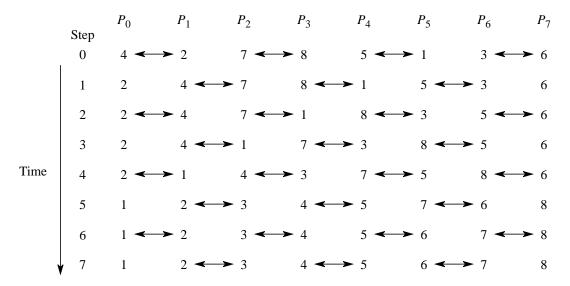


Figure 9.10 Odd-even transposition sort sorting eight numbers.

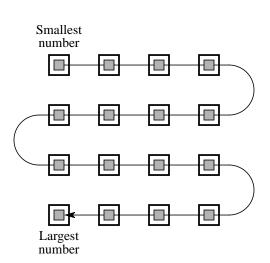


Figure 9.11 Snakelike sorted list.

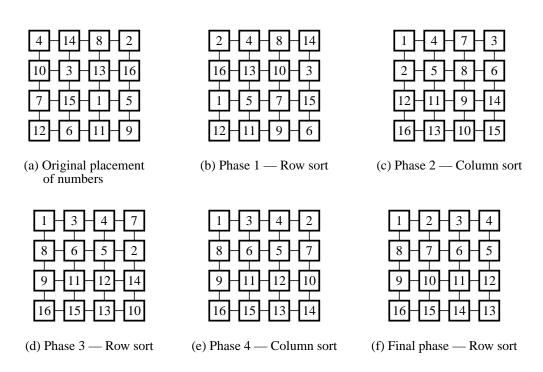
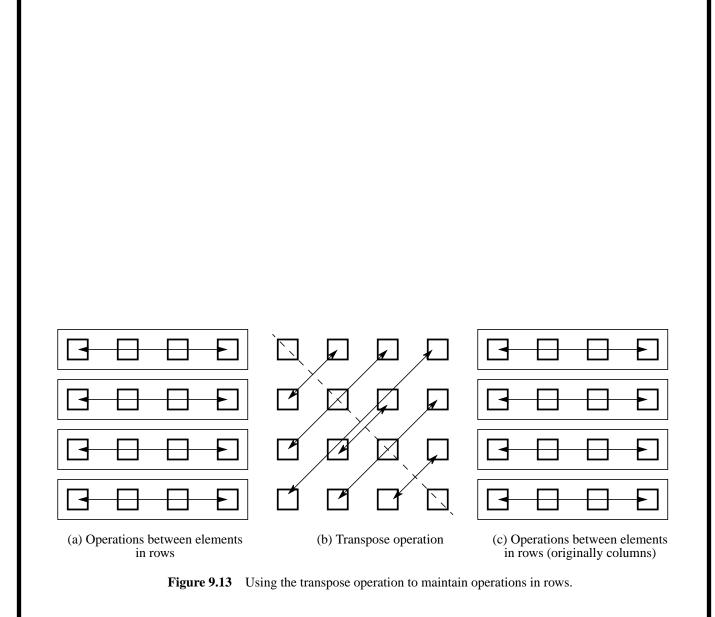


Figure 9.12 Shearsort.



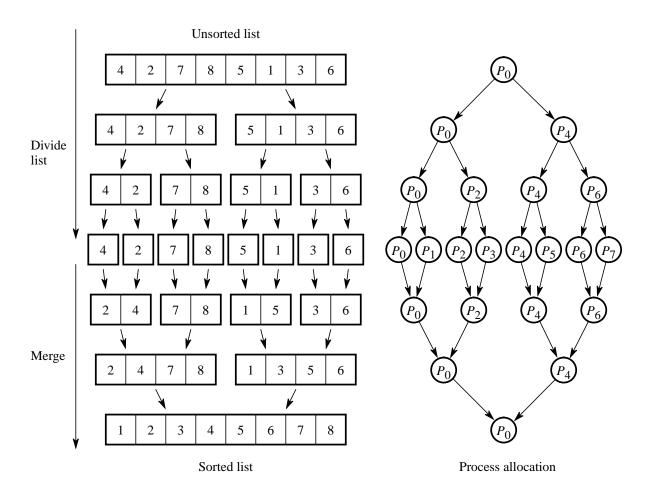


Figure 9.14 Mergesort using tree allocation of processes.

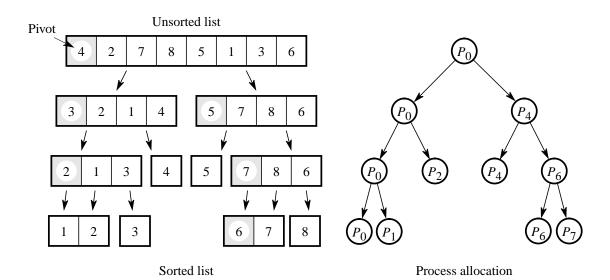


Figure 9.15 Quicksort using tree allocation of processes.

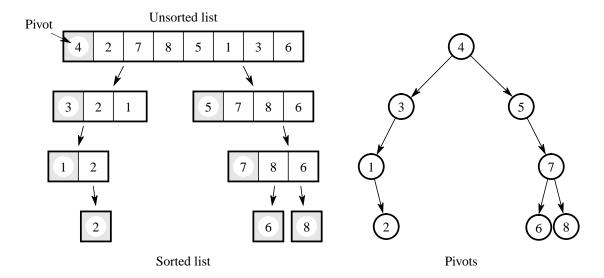
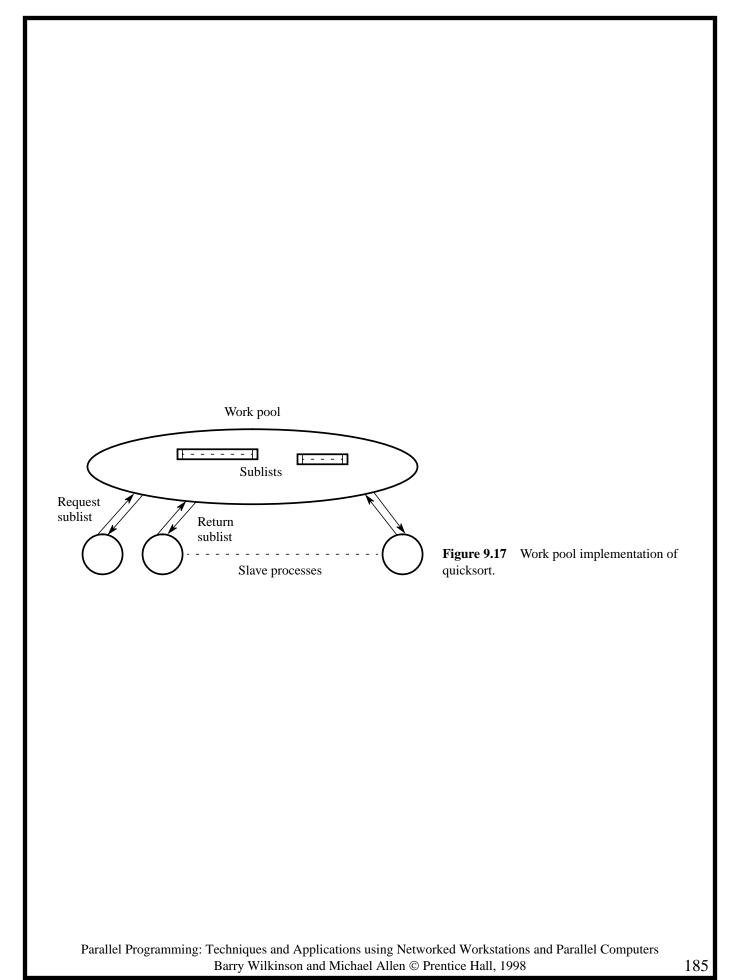


Figure 9.16 Quicksort showing pivot withheld in processes.



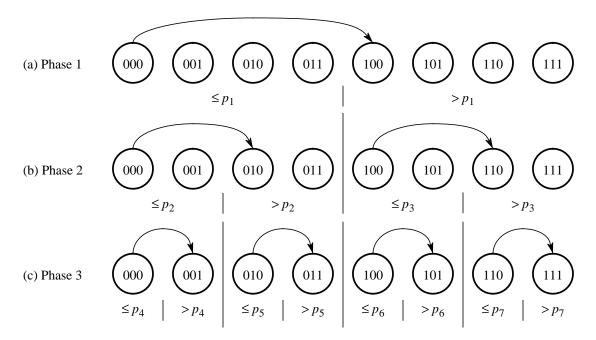


Figure 9.18 Hypercube quicksort algorithm when the numbers are originally in node 000.

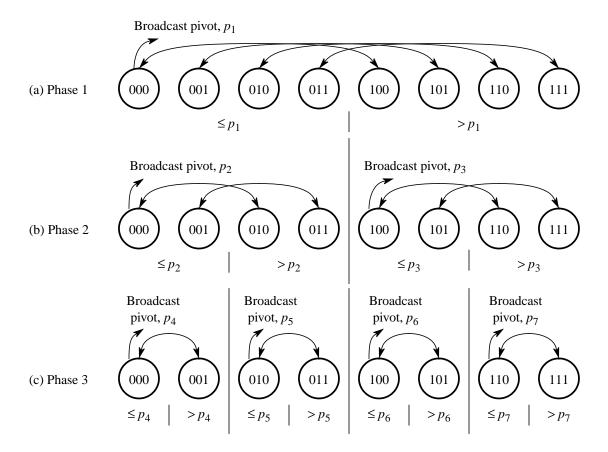
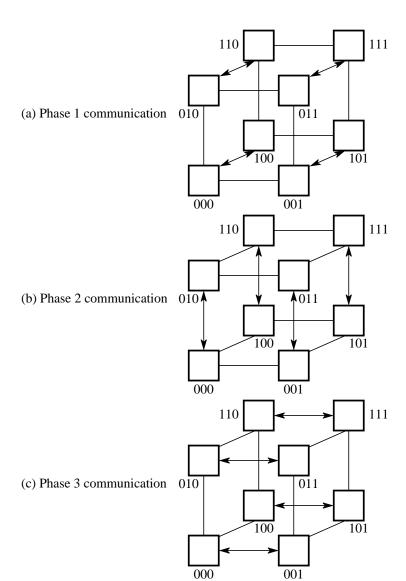


Figure 9.19 Hypercube quicksort algorithm when numbers are distributed among nodes.



**Figure 9.20** Hypercube quicksort communication.

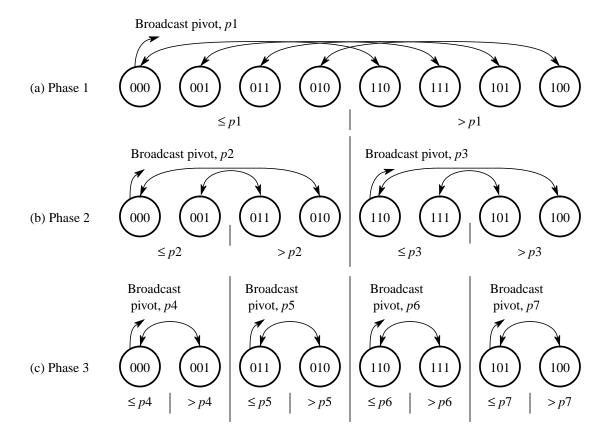
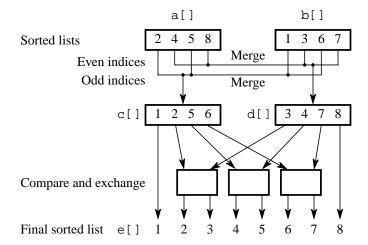


Figure 9.21 Quicksort hypercube algorithm with Gray code ordering.



**Figure 9.22** Odd-even merging of two sorted lists.

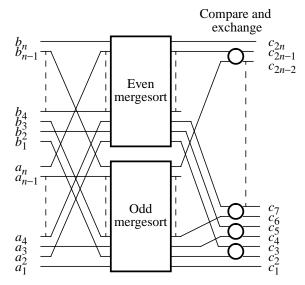


Figure 9.23 Odd-even mergesort.

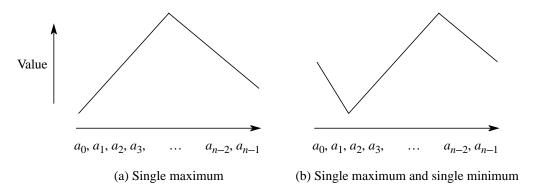
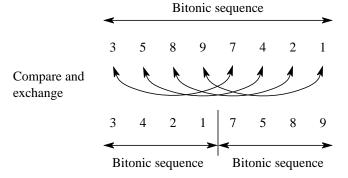


Figure 9.24 Bitonic sequences.



**Figure 9.25** Creating two bitonic sequences from one bitonic sequence.

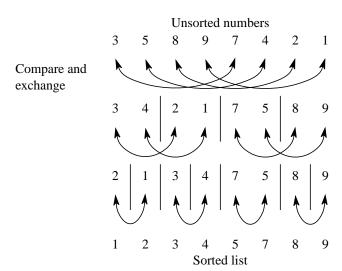


Figure 9.26 Sorting a bitonic sequence.

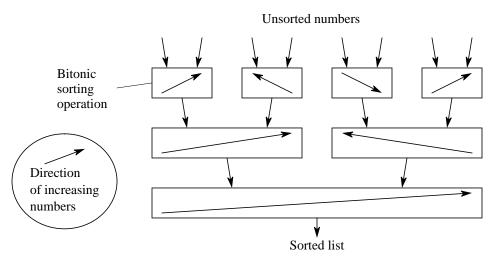
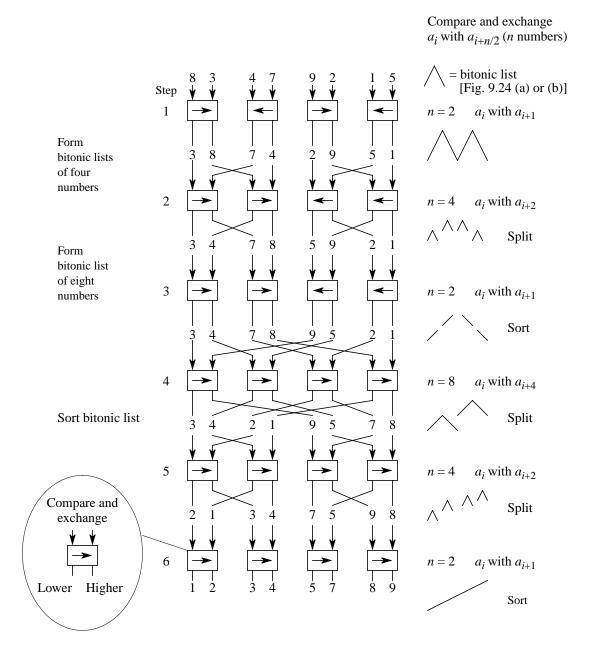
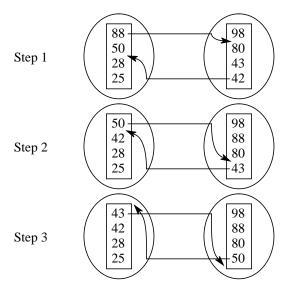


Figure 9.27 Bitonic mergesort.

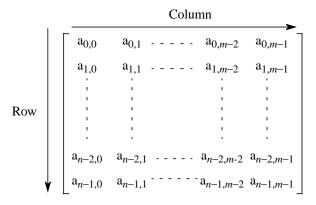


**Figure 9.28** Bitonic mergesort on eight numbers.



Terminates when insertions at top/bottom of lists

**Figure 9.29** Compare-and-exchange algorithm for Problem 9-5.



**Figure 10.1** An  $n \times m$  matrix.

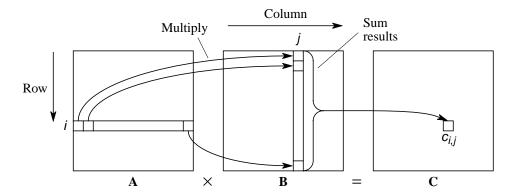
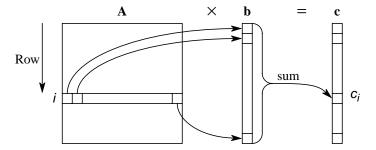


Figure 10.2 Matrix multiplication,  $C = A \times B$ .



 $\begin{aligned} & \textbf{Figure 10.3} & & \text{Matrix-vector multiplication} \\ & c = A \times b. \end{aligned}$ 

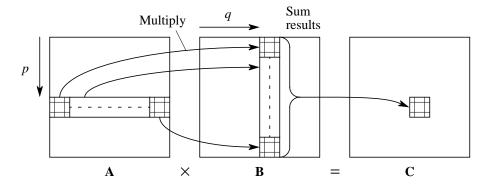


Figure 10.4 Block matrix multiplication.

$$\begin{bmatrix} a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} \\ a_{1,0} & a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,0} & a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,0} & a_{3,1} & a_{3,2} & a_{3,3} \end{bmatrix} \times \begin{bmatrix} b_{0,0} & b_{0,1} & b_{0,2} & b_{0,3} \\ b_{1,0} & b_{1,1} & b_{1,2} & b_{1,3} \\ b_{2,0} & b_{2,1} & b_{2,2} & b_{2,3} \\ b_{3,0} & b_{3,1} & b_{3,2} & b_{3,3} \end{bmatrix}$$

$$(a) \text{ Matrices}$$

$$A_{0,0} \quad B_{0,0} \quad A_{0,1} \quad B_{1,0}$$

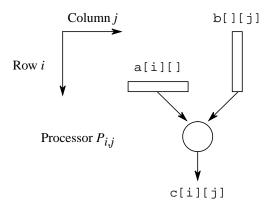
$$\begin{bmatrix} a_{0,0} & a_{0,1} \\ a_{1,0} & a_{1,1} \end{bmatrix} \times \begin{bmatrix} b_{0,0} & b_{0,1} \\ b_{1,0} & b_{1,1} \end{bmatrix} + \begin{bmatrix} a_{0,2} & a_{0,3} \\ a_{1,2} & a_{1,3} \end{bmatrix} \times \begin{bmatrix} b_{2,0} & b_{2,1} \\ b_{3,0} & b_{3,1} \end{bmatrix}$$

$$= \begin{bmatrix} a_{0,0}b_{0,0} + a_{0,1}b_{1,0} & a_{0,0}b_{0,1} + a_{0,1}b_{1,1} \\ a_{1,0}b_{0,0} + a_{1,1}b_{1,0} & a_{1,0}b_{0,1} + a_{1,1}b_{1,1} \end{bmatrix} + \begin{bmatrix} a_{0,2}b_{2,0} + a_{0,3}b_{3,0} & a_{0,2}b_{2,1} + a_{0,3}b_{3,1} \\ a_{1,2}b_{2,0} + a_{1,3}b_{3,0} & a_{1,2}b_{2,1} + a_{1,3}b_{3,1} \end{bmatrix}$$

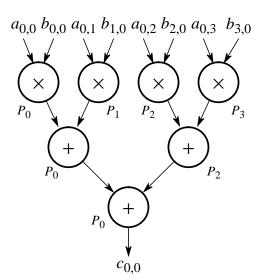
$$= \begin{bmatrix} a_{0,0}b_{0,0} + a_{0,1}b_{1,0} + a_{0,2}b_{2,0} + a_{0,3}b_{3,0} & a_{0,0}b_{0,1} + a_{0,1}b_{1,1} + a_{0,2}b_{2,1} + a_{0,3}b_{3,1} \\ a_{1,0}b_{0,0} + a_{1,1}b_{1,0} + a_{0,2}b_{2,0} + a_{1,3}b_{3,0} & a_{1,0}b_{0,1} + a_{1,1}b_{1,1} + a_{0,2}b_{2,1} + a_{0,3}b_{3,1} \\ a_{1,0}b_{0,0} + a_{1,1}b_{1,0} + a_{1,2}b_{2,0} + a_{1,3}b_{3,0} & a_{1,0}b_{0,1} + a_{1,1}b_{1,1} + a_{1,2}b_{2,1} + a_{1,3}b_{3,1} \end{bmatrix}$$

$$= C_{0,0}$$
(b) Multiplying  $A_{0,0} \times B_{0,0}$  to obtain  $C_{0,0}$ 

**Figure 10.5** Submatrix multiplication.



**Figure 10.6** Direct implementation of matrix multiplication.



**Figure 10.7** Accumulation using a tree construction.

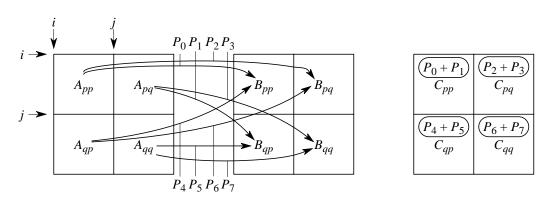
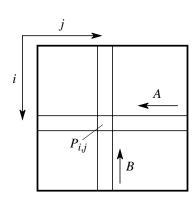
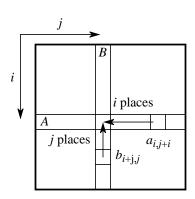


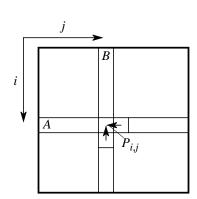
Figure 10.8 Submatrix multiplication and summation.



**Figure 10.9** Movement of A and B elements.



**Figure 10.10** Step 2 — Alignment of elements of A and B.



**Figure 10.11** Step 4 — One-place shift of elements of A and B.

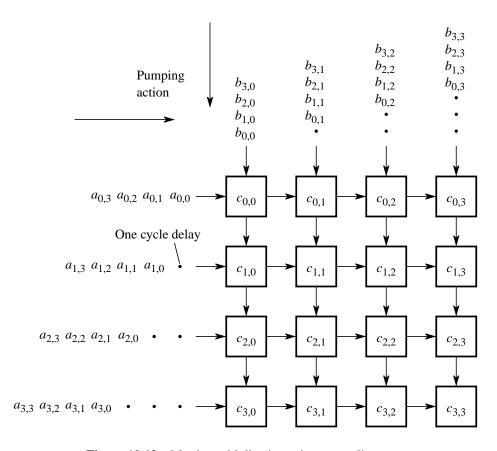
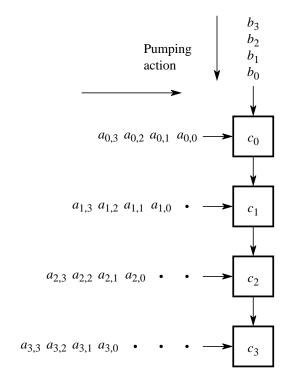


Figure 10.12 Matrix multiplication using a systolic array.



**Figure 10.13** Matrix-vector multiplication using a systolic array.

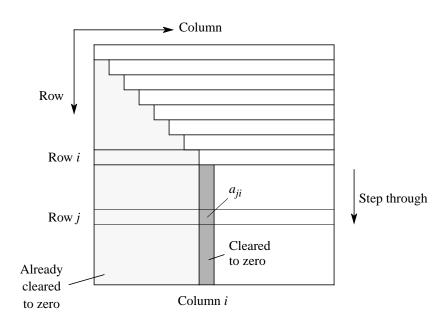


Figure 10.14 Gaussian elimination.

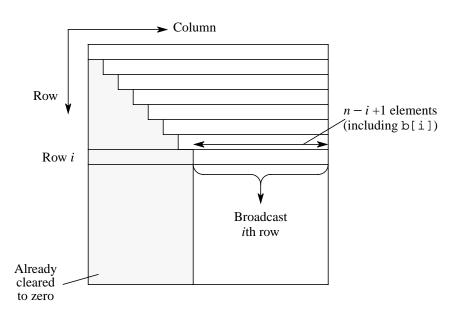
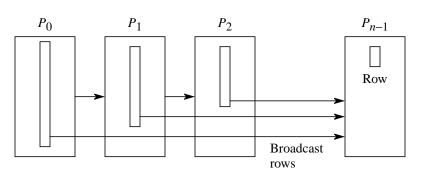
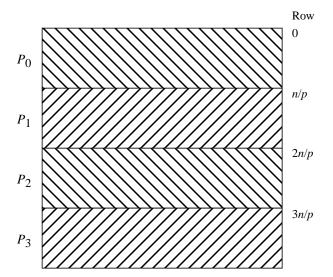


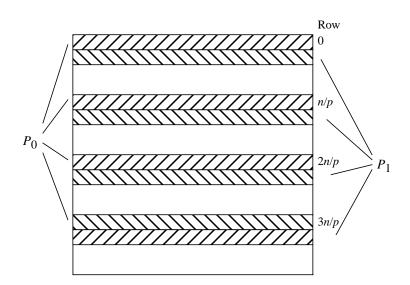
Figure 10.15 Broadcast in parallel implementation of Gaussian elimination.



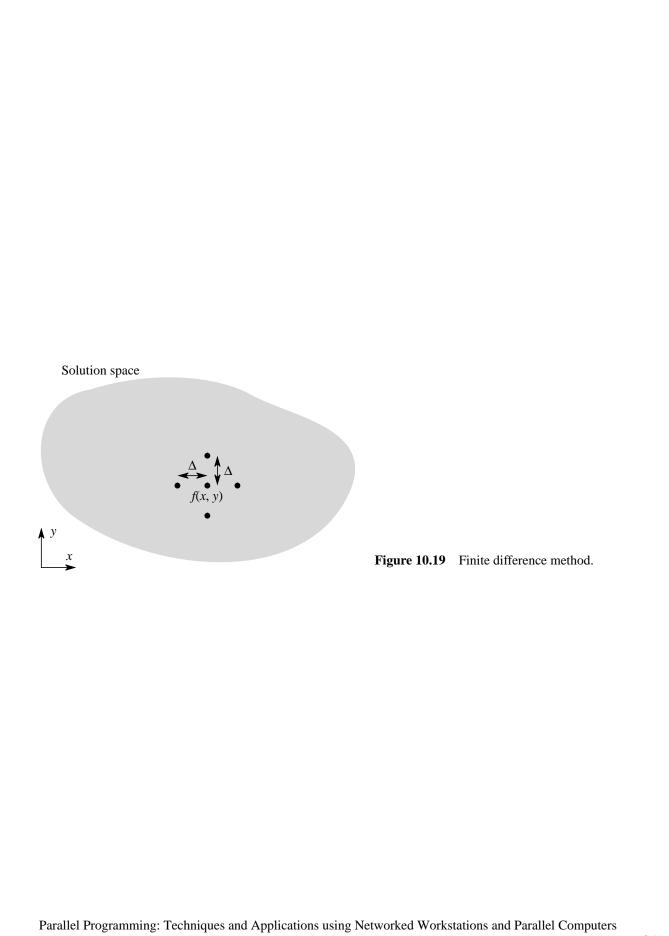
**Figure 10.16** Pipeline implementation of Gaussian elimination.



**Figure 10.17** Strip partitioning.



**Figure 10.18** Cyclic partitioning to equalize workload.



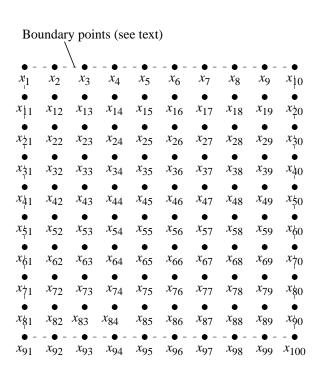


Figure 10.20 Mesh of points numbered in natural order.

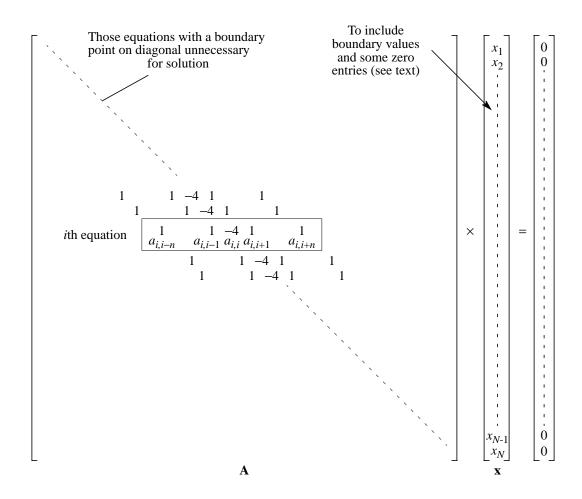


Figure 10.21 Sparse matrix for Laplace's equation.

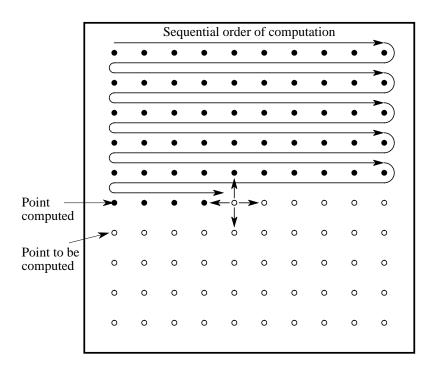
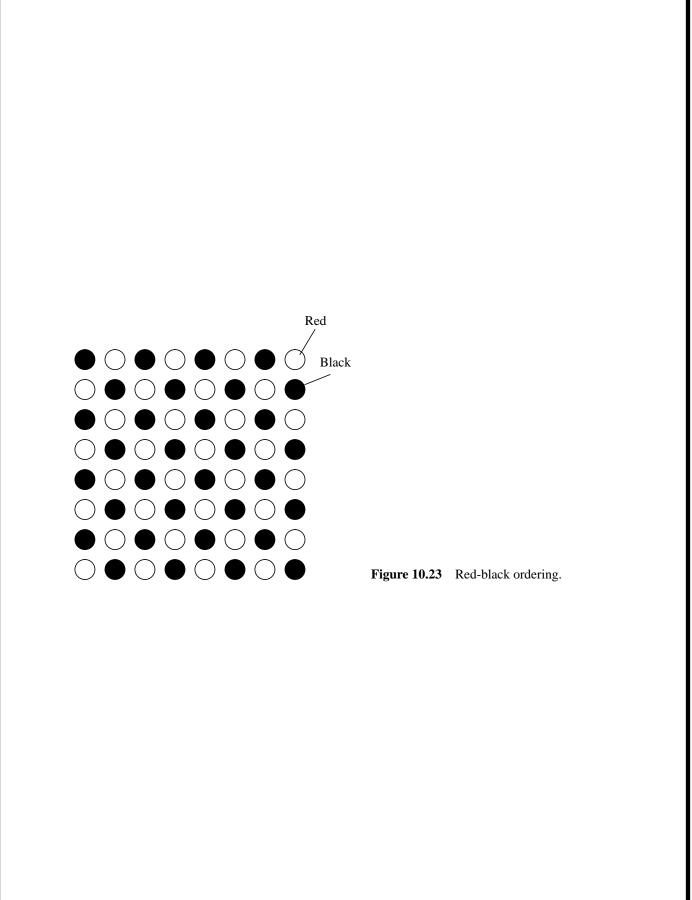
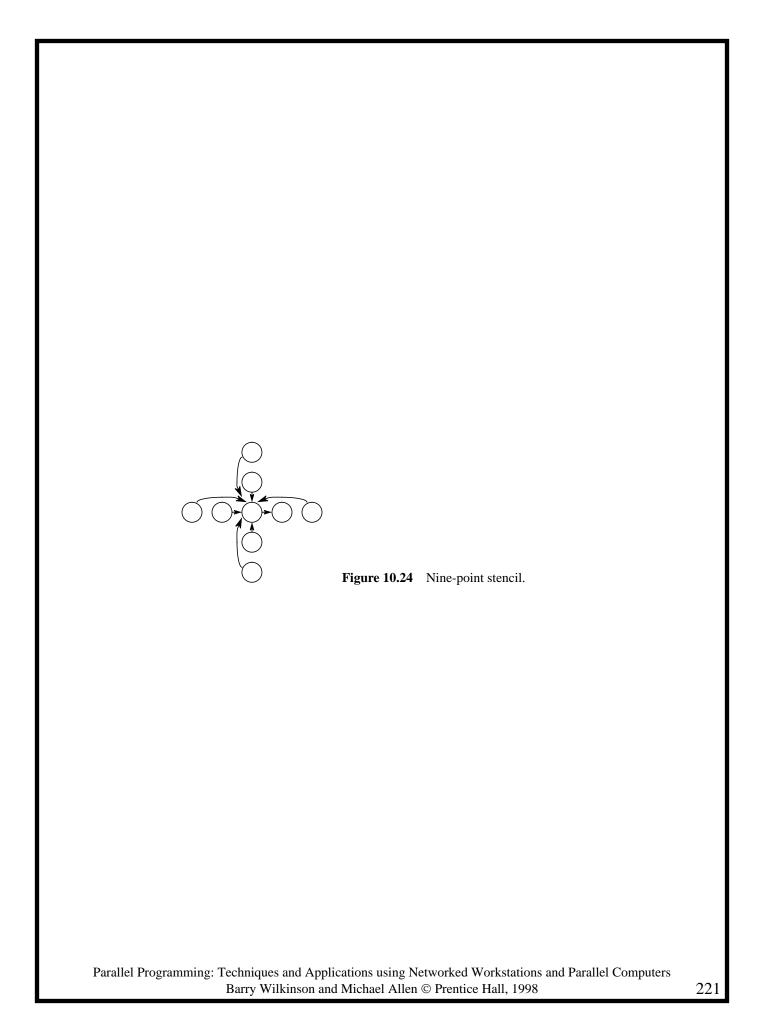
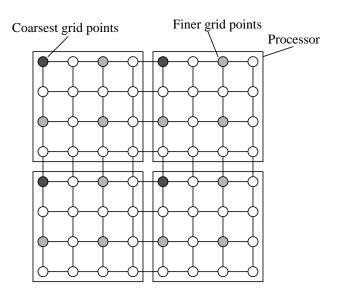


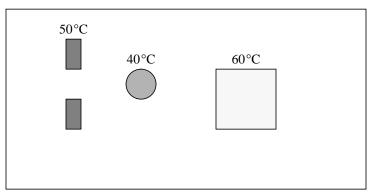
Figure 10.22 Gauss-Seidel relaxation with natural order, computed sequentially.







**Figure 10.25** Multigrid processor allocation.



Ambient temperature at edges of board = 20 °C

**Figure 10.26** Printed circuit board for Problem 10-18.

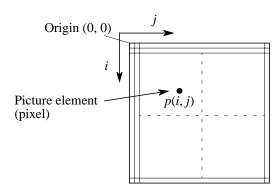


Figure 11.1 Pixmap.

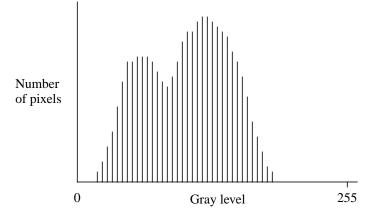
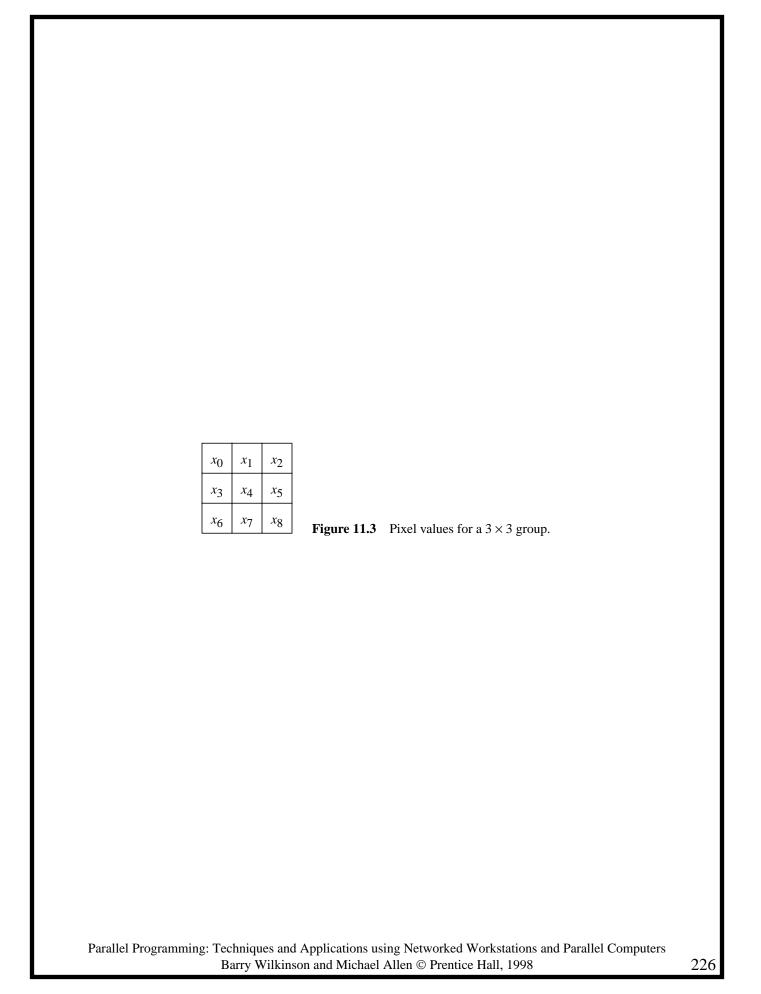
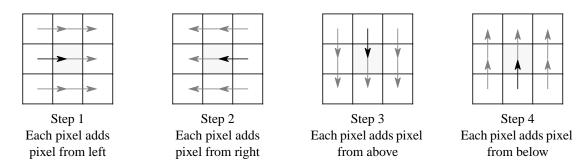


Figure 11.2 Image histogram.





**Figure 11.4** Four-step data transfer for the computation of mean.

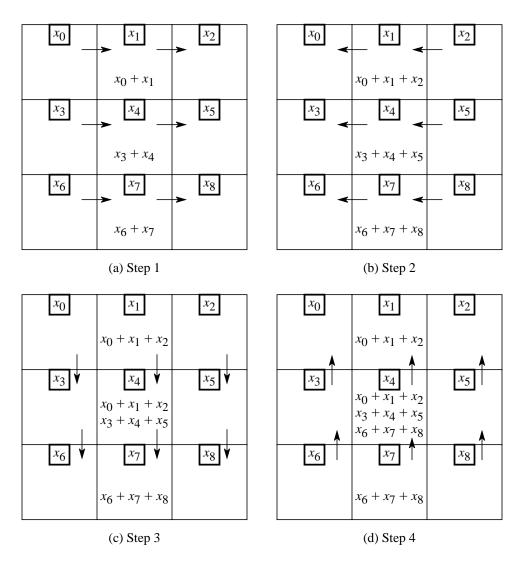


Figure 11.5 Parallel mean data accumulation.

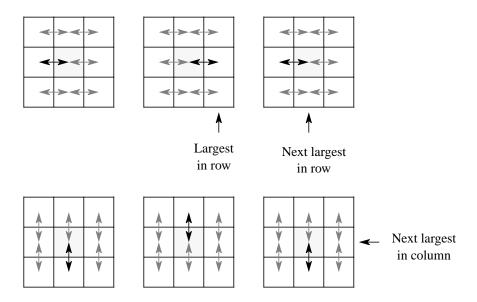
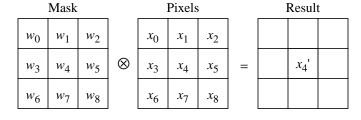


Figure 11.6 Approximate median algorithm requiring six steps.



**Figure 11.7** Using a  $3 \times 3$  weighted mask.

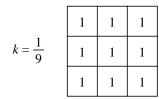


Figure 11.8 Mask to compute mean.

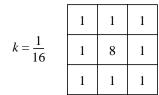
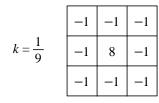
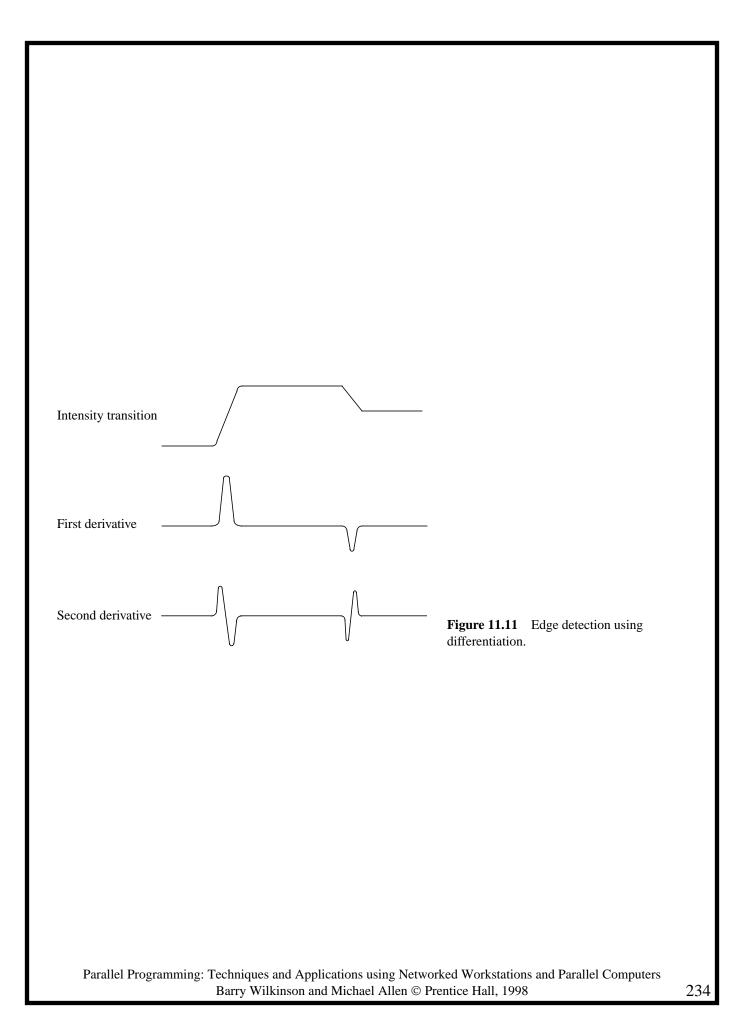
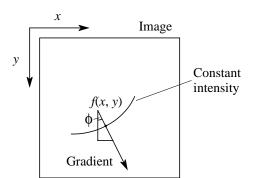


Figure 11.9 A noise reduction mask.

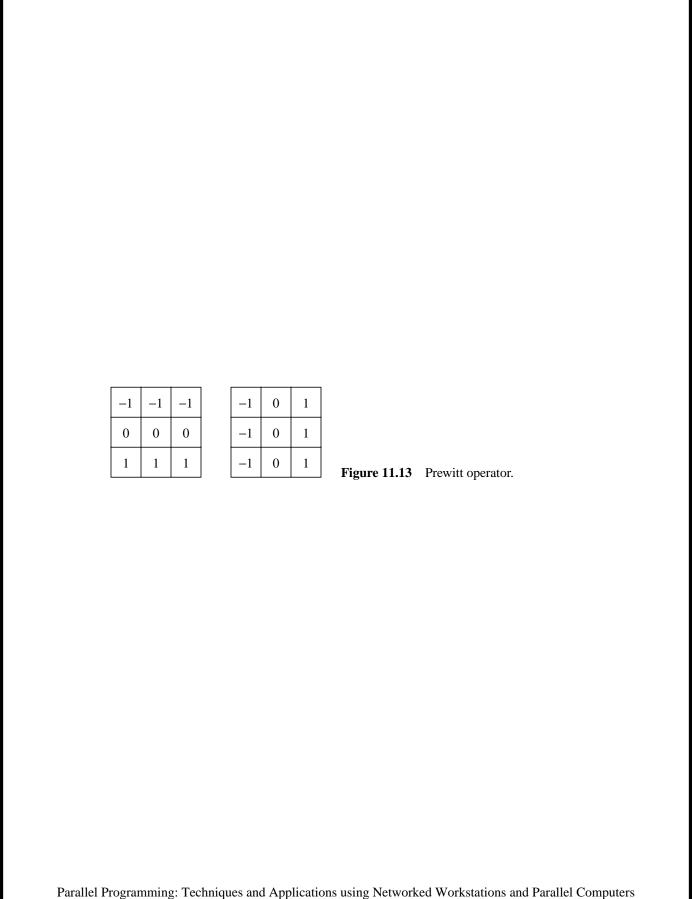


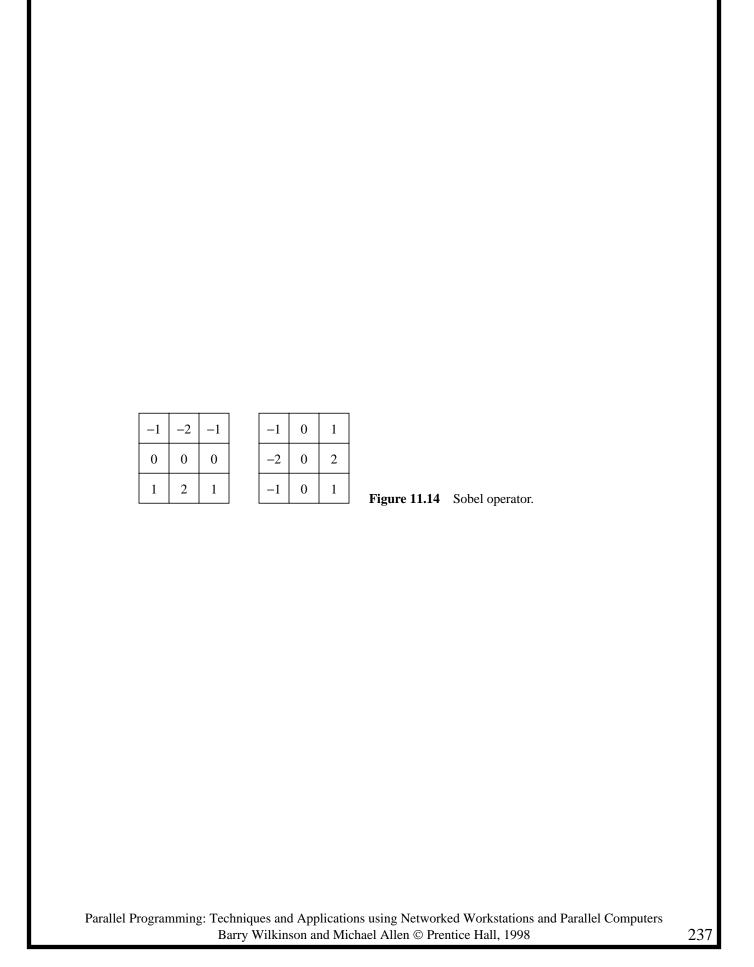
**Figure 11.10** High-pass sharpening filter mask.





**Figure 11.12** Gray level gradient and direction.





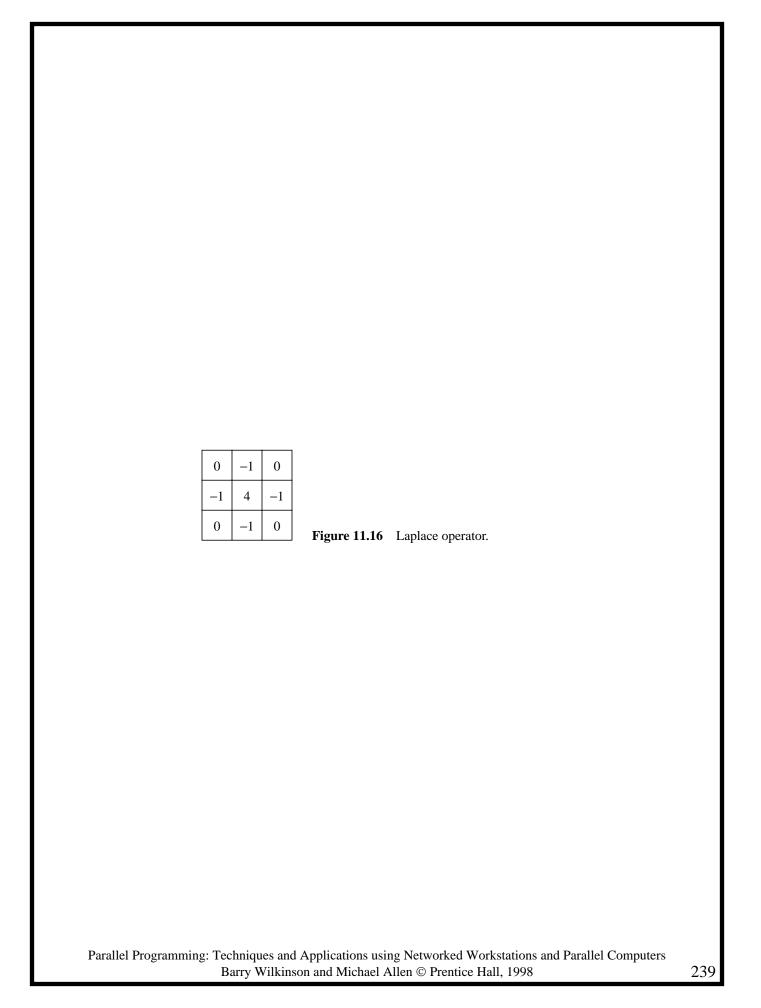


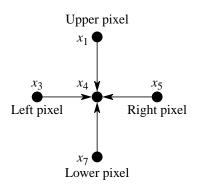


(a) Original image (Annabel)

(b) Effect of Sobel operator

 $Figure \ 11.15 \quad \hbox{Edge detection with Sobel operator.}$ 

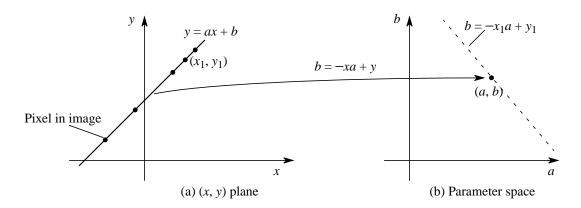




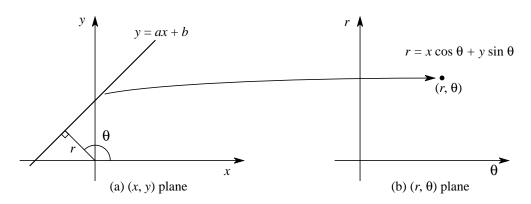
**Figure 11.17** Pixels used in Laplace operator.



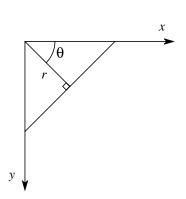
Figure 11.18 Effect of Laplace operator.



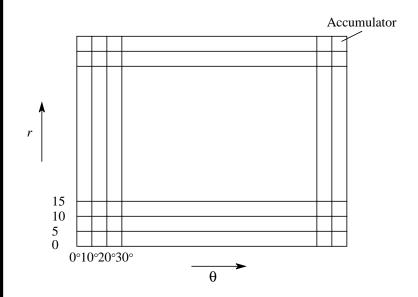
**Figure 11.19** Mapping a line into (a, b) space.



**Figure 11.20** Mapping a line into  $(r, \theta)$  space.



**Figure 11.21** Normal representation using image coordinate system.



**Figure 11.22** Accumulators,  $acc[r][\theta]$ , for the Hough transform.

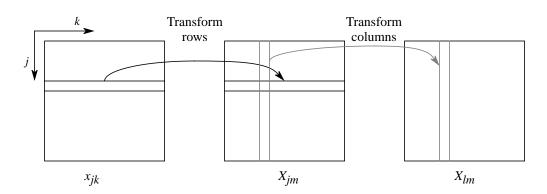


Figure 11.23 Two-dimensional DFT.

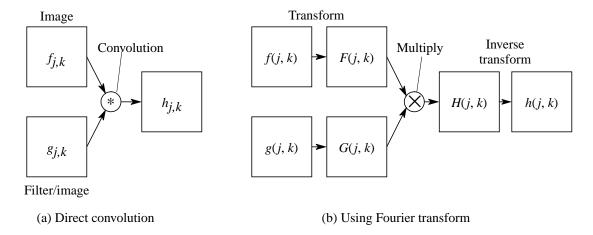
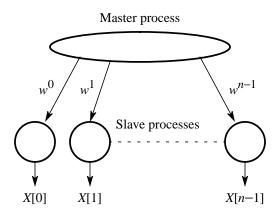
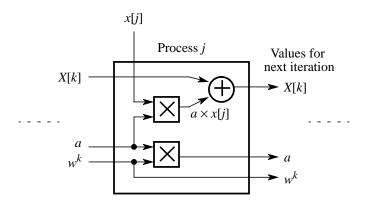


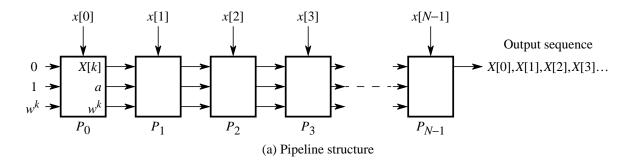
Figure 11.24 Convolution using Fourier transforms.

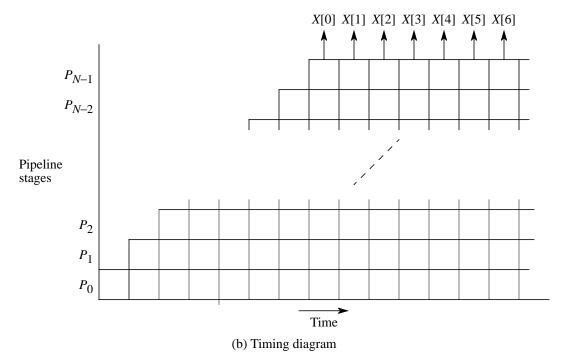


**Figure 11.25** Master-slave approach for implementing the DFT directly.

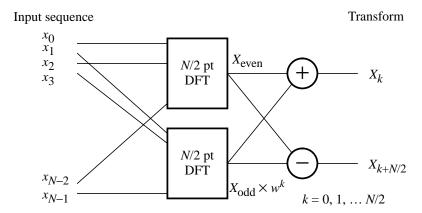


**Figure 11.26** One stage of a pipeline implementation of DFT algorithm.

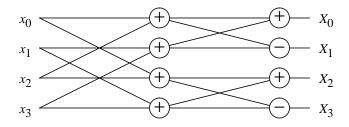




**Figure 11.27** Discrete Fourier transform with a pipeline.



**Figure 11.28** Decomposition of *N*-point DFT into two *N*/2-point DFTs.



**Figure 11.29** Four-point discrete Fourier transform.

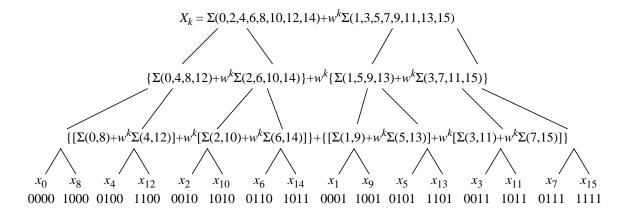


Figure 11.30 Sixteen-point DFT decomposition.

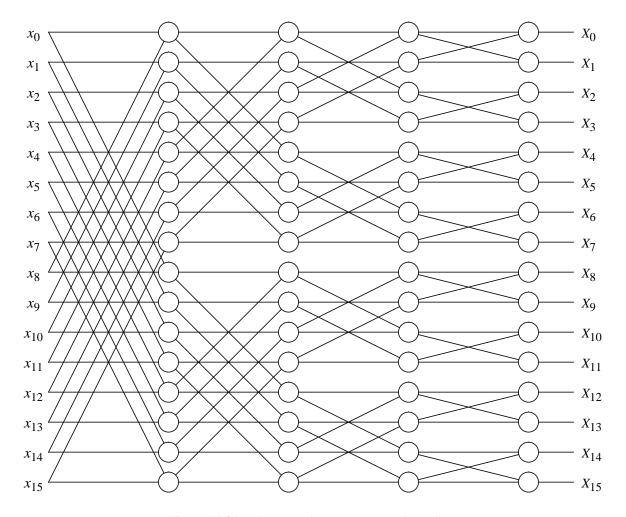


Figure 11.31 Sixteen-point FFT computational flow.

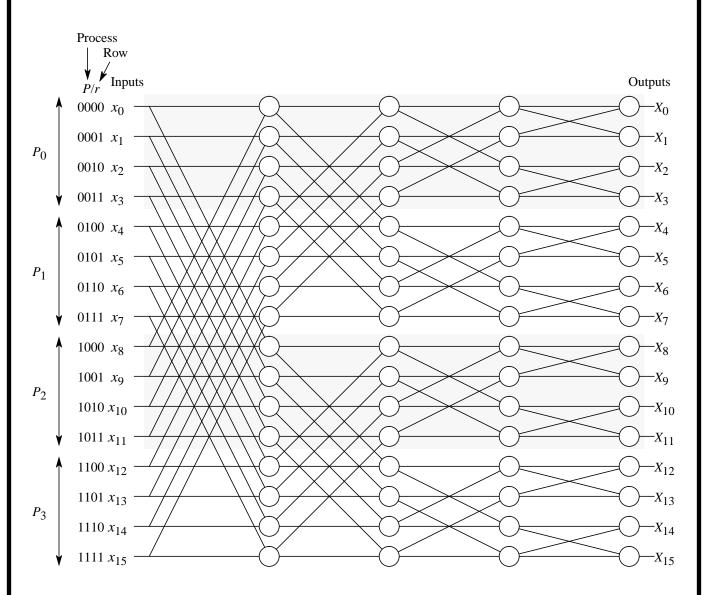
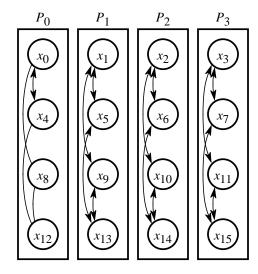
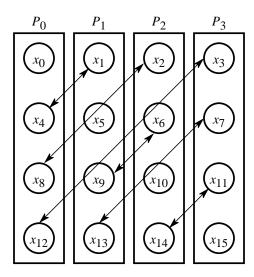


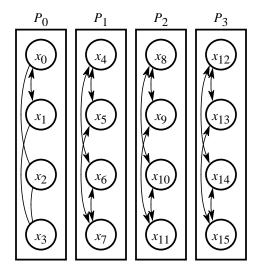
Figure 11.32 Mapping processors onto 16-point FFT computation.



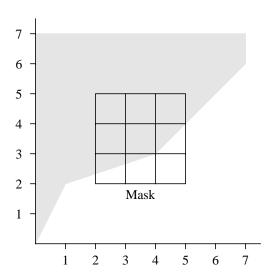
**Figure 11.33** FFT using transpose algorithm — first two steps.



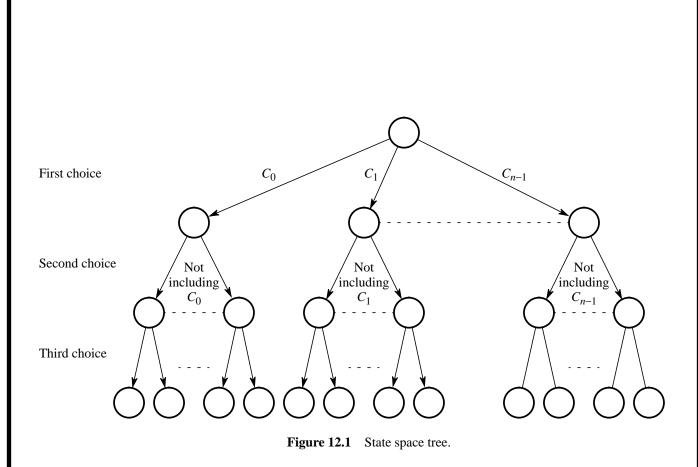
**Figure 11.34** Transposing array for transpose algorithm.



**Figure 11.35** FFT using transpose algorithm — last two steps.



**Figure 11.36** Image for Problem 11-3.



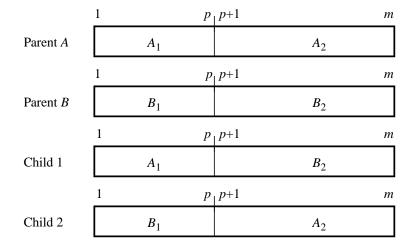
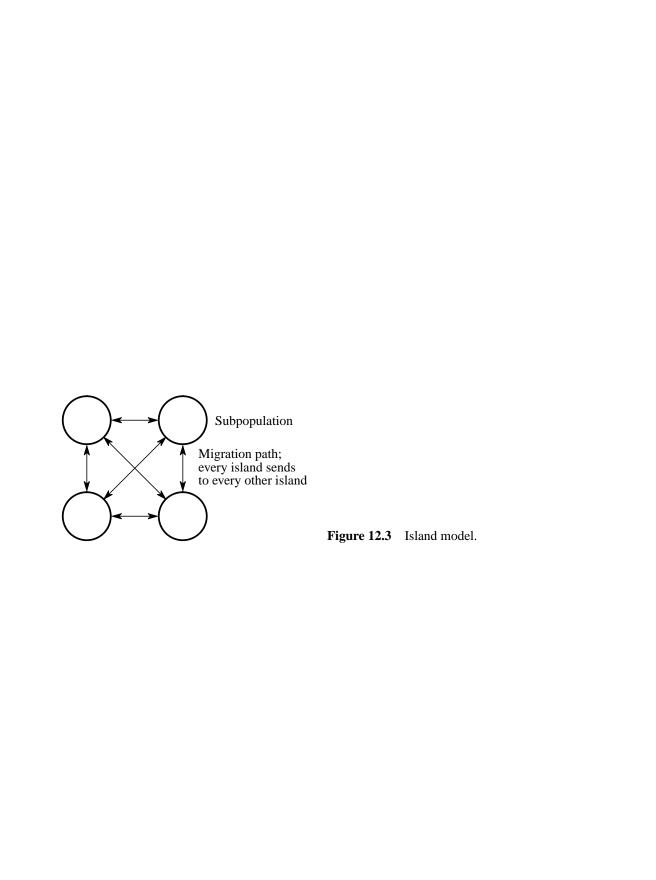
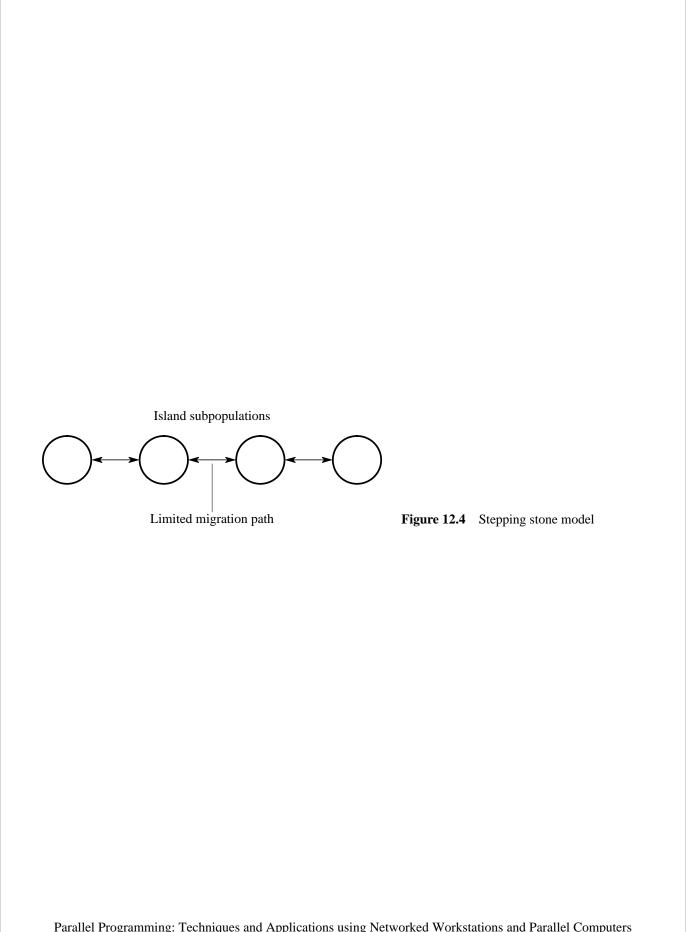


Figure 12.2 Single-point crossover.





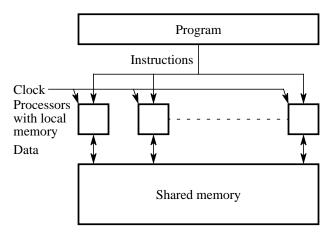


Figure D.1 PRAM model.

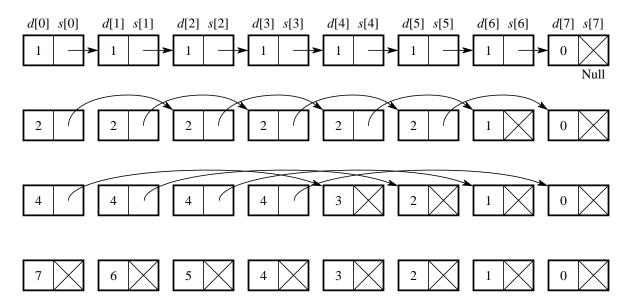
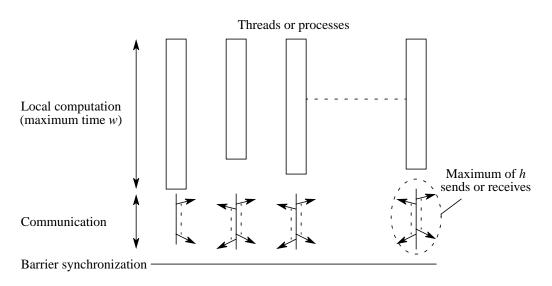
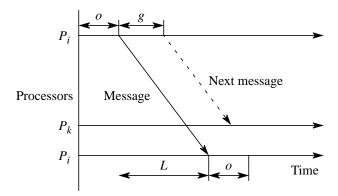


Figure D.2 List ranking by pointer jumping.



**Figure D.3** A view of the bulk synchronous parallel model.



**Figure D.4** LogP parameters.

