Optimal Policies in Energy Harvesting Two-Way Channels with Processing Costs

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***Abstract*— We consider a two-way communication channel in which both users rely solely on energy harvested from nature. Each user incurs a processing cost per unit time as long as it communicates; that is, each user's energy consumption includes energy spent for transmission and energy spent for processing. We maximize the sum throughput by a given deadline subject to energy causality constraints. We first show that the optimal power policy is bursty; the two users communicate only during a portion of the time that is uniquely determined by their available energies and processing costs. We show that it is optimal for the two users to be fully synchronized; they turn on and exchange data during the same portion of time, and then turn off together. We first solve the single energy arrival case, and then extend it to solve the multiple energy arrival throughput maximization problem. We show that it is optimal for the users to communicate in a *deferred* fashion; users postpone their energy consumption to utilize later time slots first. We present an algorithm that gives the optimal deferred policy by iteratively applying a modified version of the single energy arrival result in a backward manner.**

I. Introduction

We consider an energy harvesting communication system where users harvest energy from nature over the course of communication to sustain their operation, and energy is consumed for data transmission and processing costs: each user incurs a processing cost per unit time for the duration that it communicates. Our goal is to design an optimal transmission policy that maximizes the total throughput under energy causality constraints taking into account processing costs.

Recent literature on energy harvesting communications has considered a wide variety of system models and determined the corresponding optimum energy management policies. Initial references [1]–[4] characterize the optimal transmit policies for a single-user channel with and without fading, and for infinite and finite-sized batteries. This line of research has been extended to multiple access channels [5], [6], broadcast channels [7]–[9], interference channels [10], cooperative multiple access channels [11], two-hop channels [12]–[17], two-way channels [18], diamond channels [19], energy sharing and energy cooperation concepts [20]–[22], battery imperfections [23], [24], sensor networks [25]–[27], MIMO systems [28], temperature constrained sensor operations [29], delay minimization scenarios [30], [31], etc.; see [32], [33] for a review of offline power scheduling in energy harvesting systems.

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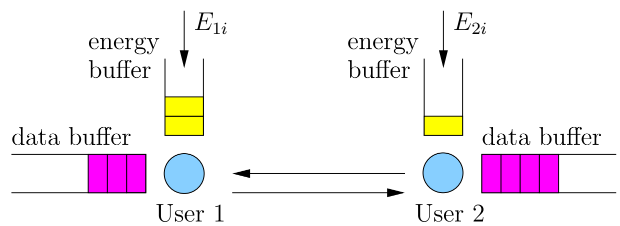

Fig. 1. Two-way channel with an energy harvesting transmitter and receiver.

Power consumption at a transmitter includes power spent for transmission as well as processing power spent for the circuitry. Depending on the energy availability and the communication distance, processing costs at the transmitter could be a significant system factor. References [34]–[38] study the impact of processing costs on energy harvesting communications. Decoding power at the receiver can also be a significant system factor [39]–[42]. The differentiating aspect regarding processing costs and decoding costs in the existing literature is as follows: the processing cost is modeled as a constant power spent per unit time whenever the transmitter is on [43], whereas the decoding cost at a receiver is modeled as an increasing convex function of the incoming rate to be decoded [40], [41]. The energy harvesting two-way channel with decoding costs is considered recently in [44]. In this paper, we focus on the processing costs.

Reference [43] shows that when processing cost is considered, the optimal transmission scheme over a Gaussian channel is bursty; the transmitter-receiver pair may only operate for a portion of the time, during which the transmitter uses Gaussian signals. Reference [43] also considers a parallel Gaussian channel, and shows that the maximum rate is achieved by *glue-pouring*, a modified version of the classical water-filling. Glue-pouring allows channels to be partially filled in time. Reference [35] considers a single-user energy harvesting system with processing costs and presents a *directional glue-pouring algorithm* to maximize the throughput. This result highlights an interesting aspect of energy harvesting communications: while the initial results in [1]–[3] found that, in energy harvesting systems, the transmitters should aim to maintain longest possible stretches of constant power subject to energy causality constraints, which resulted in *directional water-filling* [3], when the transmitter incurs a cost for being on per unit time, the optimal scheme needs to balance long stretches of constant power with the processing cost to be incurred during those stretches via *directional glue-pouring* [35].

In this paper, we consider a two-way channel [45] in which both users are energy harvesting, see Fig. 1. The communication is full duplex, and both users incur processing costs. We characterize the sum rate maximizing optimal transmit policy for a given deadline, subject to energy causality constraints and processing costs. We first consider the formulation for a single energy arrival. In this case, we show that it is optimal for the two users to be fully synchronized; the two users should be switched on for the same portion of the time during which they both exchange data, and then they switch off together. Then, we generalize this to the case of multiple energy arrivals, and show that any throughput optimal policy can be transformed into a *deferred* policy, in which users postpone their energy consumption to fill out later slots first. We find the optimal deferred policy by iteratively applying a modified version of the single energy arrival result in a backward manner.

II. Single Energy Arrival

In this section, we consider the case where users 1 and 2 harvest energy only once in the amounts E_1 and E_2, respectively. The physical layer is Gaussian with unit-variance noise at both users. In a Gaussian two-way channel, the sum rate equals the sum of two single-user capacities [45]. We note that due to the processing costs, it might be optimal for the users to be turned on for only a portion of the time. In this case, the transmission scheme becomes *bursty* [43].

In this two-way setting, we incorporate the processing costs into our problem as follows: each user incurs a processing cost when it is on for either transmitting or receiving or both. At this point, it is not clear whether it is optimal for the two users to be fully synchronized, i.e., switch on/off simultaneously. For instance, it might be the case that the second user's energy is higher, and therefore it uses the channel for a larger portion of the time $\theta_2 > \theta_1$. In this case, the first user stops sending after θ_1 portion of the time, but stays on for an extra $\theta_2 - \theta_1$ amount of time to receive the rest of the second user's data. The same argument could hold for the first user if, e.g., the first user's energy is larger. Therefore, each user stays *on* for a $\max\{\theta_1, \theta_2\}$ amount of time for the general case of $\theta_1 \neq \theta_2$. We formulate the problem as follows

$$\begin{aligned} \max_{\theta_1,\theta_2,p_1,p_2} \quad & \frac{\theta_1}{2}\log(1+p_1) + \frac{\theta_2}{2}\log(1+p_2) \\ \text{s.t.} \quad & \theta_1 p_1 + \max\{\theta_1,\theta_2\}\epsilon_1 \leq E_1 \\ & \theta_2 p_2 + \max\{\theta_1,\theta_2\}\epsilon_2 \leq E_2 \\ & 0 \leq \theta_1, \theta_2 \leq 1 \end{aligned} \tag{1}$$

where $\frac{1}{2}\log(1+p)$ is the Shannon capacity formula [45], and ϵ_1 and ϵ_2 are the processing costs per unit time for the first and the second user, respectively. Throughout this paper $\log$ denotes the natural logarithm.

We have the following two lemmas regarding this problem: Lemma 1 states that both users need to use up all of their available energies. Lemma 2 states that both users need to be fully synchronized, i.e., they need to turn on for exactly the same duration of time, and turn off together. Whenever a user is turned on, it both sends and receives data.

Lemma 1 *In the optimal solution of (1), both users exhaust their available energies.*

Proof: We show this by contradiction. Assume for instance that the first user does not use up all of its energy, i.e., the first constraint holds with strict inequality

$$\theta_1^* p_1^* + \max\{\theta_1^*, \theta_2^*\}\epsilon_1 < E_1 \tag{2}$$

Then, we can always increase p_1^* until equality holds, and thereby strictly increase the objective function. The same argument holds for the second user. ■

Lemma 2 *In the optimal solution of (1), we have $\theta_1^* = \theta_2^*$.*

Proof: We show this by contradiction. Assume without loss of generality that it is optimal to have $\theta_1 < \theta_2$. By Lemma 1, we have the powers given by

$$p_1 = \frac{E_1 - \theta_2\epsilon_1}{\theta_1}, \quad p_2 = \frac{E_2}{\theta_2} - \epsilon_2 \tag{3}$$

Therefore, we rewrite (1) as:

$$\begin{aligned} \max_{\theta_1,\theta_2} \quad & \frac{\theta_1}{2}\log\left(1 + \frac{E_1 - \theta_2\epsilon_1}{\theta_1}\right) + \frac{\theta_2}{2}\log\left(1 + \frac{E_2}{\theta_2} - \epsilon_2\right) \\ \text{s.t.} \quad & 0 \leq \theta_1 \leq \theta_2 \leq \theta_m \end{aligned} \tag{4}$$

where $\theta_m \triangleq \min\{1, \frac{E_1}{\epsilon_1}, \frac{E_2}{\epsilon_2}\}$ ensures the positivity of the powers. Next, we note that the first term in the objective function above is monotonically increasing in θ_1, and therefore its value is maximized at the boundary of the feasible set, i.e., at $\theta_1 = \theta_2$, which gives a contradiction. ■

By Lemma 2, problem (1) now reduces to a problem with only one time variable $\theta \triangleq \theta_1 = \theta_2$ as follows:

$$\begin{aligned} \max_{\theta,p_1,p_2} \quad & \frac{\theta}{2}\log(1+p_1) + \frac{\theta}{2}\log(1+p_2) \\ \text{s.t.} \quad & \theta(p_1+\epsilon_1) \leq E_1 \\ & \theta(p_2+\epsilon_2) \leq E_2 \\ & 0 \leq \theta \leq 1 \end{aligned} \tag{5}$$

We will solve (5), and its most general multiple energy arrival version, in the rest of this paper. We first note that the problem is non-convex. Applying the change of variables:

$$\bar{p}_1 \triangleq \theta p_1, \quad \bar{p}_2 \triangleq \theta p_2 \tag{6}$$

we get the following equivalent problem:

$$\begin{aligned} \max_{\theta,\bar{p}_1,\bar{p}_2} \quad & \frac{\theta}{2}\log\left(1 + \frac{\bar{p}_1}{\theta}\right) + \frac{\theta}{2}\log\left(1 + \frac{\bar{p}_2}{\theta}\right) \\ \text{s.t.} \quad & \bar{p}_1 + \theta\epsilon_1 \leq E_1 \\ & \bar{p}_2 + \theta\epsilon_2 \leq E_2 \\ & 0 \leq \theta \leq 1 \end{aligned} \tag{7}$$

which is convex, as the objective function is now concave because it is the perspective of a concave function [46], and the constraints are affine in both variables. The Lagrangian is:

$$\mathcal{L} = -\frac{\theta}{2}\log\left(1+\frac{\bar{p}_1}{\theta}\right) - \frac{\theta}{2}\log\left(1+\frac{\bar{p}_2}{\theta}\right) + \lambda_1\left(\bar{p}_1+\theta\epsilon_1-E_1\right) + \lambda_2\left(\bar{p}_2+\theta\epsilon_2-E_2\right) + \omega\left(\theta-1\right) \quad (8)$$

where λ_1, λ_2, and ω are the non-negative Lagrange multipliers. Note that we do not introduce Lagrange multipliers for the constraints $\theta, \bar{p}_1, \bar{p}_2 \geq 0$ as they are always satisfied with strict inequality at the optimal solution. Differentiating with respect to $\bar{p}_1$, $\bar{p}_2$ and θ, we obtain the KKT optimality conditions [46]:

$$\frac{\bar{p}_1}{\theta} = \frac{1}{\lambda_1} - 1 \quad (9)$$

$$\frac{\bar{p}_2}{\theta} = \frac{1}{\lambda_2} - 1 \quad (10)$$

$$\lambda_1(\epsilon_1-1) + \log\lambda_1 + \lambda_2(\epsilon_2-1) + \log\lambda_2 = -2-\omega \quad (11)$$

along with the complementary slackness conditions:

$$\lambda_1\left(\bar{p}_1+\theta\epsilon_1-E_1\right) = 0 \quad (12)$$

$$\lambda_2\left(\bar{p}_2+\theta\epsilon_2-E_2\right) = 0 \quad (13)$$

$$\omega\left(\theta-1\right) = 0 \quad (14)$$

The next lemma will help characterize the optimal solution of the problem.

Lemma 3 *The optimal solution of λ_1 and λ_2 is given by*

$$\lambda_1^* = \frac{1}{(E_1/\theta^*)-(\epsilon_1-1)} \quad (15)$$

$$\lambda_2^* = \frac{1}{(E_2/\theta^*)-(\epsilon_2-1)} \quad (16)$$

Proof: The proof follows by using (9) and (10) together with the results of Lemma 1. ■

Substituting the result of Lemma 3 in (11), we obtain the following equation in θ:

$$f_1(\theta)\cdot f_2(\theta) = e^{-(2+\omega)} \quad (17)$$

where the function $f_j(\theta)$, for $j=1,2$, is defined as

$$f_j(\theta) = \frac{e^{(\epsilon_j-1)/((E_j/\theta)-(\epsilon_j-1))}}{(E_j/\theta)-(\epsilon_j-1)} \quad (18)$$

One can show that $f_j(\theta)$ is monotonically increasing in θ. In particular, if $\epsilon_j < 1$, the function is increasing for all values of θ. On the other hand, if $\epsilon_j \geq 1$, the function is increasing only if $\theta \leq E_j/\left(\epsilon_j-1\right)$, which is already satisfied according to the jth user power constraint. Therefore, $f_j(\theta)$ is increasing over the feasible range of θ, and (17) has a unique solution in θ, for a given ω, which we denote by $\bar{\theta}(\omega)$.

We now find the optimal burstiness factor θ^* as follows. First, assume that $\theta^* < 1$. By complementary slackness in (14), we have $\omega = 0$. Therefore, θ^* is given by the solution to (17), $\bar{\theta}(0)$, if it exists in the feasible range. However, if $\bar{\theta}(0) > 1$, we then need to increase the value of ω such that the right hand side of (17) decreases in order to have a feasible solution. Since ω is now strictly positive, by (14) we have $\theta^* = 1$. Therefore, θ^* is given by:

$$\theta^* = \min\left\{\bar{\theta}(0), 1\right\} \quad (19)$$

We note that, the value of θ^* can be strictly less than 1, which leads to *bursty transmission* from the two users. The amount of burstiness depends on the available energies at both users and their processing costs, the relation among which is captured by the functions f_1 and f_2 in (17). The two users' energies and processing costs affect each other; one user having relatively low energy or relatively high processing cost can decrease the value of θ^*, i.e., increase the amount of burstiness in the channel. Finally, once the optimal θ^* is found, the optimal powers of the users are found by substituting θ^* in (15) and (16) in Lemma 3 to find λ_1^* and λ_2^*, and further substituting them in (9) and (10).

III. Multiple Energy Arrivals

We now extend our results to the case of multiple energy arrivals. At the beginning of slot i, energies arrive in amounts E_{1i} and E_{2i} at the first and the second user, respectively, ready to be used in the same time slot or saved in the batteries for future use. During time slot i, the two users can be turned on for a θ_i portion of the time. Our goal is to maximize the total throughput by a given deadline N. The users have to be synchronized. If not, then given the optimal energy distribution, we can synchronize both users in each slot independently, which gives higher throughput, as discussed in the single energy arrival scenario. Then, the problem becomes:

$$\begin{aligned} \max_{\boldsymbol{\theta},\mathbf{p}_1,\mathbf{p}_2} \quad & \sum_{i=1}^{N} \frac{\theta_i}{2}\log(1+p_{1i}) + \frac{\theta_i}{2}\log(1+p_{2i}) \\ \text{s.t.} \quad & \sum_{i=1}^{k} \theta_i(p_{1i}+\epsilon_1) \leq \sum_{i=1}^{k} E_{1i}, \quad \forall k \\ & \sum_{i=1}^{k} \theta_i(p_{2i}+\epsilon_2) \leq \sum_{i=1}^{k} E_{2i}, \quad \forall k \\ & 0 \leq \theta_i \leq 1, \quad \forall i \end{aligned} \quad (20)$$

Similar to the single energy arrival case, we apply the change of variables $\bar{p}_{1i} = \theta_i p_{1i}$ and $\bar{p}_{2i} = \theta_i p_{2i}$, for all i, to get the following equivalent convex optimization problem:

$$\begin{aligned} \max_{\boldsymbol{\theta},\bar{\mathbf{p}}_1,\bar{\mathbf{p}}_2} \quad & \sum_{i=1}^{N} \frac{\theta_i}{2}\log\left(1+\frac{\bar{p}_{1i}}{\theta_i}\right) + \frac{\theta_i}{2}\log\left(1+\frac{\bar{p}_{2i}}{\theta_i}\right) \\ \text{s.t.} \quad & \sum_{i=1}^{k} \bar{p}_{1i}+\theta_i\epsilon_1 \leq \sum_{i=1}^{k} E_{1i}, \quad \forall k \\ & \sum_{i=1}^{k} \bar{p}_{2i}+\theta_i\epsilon_2 \leq \sum_{i=1}^{k} E_{2i}, \quad \forall k \\ & \bar{p}_{1i} \geq 0, \quad \bar{p}_{2i} \geq 0, \quad 0 \leq \theta_i \leq 1, \quad \forall i \end{aligned} \quad (21)$$

The Lagrangian for this problem is:

$$\mathcal{L} = -\left(\sum_{i=1}^{N}\frac{\theta_i}{2}\log\left(1+\frac{\bar{p}_{1i}}{\theta_i}\right)+\frac{\theta_i}{2}\log\left(1+\frac{\bar{p}_{2i}}{\theta_i}\right)\right)$$
$$+\sum_{j=1}^{N}\lambda_{1j}\left(\sum_{i=1}^{j}\bar{p}_{1i}+\theta_i\epsilon_1-\sum_{i=1}^{j}E_{1i}\right)-\sum_{i=1}^{N}\eta_{1i}\bar{p}_{1i}$$
$$+\sum_{j=1}^{N}\lambda_{2j}\left(\sum_{i=1}^{j}\bar{p}_{2i}+\theta_i\epsilon_2-\sum_{i=1}^{j}E_{2i}\right)-\sum_{i=1}^{N}\eta_{2i}\bar{p}_{2i}$$
$$+\sum_{i=1}^{N}\omega_i\left(\theta_i-1\right)-\sum_{i=1}^{N}\nu_i\theta_i \quad (22)$$

where λ_{1i}, η_{1i}, λ_{2i}, η_{2i}, ω_i, ν_i are the non-negative Lagrange multipliers. Differentiating with respect to $\bar{p}_{1i}$ and $\bar{p}_{2i}$, we obtain the following KKT optimality conditions:

$$\frac{\bar{p}_{1i}}{\theta_i} = \left(\frac{1}{\sum_{j=i}^{N}\lambda_{1j}}-1\right)^+ \quad (23)$$
$$\frac{\bar{p}_{2i}}{\theta_i} = \left(\frac{1}{\sum_{j=i}^{N}\lambda_{2j}}-1\right)^+ \quad (24)$$

along with the complementary slackness conditions:

$$\lambda_{1j}\left(\sum_{i=1}^{j}\bar{p}_{1i}+\theta_i\epsilon_1-\sum_{i=1}^{j}E_{1i}\right)=0, \quad \forall j \quad (25)$$
$$\lambda_{2j}\left(\sum_{i=1}^{j}\bar{p}_{2i}+\theta_i\epsilon_2-\sum_{i=1}^{j}E_{2i}\right)=0, \quad \forall j \quad (26)$$
$$\eta_{1i}\bar{p}_{1i}=0, \quad \forall i \quad (27)$$
$$\eta_{2i}\bar{p}_{2i}=0, \quad \forall i \quad (28)$$
$$\omega_i\left(\theta_i-1\right)=0, \quad \forall i \quad (29)$$
$$\nu_i\theta_i=0, \quad \forall i \quad (30)$$

We now have the following two lemmas that characterize the optimal power policy.

Lemma 4 *In the optimal policy, powers of both users are non-decreasing over time.*

Proof: The proof follows by noting that in (23) and (24), the terms $\sum_{j=i}^{N}\lambda_{1j}$ and $\sum_{j=i}^{N}\lambda_{2j}$ are non-increasing over time by the non-negativity of $\{\lambda_{1i},\lambda_{2i}\}$. ■

Lemma 5 *In the optimal policy, if a user's energy is saved from one time slot to the next, then the powers spent by this user in the two slots have to be equal.*

Proof: The proof follows by the complementary slackness conditions; whenever, say, user 1's energy is saved in slot i, $\lambda_{1i}=0$, and thus $\sum_{j=i}^{N}\lambda_{1j}=\sum_{j=i+1}^{N}\lambda_{1j}$, i.e., the denominator in (23) does not change over time slots on which energy transfer occurs. ■

Next, we note that the optimal solution of problem (21) is not unique. For instance, assume that one solution of the problem required some energy to be transferred from the ith to the $(i+1)$st slot at both users, and that the optimal values of θ_i and θ_{i+1} are both less than 1. By Lemma 5, since we transferred some energy between the two slots, we must have equal powers in both slots. Now, if we transfer an extra amount of energy between the two slots, this allows us to do the following: 1) decrease the value of θ_i and increase that of θ_{i+1}, and 2) change the values of $\bar{p}_{ji}$ and $\bar{p}_{j(i+1)}$, $j=1,2$, correspondingly so that we obtain the same values of powers at the two slots as before. This leaves us with the same value for the objective function, as what we did is that we changed the values of the pre-log factors in a feasible manner while keeping the values inside the logs as they were. We can keep doing this until either slot $i+1$ is completely filled, i.e., $\theta_{i+1}=1$, or all of the energy is transferred from slot i, i.e., $\theta_i=0$.

We coin this type of policies as *deferred* policies; no new time slots are opened until all time slots in the future are completely filled, i.e., $0<\theta_i\leq 1$ iff $\theta_k=1$, $\forall k=i+1,\dots,N$. Consequently, $\{\theta_i\}_{i=1}^{N}$ will be non-decreasing. There can only be one unique optimal deferred policy for problem (21). In the sequel, we determine that policy.

A. Optimal Deferred Policy

Finding the optimal deferred policy relies on the fact that, by energy causality, energies can only be used after they have been harvested. To this end, we begin from the last slot, and make sure that it is completely filled, i.e., it has no burstiness, before opening up a previous slot. We apply a modified version of the single energy arrival result iteratively in a backward manner through two main phases: 1) deferring, and 2) refinement. These are illustrated as follows.

We first start by the deferring phase. The goal of this phase is to determine an initial feasible deferred policy. In the refinement phase, the optimality of such policy is investigated. We set the energy state of each slot as $\{S_{jk}=E_{jk}\}$, $j=1,2$, and start from the last slot and move backwards. In the kth slot, we start by examining the use of the kth slot energies in the kth slot only. This is done using the results of the single energy arrival (17). If the resulting $\theta_k<1$, then we transfer some energy from previous slots forward to the kth slot until either it is completely filled, i.e., $\theta_k=1$, or all previous slots' energies are exhausted. We test the possibility of the former condition by moving all energy from a previous slot $l<k$, and re-solving for θ_k. If the result is unity, then the energies of slot l can for sure fill out slot k. Next, we show how much energy is actually needed to do so.

We have two conditions to satisfy: 1) $\theta_k=1$, and 2) powers of user j in slots l and k are equal, $p_{jl}=p_{jk}\triangleq p'_j$, if user j transfers energy from slot l to k (according to Lemma 5). Let us denote the burstiness in slot l by θ'. Hence, if both users transfer energy, the optimal policy is found by solving the following problem

$$\max_{\theta',p'_1,p'_2} \quad \frac{1+\theta'}{2}\log(1+p'_1)+\frac{1+\theta'}{2}\log(1+p'_2)$$
$$\text{s.t.} \quad (1+\theta')(p'_1+\epsilon_1)=S_{1l}+S_{1k}$$

$$
\begin{aligned}
&(1+\theta')(p'_2+\epsilon_2) = S_{2l} + S_{2k} \\
&0 \le \theta' \le 1
\end{aligned} \tag{31}
$$

Following the same analysis as in the single energy arrival case, we solve the following equation for θ':

$$
f_1\left(1+\theta'\right) \cdot f_2\left(1+\theta'\right) = e^{-2} \tag{32}
$$

On the other hand, if only the first user transfers energy, the optimal policy is found by replacing the second constraint in problem (31) by $\theta'(p_{2l}+\epsilon_2) = S_{2l}$, where $p_{2k} = S_{2k} - \epsilon_2$ in this case. This gives the following to solve for θ':

$$
f_1\left(1+\theta'\right) \cdot f_2\left(\theta'\right) = e^{-2} \tag{33}
$$

Similarly, if the transfer is done only from the second user we solve:

$$
f_1\left(\theta'\right) \cdot f_2\left(1+\theta'\right) = e^{-2} \tag{34}
$$

In all the three cases of energy transfer above, the equations to solve have an increasing left hand side, and hence a unique solution. Finally, the optimal policy is the one that gives the maximum sum throughput among the feasible ones. It is worth noting that, by the concavity of the objective function, transferring energy from both users is optimal if feasible, since it equalizes arguments (powers) of a concave objective function [1].

If the initially resulting $\theta_k = 1$ in the kth slot, we do directional water-filling over the future slots, which gives the optimal sum rate [3]. Next, we check if energy should be transferred from a previous slot l from the first, second, or both users, in exactly the same way as above, i.e., by solving (32)-(34). If energy transfer (from either or both users) is feasible and gives a higher objective function, we do directional water-filling again from slot k over future slots, followed by repeating the above energy transfer checks once more. These inner iterations stop if either no energy transfer occurs, or no directional water-filling occurs. The deferring phase ends after examining the first slot. During this phase, we record how much energy is being moved forward to fill up future slots. Meters are put in between slots for that purpose. Let m_{1i} and m_{2i} denote the amount of energy transferred forward from slot i to slot $i+1$ for the first and the second user, respectively. We use the values stored in these meters in the second, refinement, phase as follows.

In the refinement phase, the goal is to check whether the currently reached energy distribution is optimal. One reason it might not be optimal is that during the deferring phase, some excess amounts of energy can be transferred from, e.g., slot k forward unnecessarily without taking into account the energies available before slot k. We check the optimality of the deferring phase policy by performing two-slot updates starting from the last two slots going backwards. During the updates, energy can be drawn back from future slots if this increases the objective function as long as it does not violate causality. This can be done by checking the values stored in the meters in between the slots. A positive value stored in, e.g., m_{1i} means that there can be bidirectional energy transfer between slots i and $i+1$ for the first user. We start by updating slots $(N-1, N)$, followed by $(N-2, N-1)$, and so on. Once we update slots $(1,2)$, we have one iteration. Iterations continue until there is no further change in the meters' values, or equivalently the energy state of each slot stays the same. The details on how to do the updates for some given two slots are as follows.

Algorithm 1 Optimal deferred policy

Phase 1: Deferring

Set $\mathbf{S}_1 = \mathbf{E}_1$, $\mathbf{S}_2 = \mathbf{E}_2$, $\mathbf{m}_1 = \mathbf{m}_2 = \mathbf{0}$, and $k = N$
while $k \ge 1$ **do**
    Using energies $\{S_{1k}, S_{2k}\}$, solve for θ_k using (17)
    if $\theta_k < 1$ **then**
        repeat
            Transfer all energy from slot $k-l$ to slot k
            Re-solve for θ_k using (17)
            if Slot k is completely filled **then**
                Find energy needed to fill it using (32)-(34)
            else $l \leftarrow \min\{l+1, k-1\}$
            end if
        until $\theta_k = 1$, or all previous energies are exhausted
    else
        repeat
            Directional water-filling over slots $\{k, \dots, N\}$
            Check for energy transfer using (32)-(34)
        until No water-filling or energy transfer occur
    end if
    Update the energy state values $\mathbf{S}_1$ and $\mathbf{S}_2$
    Update the meters' values $\mathbf{m}_1$ and $\mathbf{m}_2$
    $k \leftarrow k-1$
end while

Phase 2: Refinement

repeat
    for $k = 0 : N-2$ **do**
        Update the energy status of slots $(N-k-1, N-k)$:
        if $\theta_{N-k-1} = 1$ **then** update by solving (36)
        else update by solving (38)
        end if
    end for
until Meters' values do not change
$\mathbf{p}_1^* = \mathbf{S}_1$, and $\mathbf{p}_2^* = \mathbf{S}_2$.

In case the update involves two completely-filled slots, we solve the following problem

$$
\begin{aligned}
\max_{\mathbf{p},\mathbf{D}} \quad & \frac{1}{2}\log(1+p_{11}) + \frac{1}{2}\log(1+p_{21}) \\
& + \frac{1}{2}\log(1+p_{12}) + \frac{1}{2}\log(1+p_{22}) \\
\text{s.t.} \quad & p_{11}+\epsilon_1 = S_{11}+D_1, \quad p_{21}+\epsilon_2 = S_{21}+D_2 \\
& p_{12}+\epsilon_1 = S_{12}-D_1, \quad p_{22}+\epsilon_2 = S_{22}-D_2 \\
& 0 \le D_1 \le m_1, \quad 0 \le D_2 \le m_2
\end{aligned} \tag{35}
$$

where D_j is the amount of energy to be drawn back, if any, from the second to the first slot of user j, and m_j is the value of the meter in between its slots. Substituting the powers in terms of D_1 and D_2, the problem now decomposes into two, one for each user as follows

$$\max_{D_j} \quad \frac{1}{2}\log(1+S_{j1}-\epsilon_j+D_j)+\frac{1}{2}\log(1+S_{j2}-\epsilon_j-D_j)$$
$$\text{s.t.} \quad 0 \le D_j \le \min\{m_j, S_{j2}-\epsilon_j\} \tag{36}$$

which can be solved by a first derivative analysis over the feasible region. On the other hand, if the two-slot update includes a bursty, not completely filled, first slot we then solve

$$\begin{aligned}
\max_{\mathbf{p},\mathbf{D},\theta} \quad & \frac{\theta}{2}\log(1+p_{11})+\frac{\theta}{2}\log(1+p_{21}) \\
& +\frac{1}{2}\log(1+p_{12})+\frac{1}{2}\log(1+p_{22}) \\
\text{s.t.} \quad & \theta(p_{11}+\epsilon_1)=S_{11}+D_1, \quad \theta(p_{21}+\epsilon_2)=S_{21}+D_2 \\
& p_{12}+\epsilon_1=S_{12}-D_1, \quad p_{22}+\epsilon_2=S_{22}-D_2 \\
& 0\le D_1\le m_1, \quad 0\le D_2\le m_2 \\
& 0\le\theta\le 1
\end{aligned} \tag{37}$$

which can be rewritten as

$$\begin{aligned}
\max_{\mathbf{D},\theta} \quad & \frac{\theta}{2}\log\left(1-\epsilon_1+\frac{S_{11}+D_1}{\theta}\right) \\
& +\frac{1}{2}\log\left(1-\epsilon_1+S_{12}-D_1\right) \\
& +\frac{\theta}{2}\log\left(1-\epsilon_2+\frac{S_{21}+D_2}{\theta}\right) \\
& +\frac{1}{2}\log(1-\epsilon_2+S_{22}-D_2) \\
\text{s.t.} \quad & (\theta\epsilon_j-S_{j1})^+\le D_j\le\min\{m_j,S_{j2}-\epsilon_j\}, \quad j=1,2 \\
& 0\le\theta\le 1
\end{aligned} \tag{38}$$

where the lower bound on D_j is to ensure non-negativity of powers. We solve the above problem by first fixing the parameter θ and solving for D_1 and D_2. Note that when θ is fixed, the problem decomposes into two independent problems in D_1 and D_2 that can be solved by a first derivative analysis over the feasible region. We then find the optimal θ by one dimensional line search over the interval $[0,1]$. This concludes our discussion on the refinement phase.

We summarize the solution in Algorithm 1. Based on the above analysis, we now have the following theorem

Theorem 1 *Algorithm 1 is feasible and gives the optimal deferred policy for problem (21).*

IV. Numerical Examples

In this section, we provide some numerical examples to further illustrate how the proposed algorithm works. We begin by considering a two-slot system to show the basic idea of the deferred policy. Energies arrive with amounts $\mathbf{E}_1 = [0.5, 1]$ and $\mathbf{E}_2 = [1, 1]$, at the first and the second user, respectively. The processing costs at the first and the second user are $\epsilon_1 = 0.5$ and $\epsilon_2 = 0.4$, respectively. In Fig. 2, we present one feasible, and two optimal, power policies. The height of the water levels in blue represents the actual transmit powers $\{p_{1i}, p_{2i}\}$, while the width represents the burstiness $\{\theta_i\}$, for $i = 1, 2$. On the top, we solve for each slot independently using the single arrival result. This gives a non-deferred policy with $\boldsymbol{\theta} = [0.47, 0.65]$, $\mathbf{p}_1 = [0.57, 1.04]$, $\mathbf{p}_2 = [1.75, 1.14]$, and a sum throughput equal to 0.541. Applying Algorithm 1, we then transfer all the energy from the 1st to the 2nd slot and re-solve for θ_2 using (17). The result is $\theta_2 = 1$, which means that the 1st slot's energies are capable of totally filling the 2nd slot. We therefore compute the exact amount needed to do so by setting $\theta_2 = 1$ and solving for $\theta_1 = \theta'$ using (32), i.e., we assume both users transfer energy. This gives $\theta_1 = 0.122$, $\mathbf{p}_1^* = [0.84, 0.84]$, $\mathbf{p}_2^* = [1.39, 1.39]$. The resulting powers and burstiness are feasible, and are therefore optimal, with a sum throughput equal to 1.656. We show the optimal deferred policy at the middle of Fig. 2. Finally, at the bottom of Fig. 2, we show another optimal, yet non-deferred, power policy. This is simply done by shifting some of the water back, in a feasible manner, from slot 2 to slot 1. Namely, we increase the value of θ_1 to 0.35 and decrease that of θ_2 to 0.772, with the same transmit powers. This is a feasible non-deferred policy, and gives the same objective function of 1.656. This shows the non-uniqueness of the solution of problem (21), unless we focus on deferred policies.

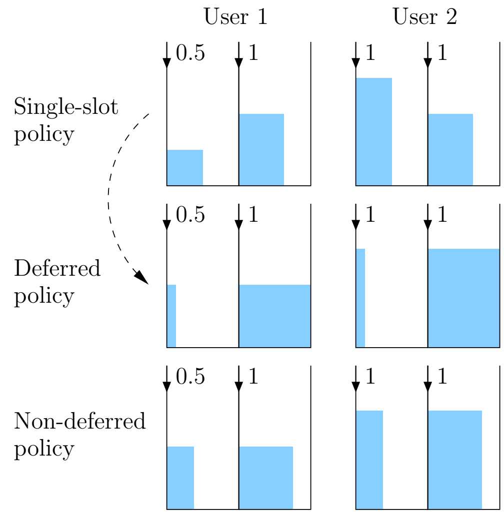

Fig. 2. Numerical example of a deferred policy in a two-slot system.

Next, we use the same ideas above to solve a more invloved four-slot system, with energy arrivals $\mathbf{E}_1 = [0.7, 2.5, 0.8, 0.6]$ and $\mathbf{E}_2 = [0.8, 3, 1.3, 0.4]$; processing costs $\epsilon_1 = 0.5$ and $\epsilon_2 = 0.2$. In Fig. 3, the left (resp., right) hand side represents the first (resp., second) user's power allocations over the slots. We illustarte the deferring phase of Algorithm 1 through four main steps, (a) through (d). In step (a), we start by considering the last time slot to get $p_1 = 1.04$, $p_2 = 0.83$, and $\theta = 0.39$. Therefore, in step (b), we transfer some energy from slot 3 to slot 4 to fill it out. This leaves us with $\mathbf{p}_1 = [0.71, 0.71]$, $\mathbf{p}_2 = [1.27, 1.27]$, and $\theta = [0.16, 1]$. As we can see, the powers are equal in both slots for each user as stated in Lemma 5. In

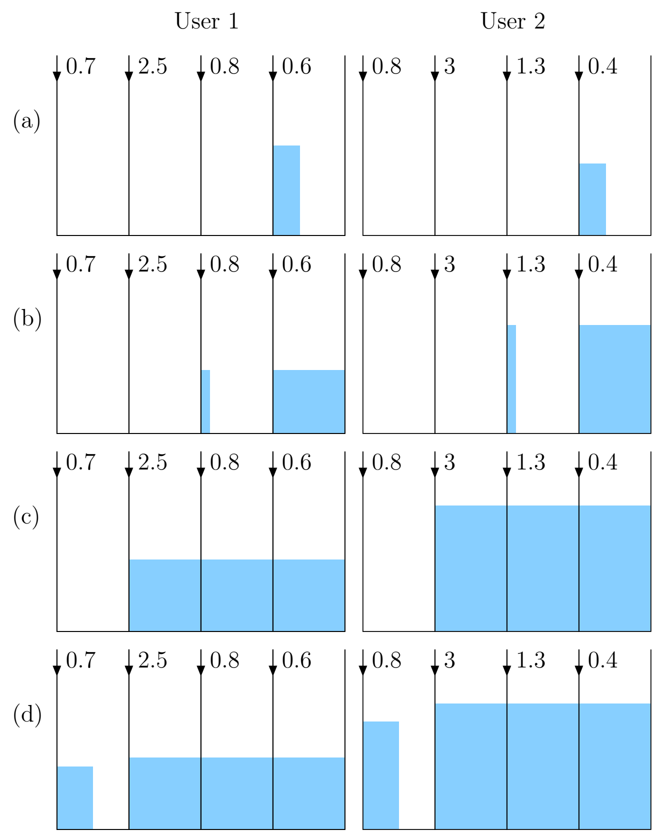

Fig. 3. Numerical example of applying Algorithm 1 on a four-slot system.

step (c), we first fill out slot 3 using some energy from slot 2, but still, this leaves a large amount of energy in slot 2 that is capable of filling it all, and doing directional water-filling over slots 3 and 4. This leaves us with three completely filled slots at both users with power allocations $\mathbf{p}_1 = [0.8, 0.8, 0.8]$, and $\mathbf{p}_2 = [1.37, 1.37, 1.37]$. We then check for possible energy transfer from slot 1, which is found infeasible, or decreasing the objective function. Therefore, finally in step (d), we do the power allocations for slot 1, which leaves us with the following (initial) deferred policy: $\mathbf{p}_1 = [0.74, 0.8, 0.8, 0.8]$, $\mathbf{p}_2 = [1.22, 1.37, 1.37, 1.37]$, and $\boldsymbol{\theta} = [0.56, 1, 1, 1]$. We then apply the refinement phase of Algorithm 1, and find that no further changes are needed and that the initial deferred policy is optimal. As we see, powers are non-decreasing as stated in Lemma 4, and $\{\theta_i\}$ is also non-decreasing; an attribute of the deferred policy.

Finally, we consider another four-slot system, with energy arrivals $\mathbf{E}_1 = [0.9, 0.1, 3, 0.8]$ and $\mathbf{E}_2 = [0.8, 1.5, 2, 2]$; processing costs $\epsilon_1 = 0.3$ and $\epsilon_2 = 0.6$. After the deferring phase of Algorithm 1, we get the following energy distribution for the initial deferred policy: $\mathbf{S}_1 = [0, 1, 1.9, 1.9]$ and $\mathbf{S}_2 = [0, 1.54, 2.38, 2.38]$. We then apply the two-slot updates in the refinement phase of the algorithm. We reach the optimal deferred policy after 5 iterations, which is given by $\mathbf{p}_1^* = [0.67, 0.67, 1.6, 1.6]$, $\mathbf{p}_2^* = [1.47, 1.47, 1.47, 1.47]$, and $\boldsymbol{\theta}^* = [0.033, 1, 1, 1]$.

V. Conclusion

We considered an energy harvesting two-way communication channel in which both users harvest energy from nature, and incur an amount of energy cost to account for their circuitry processing costs. We showed that the throughput maximizing policy is bursty; the two users only communicate for a portion of the time, depending on their energies and processing costs. We showed that it is optimal for the two users to be fully synchronized; they turn on and exchange data during the same portion of the time, and they turn off together. We first investigated the single energy arrival scenario, and then generalized the solution to the case of multiple energy arrivals. In particular, we showed that it is optimal to communicate using deferred policies, in which a new time slot is utilized only if all future time slots are completely filled. We presented an algorithm that gives such a deferred policy by iteratively applying a modified version of the single energy arrival result in a backward manner.

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