A Feedback- Soft Sensing-Based Access Scheme for Cognitive Radio Networks

Ahmed M. Arafa, Karim G. Seddik, Ahmed K. Sultan, Tamer ElBatt, and Amr A. El-Sherif

***Abstract*—In this paper, we examine a cognitive spectrum access scheme in which secondary users exploit the primary feedback information. We consider an overlay secondary network employing a random access scheme in which secondary users access the channel by certain access probabilities that are functions of the spectrum sensing metric. In setting our problem, we assume that secondary users can eavesdrop on the primary link's feedback. We study the cognitive radio network from a queuing theory point of view. Access probabilities are determined by solving a secondary throughput maximization problem subject to a constraint on the primary queues' stability. First, we formulate our problem which is found to be non-convex. Yet, we solve it efficiently by exploiting the structure of the secondary throughput equation. Our scheme yields improved results in, both, the secondary user throughput and the primary user packet delay as compared to the scheme where no feedback information is exploited. In addition, it comes very close to the *optimal* genie-aided scheme in which secondary users act upon the presumed perfect knowledge of the primary users' activity.**

***Index Terms*—ARQ feedback, cognitive radio, Markov chains, soft sensing.**

I. Introduction

COGNITIVE Radio technology is a communication paradigm that emerged in order to solve the spectrum scarcity problem by allowing secondary users (SU or unlicensed users) to exploit the under-utilized spectrum of the primary users (PU or licensed users). Coexistence of such SUs along with PUs is allowed provided that minimal or no harm is caused upon the primary network, and that a minimum quality of service is guaranteed for PUs. In a typical cognitive radio setting, the cognitive transmitter senses the primary activity and decides on accessing the channel based on the sensing outcome. This setting is problematic in the sense that cognitive users are not aware of their impact on the primary network, besides the usual sensing errors.

This, in turn, induced two ideas to alleviate these hurdles. The first is to decrease the sensing error rate. Among the various techniques proposed in the literature, we focus on the concept of "soft sensing" in [2], where the authors introduced a novel design in which the value of the test statistic is used as a confidence measure for the sensing outcome. This value is then used to specify a channel access probability for the secondary network. The access probabilities as functions of the sensing metric are obtained by solving an optimization problem formulated to maximize the secondary throughput given a constraint on the primary queue stability. Soft sensing was first introduced in [3], however, the focus was on physical layer power adaptation to maximize the capacity of the secondary link.

The second idea proposed in the literature is to make the SU aware of the primary activity by leveraging the feedback sent from the primary receiver to the primary transmitter and optimizing its transmission strategy based on its effect on the primary receiver. For instance, in [4], the SU observes the automatic repeat request (ARQ) from the primary receiver. The ARQ feedback messages reflect the PU's achieved packet rate. The cognitive radio's objective is to maximize the secondary throughput under the constraint of guaranteeing a certain packet rate for the PU. In [5], the authors use a partially observable Markov decision process (POMDP) to devise an optimized admission control policy. Their problem is formulated such that the SU's best policy (admission to the channel, and power allocation) is chosen subject to keeping PU's NACK rate under control. Secondary power control on the basis of the primary link feedback is investigated in [6]. The objective was to maximize the SUs' utility, in a distributed manner, while maintaining a PU outage constraints. In [7], the optimal transmission policy for the SU, when the PU adopts a retransmission based error control scheme, is investigated. The policy of the SU determines how often it transmits according to the retransmission state of the packet being served by the PU. The resulting optimal strategy of the SU is proven to have a unique structure. In particular, the optimal throughput is achieved by the SU by concentrating its interference to the PU in the first transmission attempt of a packet.

Another scope of how to exploit the primary feedback messages arose from the fact that the ARQ mechanism introduces redundancy in the system, in the form of copies of the same message transmitted in subsequent time slots. The idea of exploiting this redundancy is investigated in [8] where several

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A. M. Arafa is with the Department of Electrical and Computer Engineering, University of Maryland, College Park, MD 20742, USA. This work was done when he was with the Wireless Intelligent Networks Center (WINC), Nile University, Giza, Egypt (e-mail: arafa@umd.edu).

K. G. Seddik is with the Electronics Engineering Department, American University in Cairo, AUC Avenue, New Cairo 11835, Egypt (e-mail: kseddik@aucegypt.edu).

A. K. Sultan and A. A. El-Sherif are with the Department of Electrical Engineering, Alexandria University, Alexandria 21544, Egypt (e-mail: salatino@alumni.stanford.edu, amr.elsherif@ieee.org).

T. ElBatt is with the EECE Dept., Faculty of Engineering, Cairo University, Giza, Egypt, and the Wireless Intelligent Networks Center (WINC), Nile University, Giza, Egypt (e-mail: telbatt@ieee.org).

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protocols are proposed, in which the secondary transmitter collects side-information about the primary message in the first primary transmission, which is exploited to relay the primary message, if a retransmission occurs. While in [9], the authors introduce a mechanism where the secondary receiver can perform interference cancellation during the whole primary ARQ window by decoding the primary message, thus enhancing its own outage performance. In particular, they investigate a Backward Interference Cancellation mechanism in which the secondary receiver buffers the secondary transmissions that underwent outage due to primary interference, and attempts to recover them once the knowledge about the primary message becomes available due to decoding operation in a future instant.

A simple idea was introduced in a previous work [10] in which SUs refrain from accessing the channel upon hearing a NACK from the primary receiver allowing for an interference-free primary retransmission, thereby increasing secondary throughput and decreasing primary packet delay.

In this paper, we introduce a hybrid scheme in which we capture the benefits of a feedback-based access scheme introduced on top of soft sensing, and accordingly, high sensing reliability is attained besides awareness of the primary environment. We consider a secondary network employing a random access scheme in which SUs access the channel by certain access probabilities that are functions of the sensing metric. The network is studied from a queuing theoretic perspective, and access probabilities are determined by solving an optimization problem subject to a constraint on the PU's queue stability. In addition, SUs can overhear and, hence, leverage the primary link's feedback; SUs back-off completely from accessing the channel upon hearing a NACK, and attempt accessing if an ACK, or no feedback, is overheard. This leads to significant improvements in the SU's throughput as well as the PU's packet delay. This is attributed to the high sensing reliability, due to the use of soft sensing, as well as avoiding sure collisions between primary and SUs when SUs back-off upon hearing a NACK. Our scheme is shown to outperform both soft sensing and conventional hard decision sensing not leveraging feedback information and approaches the *optimal* genie-aided scheme in which SUs have perfect knowledge of the PU's activity and, hence, make best use of it.

To summarize, 1) our work is different than the stated literature in that we optimize the SUs' access decisions based on a collision-based scenario, where, in addition to channel outage, a packet is lost when more than one transmission proceeds at a time. In addition, 2) we study the system from a queuing-theoretic perspective. Finally, 3) our proposed scheme shows that increasing, both, the sensing reliability, and the awareness of the primary activity for the SUs, leads to a nearly-optimal result regarding the SU throughput.

The rest of the paper is organized as follows. The system model is presented in Section II. A background on the soft sensing scheme is presented in Section III. The proposed feedback- and soft sensing-based access scheme is described and analyzed in Section IV. Performance results are given in Section V. Finally, conclusions are drawn and future work is presented in Section VI.

II. System Model

We consider the uplink of a TDMA system consisting of M_p PUs, along which we have M_s SUs attempting to access the channel using a random access scheme as shown in Fig. 1. Let $\mathcal{M}_p = \{1, 2, ..., M_p\}$ denote the set of all PUs, and $\mathcal{M}_s = \{1, 2, ..., M_s\}$ denote the set of all SUs.

PUs access the channel in their dedicated time slots whenever they have packets to send. On the other hand, SUs attempt to send their packets only when PUs are sensed to be idle. At the beginning of each time slot, each SU senses the channel and if found idle, it accesses the channel with a certain access probability. Simple energy detection [11] is adopted as the sensing mechanism since it does not need prior information of the primary signal or its structure. Note that in this work, we do not consider the optimization of spectrum sensing duration; we assume the sensing activity takes place at the beginning of each time slot and that it is short enough to allow SUs to sense and then to transmit their (shorter) packets for the remainder of the same time slot.

The channel is modeled as a Rayleigh flat fading channel with additive white Gaussian noise (AWGN). Thus, the received signal at node j from node q at time slot t is given by

$$y_{qj}^t = \sqrt{G_q r_{qj}^{-\gamma}} h_{qj}^t x_q^t + n_j^t, \tag{1}$$

where G_q is the transmitted power, r_{qj} is the distance between the two nodes, and γ is the path loss exponent. x_q^t is the transmitted signal, which is assumed to be drawn from any constellation with zero mean and unit variance. h_{qj}^t is the channel coefficient between the two nodes, modeled as i.i.d. circularly symmetric complex Gaussian random variable with zero mean and unit variance. The noise term n_j^t is also modeled as i.i.d. circularly symmetric complex Gaussian random variable with zero mean and variance N_0. We assume the channel is stationary and independent from slot to slot, thus, the superscript t can be dropped.

For a transmission to be successful, the channel must not be in outage, i.e. the received SNR should not be smaller than a pre-specified threshold ζ. From the signal model in (1), the outage probability between nodes q and j is given by

$$P_{qj}^o = \Pr\left\{ |h_{qj}|^2 < \frac{\zeta N_0 r_{qj}^{\gamma}}{G_q} \right\} = 1 - \exp\left(-\frac{\zeta N_0 r_{qj}^{\gamma}}{G_q} \right). \tag{2}$$

Furthermore, we adopt a collision model, in which whenever more than one transmission proceeds at a time, all packets involved are lost. A situation in which the interference is assumed to be too high for the receivers to have a decodable signal.

During each time slot, according to the aforementioned system properties, M_s SUs contend with only one PU for channel acquisition. Thus, there exists two scenarios in which collisions may occur. The first is when one or more SUs access the channel during the presence of an active PU. While the second is when more than one SU access the channel simultaneously during the absence of an active PU. Orthogonality in time, through the use of TDMA, prohibit the chance of collisions between primary packets.

Each PU has an infinite buffer for storing its incoming packets. Assuming symmetry conditions, for mathematical

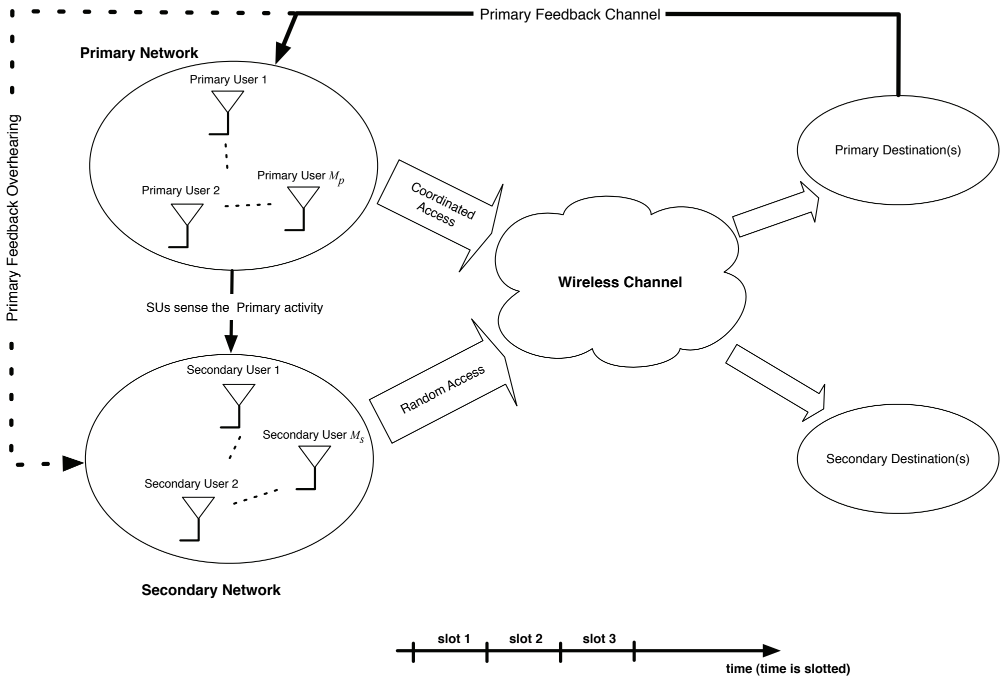

Fig. 1. The system model.

tractability, packet arrival process at any primary queue is assumed to be Bernoulli i.i.d. with an average arrival rate of λ_p packets-per-time slot. A slot duration is equal to the packet transmission time, and therefore, we assume $0 \leq \lambda_p \leq 1$ or else the queues will not be stable. Finally, we consider the case where SUs always have packets to send.

A primary ARQ protocol is implemented, in which an error-free feedback channel is leveraged by the primary receiver to send a feedback by the end of each time slot to acknowledge the reception of packets. Accordingly, an ACK is sent if a packet is correctly received, and a NACK is sent if a packet is lost (which is attributed to either primary channel outage, or collision between secondary and primary packets). In case of an idle slot, no feedback is sent.

SUs are assumed to overhear this primary feedback *perfectly*[1] and act as follows: assuming we are now in time slot t; if an ACK or no feedback is heard, the SUs behave normally, and start sensing the channel in the PU's next time slot (i.e. time slot $t + M_p$). On the other hand, if a NACK is heard, all SUs back-off in the PU's next time slot allowing for an interference-free retransmission of the erroneous primary packet. Accordingly, collisions can be avoided since the reception of a NACK triggers the PU to send in its next time slot with probability one.

In the sequel, we assume symmetry conditions, for simplicity of analysis and presentation, in which all PUs' transmit powers are equal and all distances between secondary and PUs are equal. Therefore the subscript qj is dropped in the rest of the paper.

In the next section, we present a background on the so-called soft sensing scheme which was briefly mentioned in Section I.

[1]Considering imperfect reception of the primary ARQ feedback messages is the subject of ongoing investigations.

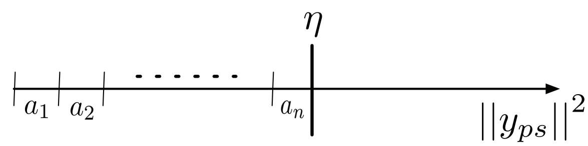

Fig. 2. Soft Sensing: Division of the interval $[0, \eta]$ into subintervals and their associated access probabilities.

III. Background: Soft Sensing-Based Access

We focus on the concept of soft sensing originally introduced in [2] which basically uses the energy statistic $\|y_{ps}\|^2$ acquired from the energy detector as a measure of reliability, where subscript p denotes the PU and s denotes the SU. The lower the value of $\|y_{ps}\|^2$ compared to the decision threshold η, the more certain SUs become that PUs are idle in the time slot in question. This observation is exploited to yield the powerful concept of soft sensing as follows:

- The interval $[0, \eta]$ is divided into n subintervals of equal length as shown in Fig. 2. Let $\mathcal{I} = \{1, 2, ..., n\}$ denote the set of all subintervals.
- For each subinterval $i \in \mathcal{I}$, an access probability a_i is assigned.
- If $\|y_{ps}\|^2$ lies in the i^{th} subinterval, the SU attempts to access the channel with probability a_i.
- If $\|y_{ps}\|^2$ value is greater than η, the SU does not access the channel.

Intuition suggests that access probabilities associated with subintervals far less than the threshold η are given higher values than those associated with ones near the threshold, i.e. their values are sorted in a descending order as i goes from

1 to n. This is mainly because of the very low probability of collision with a PU packet whenever the energy statistic lies in the subintervals close to zero, and therefore an SU should be more aggressive on accessing the channel. On the other hand, there is a higher risk of collision whenever the energy statistic lies in the subintervals close to the energy threshold, which in turn causes the SU to be less aggressive on accessing the channel.

In this work, the stability of the PU queue is studied as the performance measure. Access probabilities are chosen such that the SU throughput is maximized provided that the PU queue is stable. Stability can be loosely defined as keeping a quantity of interest bounded, in our case, the queue size. For a more general and rigorous definition of stability, see [12] and [13]. If the arrival and service processes of a queuing system are strictly stationary, one can apply Loynes' theorem to check for stability [14]. This theorem states that if the average arrival rate is less than the average service rate of a queuing system whose arrival and service processes are strictly stationary, then the queue is stable, otherwise it is unstable.

Therefore, the baseline problem, without feedback, can be formulated as maximizing the secondary throughput subject to the primary queue being stable. That is,

$$\begin{aligned} \max_{a_i, i\in\mathcal{I}} \quad & \mu_s \\ \text{subject to} \quad & \lambda_p < \mu_p, \end{aligned} \tag{3}$$

where μ_s is the SU throughput, and μ_p is the PU service rate. Next, we characterize μ_p.

Under the assumption stated before that SUs always have packets to send, the service process of the q^{th} PU at time slot t can be characterized as

$$Y_q^t = \mathbf{1}\left(A_q^t \bigcap \overline{O_{qd}^t} \bigcap_{l\in\mathcal{M}_s} \left\{\overline{\mathcal{B}\bigcap P_s}\right\}\right), \tag{4}$$

where $\mathbf{1}(\cdot)$ denotes the indicator function ($\mathbf{1}(A) = 1$ if event A occurs, and 0 otherwise), A_q^t denotes the event that time slot t is assigned to PU q, $\overline{O_{qd}^t}$ denotes the event that the link between PU q and its destination is not in outage, $\mathcal{B}$ is the event of missed detection, and P_s is the event that an SU gains access to the channel. The probability of the joint event of missed detection and permission to access the channel, is given by

$$\Pr\left\{\mathcal{B}\bigcap P_s\right\} = \sum_{i\in\mathcal{I}} p_i^1 a_i, \tag{5}$$

where p_i^1 is the probability that the energy detector's output of the received signal $\|y_{ps}\|^2$ falls in the i^{th} subinterval when the PU is present. From the received signal model of (1), p_i^1 is given by

$$p_i^1 = \exp\left(-\frac{(i-1)\eta}{2n\sigma_1^2}\right) - \exp\left(-\frac{i\eta}{2n\sigma_1^2}\right), \tag{6}$$

where σ_1^2 is the variance of the energy detector's output when the PU is present.

The average PU service rate can now be written as

$$\mu_p = E\left\{Y_q^t\right\} = \frac{1-P_{pd}^o}{M_p}\left(1 - \sum_{i\in\mathcal{I}} p_i^1 a_i\right)^{M_s}, \tag{7}$$

where $E\{\cdot\}$ is the expectation operator, and P_{pd}^o is the probability that the link between the primary transmitter and the primary receiver is in outage. Note that the SUs' missed-detection and PU's outage events are independent since we assume constant powers for all PUs. Next, we move to characterizing μ_s.

For an SU to successfully send its packet, the following events have to all take place simultaneously: it has to correctly identify the channel as idle, i.e. no false alarm occurs; it must gain access to the channel; its own link must not be in outage; all other SUs must either have a false alarm decision or have no access to the channel; and the PU's queue has to be empty. Thus, the service process of the k^{th} SU at time slot t can be characterized as shown in (8) on the next page, where $\mathcal{A}$ is the event of false alarm, and $\{Q_q^t = 0\}$ denotes the event that the q^{th} PU queue is empty, which can be determined using Little's theorem [15] to be $(1 - \lambda_p/\mu_p)$.

Similar to (5), the joint event of no false alarm and gaining channel access when the PU is not present can be expressed as

$$\Pr\left\{\overline{\mathcal{A}}\bigcap P_s\right\} = \sum_{i\in\mathcal{I}} p_i^0 a_i, \tag{9}$$

where

$$p_i^0 = \exp\left(-\frac{(i-1)\eta}{2n\sigma_0^2}\right) - \exp\left(-\frac{i\eta}{2n\sigma_0^2}\right), \tag{10}$$

where σ_0^2 is the variance of the energy detector's output when no PU is present.

Therefore, the average SU service rate is given by (11) on the next page. Fortunately, the optimization problem (3) using (7) and (11) was proved to be convex in [2]. Thus, its global optimum can be calculated efficiently via standard techniques [16].

IV. Proposed Feedback- and Soft Sensing-Based Access

In our proposed scheme, the SUs overhear each PU's ARQ feedback without error at the end of the PU's assigned time slot. Assuming operation at time slot t that is dedicated to one of the PUs, the SUs leverage the overheard feedback as follows:

- If an ACK or no feedback is heard, each SU attempts sensing and accessing the channel in the time slot $t+M_p$, which is the next time slot of the same PU.
- Otherwise, if a NACK is heard, all SUs back-off completely in time slot $t + M_p$ allowing for an interference-free retransmission of the erroneous primary packet.

The scheme prevents operation of SUs in time slots that are guaranteed to be busy, thereby increasing the PU service rate and decreasing the PU's packet delay. Accordingly, the primary queue will be empty with a higher probability which increases the throughput of the secondary network.

The Markov chain modeling the PU's queue dynamics is presented in Fig. 3. There are two classes of states the PU queue may encounter, the first is k_F, denoting the case where the PU has k packets and sending for the first time, where F stands for "First transmission". The second is k_R, denoting the case where the PU has k packets and re-transmitting, where

$$Y_k^t = \mathbf{1}\left(\bigcup_{q\in\mathcal{M}_p}\left[A_q^t \bigcap \{Q_q^t=0\} \bigcap \overline{O_{kd}^t} \bigcap \overline{\mathcal{A}} \bigcap P_s \bigcap_{l\in\mathcal{M}_s\backslash k}\left\{\mathcal{A}\bigcup\overline{P_s}\right\}\right]\right), \tag{8}$$

$$\mu_s = E\left\{Y_k^t\right\} = \left(1 - \frac{\lambda_p M_p}{\left(1-P_{pd}^0\right)\left(1-\sum_{i\in\mathcal{I}} p_i^1 a_i\right)^{M_s}}\right)(1-P_{sd}^o)\left(\sum_{i\in\mathcal{I}} p_i^0 a_i\right)\left(1-\sum_{i\in\mathcal{I}} p_i^0 a_i\right)^{M_s-1}. \tag{11}$$

R stands for "Retransmission". States k_F have stationary probability π_k, and states k_R have stationary probability ϵ_k. Note that the PU queue stability is a necessity for stationary probabilities to exist.

The second class of states is only reached after the reception of a NACK. In such case, the primary link's outage is the sole cause of packet retransmission since SUs refrain completely from transmission upon overhearing a NACK from the primary receiver. If the PU queue is empty, then, clearly the PU cannot be in the retransmission state, therefore, $\epsilon_0 = 0$.

In order to model our system, we do the analysis using *probabilistic* TDMA (see e.g. [17] and [18]). In such, every time slot is assigned to a certain PU according to a uniform distribution (i.e. with probability $1/M_p$). Hence, when the system runs for an infinite duration, all PUs will have an equal share of the channel just as in conventional TDMA. Since our channel is stationary and independent from slot to slot, the success and failure probabilities will be the same for a given PU in both the probabilistic and conventional TDMA models, and therefore, its queue evolution will be the same.

As shown in Fig. 3, a back-transition from $(k+1)_F$ in slot t to k_F in slot $t+1$ occurs when the PU does not receive any packets during slot t, which occurs with probability $1-\lambda_p$, and at the same time succeeds in transmission, which occurs with probability $\Gamma_p = \frac{1}{M_p}(1-P_{pd}^o)(1-\sum_{i\in\mathcal{I}} p_i^1 a_i)^{M_s}$, i.e. the PU got allocated to this time slot, at the same time its link was not in outage, and all SUs did not interfere with its transmission. These two events are independent, and hence, their joint probability simply boils down to their product. It is worth noting here that Γ_p has the same value as the PU service rate μ_p in the baseline no feedback scheme since they both denote the successful primary transmission probability in the same surrounding conditions. A PU will stay in state k_F in time slot $t+1$, for $k \geq 1$, if it received a new packet during time slot t, which occurs with probability λ_p, and if it simultaneously succeeded in transmission, which occurs with probability Γ_p. Again, these events are independent and hence there joint probability is equal to their multiplication, that is $\lambda_p\Gamma_p$.

On the other hand, a forward-transition from k_R in slot t to $(k+1)_R$ in slot $t+1$ occurs when the PU receives a packet in slot t, which occurs with probability λ_p, and fails in its transmission, which should now be considered in the absence of SUs. This occurs with probability $\delta = 1 - \frac{1}{M_p}(1-P_{pd}^o)$, which means that either the PU did not gain access to the slot, or it gained access, yet, its channel was in outage. It follows that the probability of a PU staying in state ϵ_k is equal to $(1-\lambda_p)\delta$. The rest of the probabilities can be derived using similar arguments.

Next, we present our system analysis and consider two performance metrics, namely the secondary throughput and primary packet delay.

A. Secondary Throughput Analysis

In this subsection, we derive an expression for the SU throughput in the proposed feedback-based scheme. The SU service event will be just the same as in (8). Due to the feedback, it is only the value of $\Pr\{Q_q^t = 0\}$ (which is equivalent in our model to π_0) that is going to change. The PU Markov chain in Fig. 3 can be analyzed, using the global balance equations, in order to get the value of π_0 which is given by

$$\pi_0 = \frac{\chi - \lambda_p}{1-\delta}, \tag{12}$$

where $\chi = \lambda_p\Gamma_p + (1-\lambda_p)(1-\delta)$ (proof in Appendix A). After some algebraic manipulations, we get the following result

$$\pi_0 = 1 - \lambda_p\left[\left(1 + \frac{M_p}{1-P_{pd}^o}\right) - \left(1 - \sum_{i\in\mathcal{I}} p_i^1 a_i\right)^{M_s}\right]. \tag{13}$$

It is worth noting that for an irreducible and aperiodic Markov chain, the queue is stable if there exists a non-zero value for the probability of the queue being empty [13]. This condition is equivalent in our model to having $\pi_0 > 0$, which leads to $\lambda_p < \chi$ (which is the same condition of $\psi < 1$ stated in (40) in Appendix A).

In order to gain more insight into the difference in the SU throughput between our proposed scheme and the no feedback one, we compute $\triangle\pi_0$, denoting the difference between π_0 in the feedback-based scheme (equation (12)), and π_0 in the no feedback one, which is given by

$$\begin{aligned}\triangle\pi_0 &= \frac{\chi - \lambda_p}{1-\delta} - \left(1 - \frac{\lambda_p}{\Gamma_p}\right)\\ &= \frac{\lambda_p\Gamma_p + (1-\lambda_p)(1-\delta) - \lambda_p}{(1-\delta)} - \left(1 - \frac{\lambda_p}{\Gamma_p}\right).\end{aligned} \tag{14}$$

After some algebraic manipulations, the following result can be reached

$$\triangle\pi_0 = \frac{\lambda_p(1-\Gamma_p)}{\Gamma_p}\left[1 - \left(1 - \sum_{i\in\mathcal{I}} p_i^1 a_i\right)^{M_s}\right], \tag{15}$$

which is non-negative for $0 \leq a_i \leq 1$, and is strictly positive except for the trivial case of $a_i = 0\ \forall i$. Therefore, the SU throughput of the proposed feedback-based scheme is always larger than that of the no feedback one for the same set of SU access probabilities. One can expect that finding access probabilities that maximize the throughput for the feedback-based scheme should give even higher SU throughput.

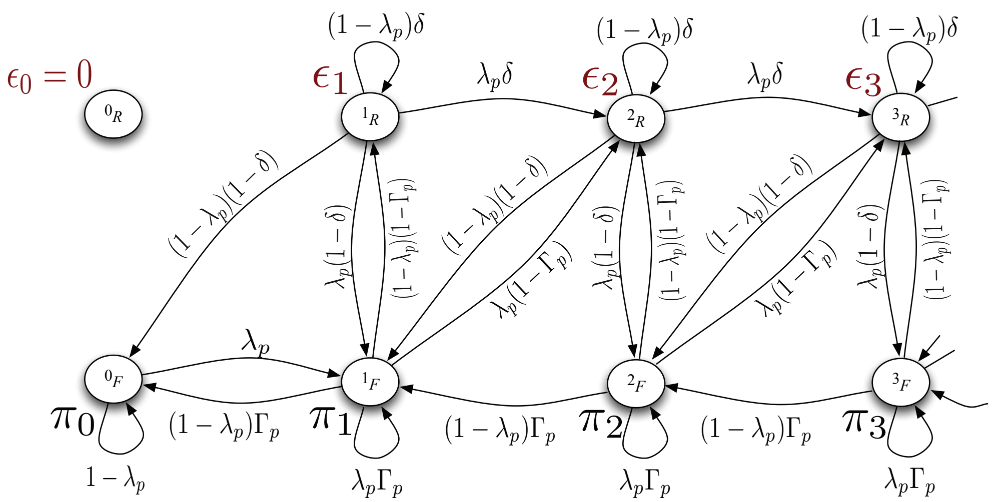

Fig. 3. Markov Chain model of the PU queue evolution.

$$\mu_s = \left(1 - \lambda_p\left[\left(1 + \frac{M_p}{1-P_{pd}^o}\right) - \left(1 - \sum_{i\in\mathcal{I}} p_i^1 a_i\right)^{M_s}\right]\right)(1-P_{sd}^0)\left(\sum_{i\in\mathcal{I}} p_i^0 a_i\right)\left(1 - \sum_{i\in\mathcal{I}} p_i^0 a_i\right)^{M_s-1}. \quad (16)$$

We can now write the formula of the SU throughput as shown in (16) on the next page. Therefore, the optimization problem is given by

$$\max_{a_i, i\in\mathcal{I}} \quad \mu_s$$
$$\text{subject to} \quad \lambda_p < \chi. \quad (17)$$

Unfortunately, the optimization problem in (17) is non-convex as we show later. For instance, because the objective function is not concave in the access probabilities. Nevertheless, It can still be solved efficiently by exploiting its structure as discussed next.

Consider the following general optimization problem

$$\max_{x} \quad f_1(x) + f_2(x)$$
$$s.t. \quad 0 \le x \le 1 \quad (18)$$

where $f_1(x)$ is a concave function in x, while $f_2(x)$ is non concave. Now let us assume without loss of generality, that the function $f_2(x)$ is bounded from below and from above by f_2^{min} and f_2^{max} respectively for any given value of x in the feasible region.

Now consider the algorithm above, where ν is the step size, and x^* is the value of x that maximizes $f_1(x)$ while satisfying the constraints. As $\nu \to 0$, the algorithm introduced here gives the same solution for (18). The proof of this is direct, as the problem is similar to the epigraph form in [16]. In the search for the optimal value of the variable x, it is made sure that f_1 is maximized and, simultaneously, f_2 has a value larger than or equal to τ, which is an iteration term that takes on the possible values of the bounded function f_2. Once the problem is solved, the value $f_1 + f_2$ is computed and compared to the largest saved value. If it is larger, it is then saved as the new largest value. If not, the algorithm continues to the next iteration. Eventually, the largest value is reached. This reformulation is relatively efficient to solve if, for each τ, the optimization problem inside the loop is convex. This requires that function f_1 is concave and the inequality constraint $f_2 > \tau$ can be cast in the form of a concave function greater than a constant [16]. We next show that (17) can be solved using the aforementioned algorithm.

OPTIMIZATION ALGORITHM

$\texttt{sum} = -\infty$

LOOP : $\tau = f_2^{min} : \nu : f_2^{max}$

$\max_x \quad f_1(x)$

$s.t. \quad 0 \le x \le 1$

$f_2(x) \ge \tau,$

$\texttt{dummy} = f_1(x^*) + f_2(x^*)$

if $\{\texttt{dummy} \ge \texttt{sum}\}$

$\texttt{sum} = \texttt{dummy}$

end if

end LOOP

First, we take the logarithm of the expression of μ_s in (16) before the maximization. This yields an equivalent problem that has the same solution since the $\log(.)$ is a monotonic function. The expression now becomes (19) on the next page. The last two terms in (19) are the logarithm of an affine function in a_i and hence are concave in a_i [16].

The first term, however, is the logarithm of a convex function in a_i (see Appendix B), which, in general, is not concave [16]. But since π_0 (the term inside the logarithm) is bounded between zero and one, we can use the *Optimization Algorithm* presented above to solve this optimization problem as follows. First, we divide the term $\log(\mu_s)$ into two parts, the first consists of the sum of all concave terms, and the second

$$\log(\mu_s) = \log\Bigg(\underbrace{1-\lambda_p\left[\left(1+\frac{M_p}{1-P^o_{pd}}\right)-\left(1-\sum_{i\in\mathcal{I}}p^1_i a_i\right)^{M_s}\right]}_{\pi_0}\Bigg) + \log(1-P^0_{sd}) + \log\left(\sum_{i\in\mathcal{I}}p^0_i a_i\right) + (M_s-1)\log\left(1-\sum_{i\in\mathcal{I}}p^0_i a_i\right). \tag{19}$$

consists of the non-concave term $\log(\pi_0)$. The two parts map into $f_1(x)$ and $f_2(x)$ in the *Optimization Algorithm* presented above, respectively. The second step is to apply the algorithm as follows

$\mathtt{sum} = -\infty$

LOOP : $\tau = 0 : \nu : 1$

$$\max_{a_i, i\in\mathcal{I}} \log\left(\sum_{i\in\mathcal{I}}p^0_i a_i\right) + (M_s-1)\log\left(1-\sum_{i\in\mathcal{I}}p^0_i a_i\right) \tag{20}$$

$s.t.\quad 0 \le a_i \le 1, \quad \forall i \in [1, n]$

$$\log\left(1-\sum_{i\in\mathcal{I}}p^1_i a_i\right) \ge \frac{1}{M_s}\log\left[f(\tau)\right]^+ \tag{21}$$

$\mathtt{dummy} = \mu_s(a^*_i)$

if $\{\mathtt{dummy} \ge \mathtt{sum}\}$

$\mathtt{sum} = \mathtt{dummy}$

end if

end LOOP,

where $f(\tau) = \frac{\tau}{\lambda_p} + \frac{M_p}{1-P^o_{pd}} - \frac{1-\lambda_p}{\lambda_p}$, and $[f(\tau)]^+$ denotes $\max(0, f(\tau))$. Accordingly, (20) is now concave and can be solved using standard convex optimization tools. It must be noted that the stability condition in (17) can be rewritten as

$$\log\left(1-\sum_{i\in\mathcal{I}}p^1_i a_i\right) \ge \frac{1}{M_s}\log\left(\left[\frac{M_p}{1-P^o_{pd}} - \frac{1-\lambda_p}{\lambda_p}\right]^+\right), \tag{22}$$

which is subsumed by the newly added constraint (21) as it corresponds to $\tau = 0$ and τ is non-negative. Therefore, we have managed to overcome the problem of non-convexity of the optimization problem in (17) via a simple algorithm which requires an exhaustive search over only one bounded parameter τ.

B. Primary Delay Analysis

In this subsection, we derive an expression for the average PU packet delay for the no feedback soft sensing scheme and for our proposed feedback-based scheme. We apply Little's formula [15],

$$D_p = \frac{E\{Q_p\}}{\lambda_p}, \tag{23}$$

to get the average delay for both schemes, where D_p is the average PU packet delay, and Q_p is the number of packets in the PU queue.

1) Baseline no Feedback Access Scheme: From the soft sensing scheme introduced in section III, one can easily show that for $k \ge 1$

$$\pi'_k = \frac{1}{1-\mu_p}\left(\frac{\lambda_p(1-\mu_p)}{(1-\lambda_p)\mu_p}\right)^k \pi'_0, \tag{24}$$

where π'_k is the probability that the PU has k packets in its queue. Thus, the expected number of packets in the queue is given by

$$E\{Q_p\} = \sum_{k=1}^{\infty} k\pi'_k = \sum_{k=1}^{\infty} k\frac{1}{1-\mu_p}\left(\frac{\lambda_p(1-\mu_p)}{(1-\lambda_p)\mu_p}\right)^k \pi'_0, \tag{25}$$

and since

$$\pi'_0 = 1 - \frac{\lambda_p}{\mu_p},$$

therefore we get

$$E\{Q_p\} = \frac{\lambda_p(1-\lambda_p)}{\mu_p-\lambda_p}. \tag{26}$$

Substituting by $E\{Q_p\}$ in (23) we finally get the average delay experienced by a PU packet

$$D_p = \frac{1-\lambda_p}{\mu_p-\lambda_p}. \tag{27}$$

2) Feedback-Based Access Scheme: The final expression of the delay formula is given by

$$D_p = \frac{(\Gamma_p-\chi)(\chi-\lambda_p)^2 + (1-\lambda_p)^2(1-\Gamma_p)\chi}{(1-\lambda_p)(1-\chi)(1-\delta)(\chi-\lambda_p)}. \tag{28}$$

(see Appendix C for proof).

V. Performance Results

In this section, we compare the performance of our proposed feedback-based scheme with two other schemes, namely the conventional (no feedback) soft sensing scheme, and the Neyman-Pearson (N-P) hard decision scheme [19]. We consider a system of $M_p = 4$ PUs and $M_s = 2$ SUs. Distance between the primary transmitters and receivers is set to 100 m, distance between the secondary transmitters and receivers is also set to 100 m, and the distance between any PU and any SU is set to 150 m. The SNR threshold ζ is 10 dB, the transmit power is 100 mW, the path loss exponent $\gamma = 3.7$, and $N_0 = 10^{-11}$ W/Hz. The energy threshold η is chosen by the N-P design for a false alarm rate of 0.1. The region below the threshold is divided into $n = 4$ regions each having a different access probability.

In Fig. 4, the SU throughput is plotted against the PU arrival rate. Different schemes are compared with respect to the upper bound acquired by perfect sensing; a scheme that

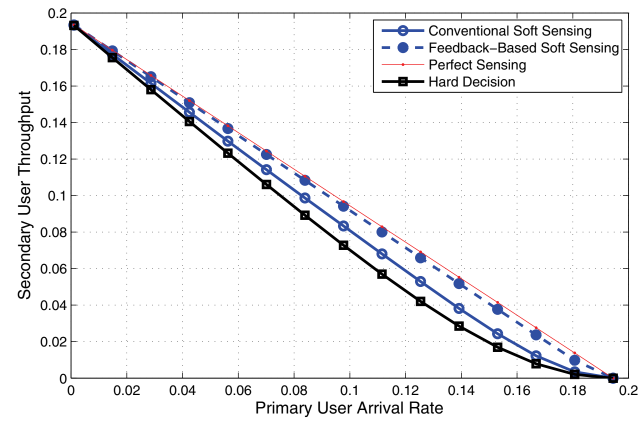

Fig. 4. Comparison between SU throughput of different schemes in a system of 4 PUs and 2 SUs.

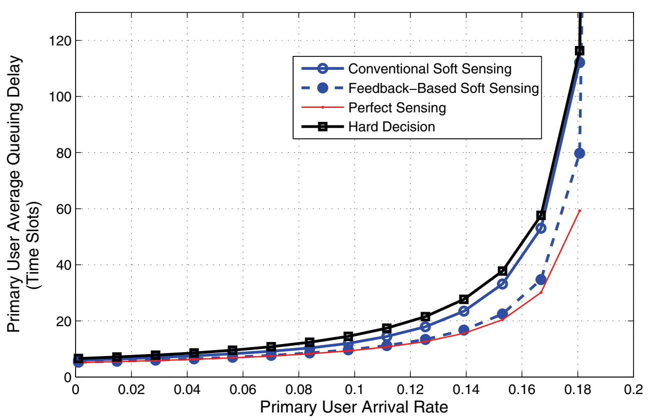

Fig. 5. Comparison between PU packet delay of different schemes in a system of 4 PUs and 2 SUs.

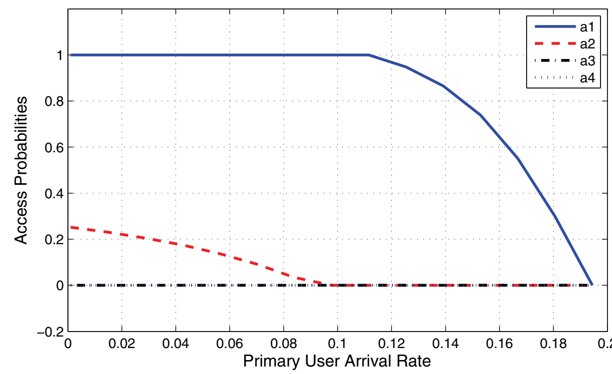

Fig. 6. SU access probabilities in a conventional soft sensing scheme in a system of 4 PUs and 2 SUs.

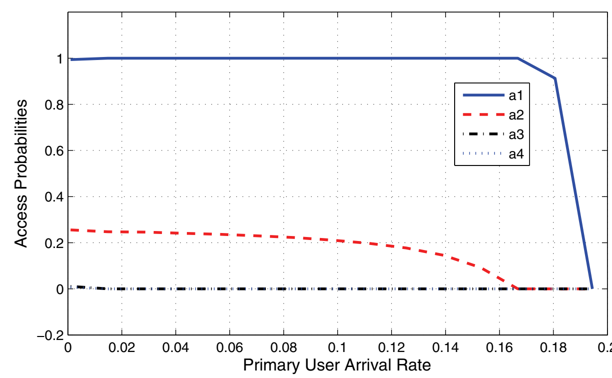

Fig. 7. SU access probabilities in a feedback-based soft sensing scheme in a system of 4 PUs and 2 SUs.

can be considered genie-aided, where the SU perfectly knows when the PU is idle. We can see that our proposed feedback-based scheme, when applied jointly with soft sensing, not only outperforms the conventional soft sensing one but also approaches the upper bound almost with equality in some regions. Also the N-P hard decision sensing scheme is plotted for completeness.

In Fig. 5, the average PU queuing delay is plotted against the PU arrival rate, we see that our proposed feedback-based scheme when applied jointly with soft sensing also outperforms the conventional soft sensing and the hard decision sensing ones in all regions.

Access probabilities for both the conventional soft sensing scheme and for the feedback-based one are plotted against the PU arrival rate in Fig. 6 and Fig. 7, respectively. From the figures, we can see that in both cases the two access probabilities closer to the decision threshold a_3 and a_4 are equal to zero for any given arrival rate. However, a_1 and a_2 have higher values in the feedback-based scheme than their counterparts in the conventional soft sensing one at relatively high arrival rates. This is attributed to the proper use of the primary feedback information by the SUs, which makes them avoid sure collisions, and thus enables them to access the channel more aggressively without affecting the PU's stability.

In order to gain more insights into how our proposed scheme performs with different number of SUs, we provide scalability results. For an arrival rate of $\lambda_p \simeq 0.1$ packets per time slot, a plot of PU packet delay for different schemes against the number of SUs is presented in Fig. 8. We see that our proposed feedback-based scheme is the nearest to the lower bound at any given number of SUs. We also notice that the PU packet delay curve converges to a certain level. This is due to the fact that the access probabilities change inversely proportional to the number of SUs in order to guarantee the stability of the PUs' queues. This opposite change also causes the PU service rate to converge to a certain level, thereby causing the delay to be constant. A plot of the PU service rate μ_p versus the number of SUs is provided in Fig. 9 for convenience.

Fig. 10 presents a result pertaining to the secondary network throughput $(M_s \times \mu_s)$ against the number of SUs, for $\lambda_p \simeq 0.1$ packets per time slot too. The network throughput seems to be slowly decreasing with the increase of SUs, however, our proposed feedback-based scheme outperforms both the conventional soft sensing and hard decision schemes at every given number of SUs and closely approaches the optimal scheme.

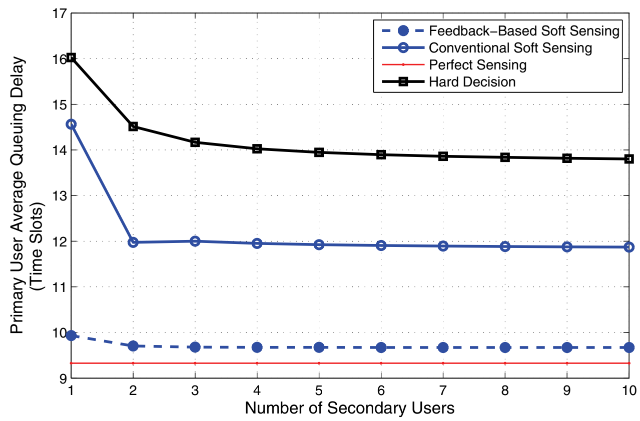

Fig. 8. PU packet delay vs. number of SUs at $\lambda_p \simeq 0.1$ and $M_p = 4$ PUs.

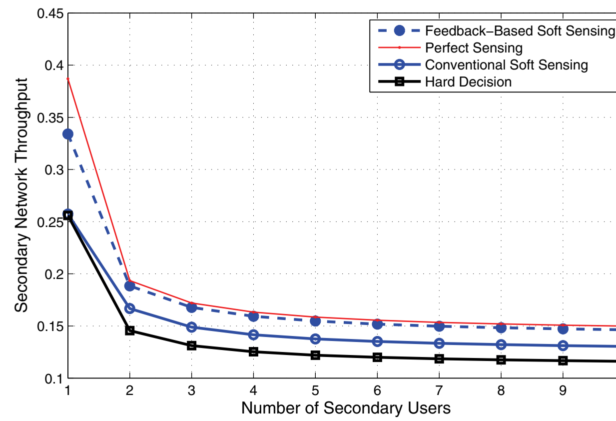

Fig. 10. Secondary network throughput vs. number of SUs at $\lambda_p \simeq 0.1$ and $M_p = 4$ PUs.

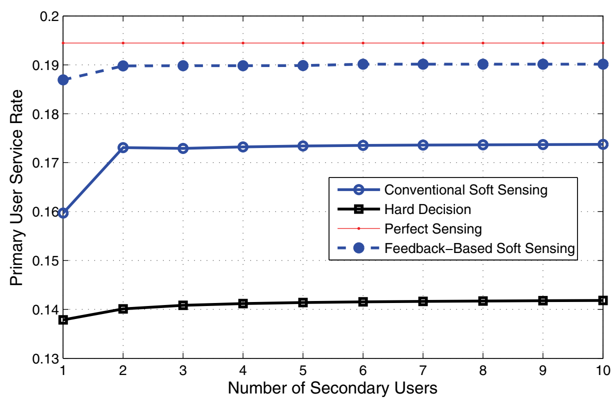

Fig. 9. PU service rate vs. number of SUs at $\lambda_p \simeq 0.1$ and $M_p = 4$ PUs.

VI. Conclusions and Future Work

In this paper, we examined a cognitive spectrum access scheme in which SUs exploit the primary feedback information. We considered a secondary network employing a random access scheme in which SUs access the channel by certain access probabilities that are functions of the sensing metric. We studied the cognitive radio network from a queuing theory point of view. Access probabilities are determined by solving an optimization problem subject to a constraint on the PU queue stability. In setting our problem, we assumed that SUs can eavesdrop on the primary link's feedback; SUs back-off completely from accessing the channel upon hearing a NACK, and attempt to access if an ACK/no feedback is overheard. This has led to significant results in both SU throughput and PU packet delay. Our proposed scheme outperforms both the soft sensing and the conventional hard decision sensing schemes and produces very close results to the *optimal* genie-aided scheme in which SUs have perfect knowledge of the activity of the PU and act upon it.

For future work, the problem can be studied after relaxing some assumptions in the system model. For instance, we assume symmetry among different links in the problem setting, at this fresh look at the problem, for mathematical tractability which enables us to focus on the fundamental concepts, derive closed form expressions for the secondary throughput and primary user delay, and transform it to a convex optimization problem. Studying the general asymmetric setting adds complexity due to increasing the problem dimensionality and number of optimization variables. However, one can investigate these asymmetrical effects on the SUs' throughput and the PUs' delay. Also, the effect of overhearing perfect primary feedback by the SUs can be relaxed in order to see whether imperfect feedback overhearing can still lead to improved results.

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Appendix A

Referring to the Markov chain in Fig. 3, we can write the global balance equation around state 0_F as follows

$$\pi_0 \lambda_p = \pi_1 \bar{\lambda}_p \Gamma_p + \epsilon_1 \bar{\lambda}_p \bar{\delta}, \tag{29}$$

where the notation $\bar{x} = 1 - x$. Writing the balance equation around state 1_R we get

$$\epsilon_1 (1 - \delta \bar{\lambda}_p) = \pi_1 \bar{\lambda}_p \bar{\Gamma}_p,$$

therefore, we have

$$\pi_1 = \epsilon_1 \frac{1 - \delta \bar{\lambda}_p}{\bar{\lambda}_p \bar{\Gamma}_p}. \tag{30}$$

Substituting from (30) in (29), we get

$$\epsilon_1 = \frac{\lambda_p \bar{\Gamma}_p}{\chi} \pi_0, \tag{31}$$

where $\chi = \lambda_p \Gamma_p + \bar{\lambda}_p \bar{\delta}$. Now using (31) in (30) we get

$$\pi_1 = \frac{\lambda_p (1 - \delta \bar{\lambda}_p)}{\bar{\lambda}_p \chi} \pi_0. \tag{32}$$

Writing the balance equation around state 1_F, we have

$$\pi_1 (1 - \lambda_p \Gamma_p) = \pi_0 \lambda_p + \epsilon_1 \lambda_p \bar{\delta} + \pi_2 \bar{\lambda}_p \Gamma_p + \epsilon_2 \bar{\lambda}_p \bar{\delta}.$$

Using (29) to substitute for the term $\pi_0\lambda_p$, we get

$$\pi_1\bar{\Gamma}_p = \epsilon_1\bar{\delta} + \pi_2\bar{\lambda}_p\Gamma_p + \epsilon_2\bar{\lambda}_p\bar{\delta}. \quad (33)$$

Using (31) and (32) into (33), we now have

$$\pi_2\bar{\lambda}_p\Gamma_p + \epsilon_2\bar{\lambda}_p\bar{\delta} = \frac{\lambda_p^2\bar{\Gamma}_p}{\bar{\lambda}_p\chi}\pi_0. \quad (34)$$

Writing the balance equation around state 2_R, we get

$$\epsilon_2(1-\delta\bar{\lambda}_p) = \epsilon_1\lambda_p\delta + \pi_1\lambda_p\bar{\Gamma}_p + \pi_2\bar{\lambda}_p\bar{\Gamma}_p,$$

but since from (31) and (32) we have

$$\epsilon_1\lambda_p\delta + \pi_1\lambda_p\bar{\Gamma}_p = \frac{\lambda_p^2\bar{\Gamma}_p}{\bar{\lambda}_p\chi}\pi_0,$$

therefore

$$\epsilon_2(1-\delta\bar{\lambda}_p) - \pi_2\bar{\lambda}_p\bar{\Gamma}_p = \frac{\lambda_p^2\bar{\Gamma}_p}{\bar{\lambda}_p\chi}\pi_0. \quad (35)$$

From (34) and (35) we can get the following

$$\epsilon_2 = \frac{\bar{\lambda}_p}{\lambda_p}\pi_2. \quad (36)$$

Therefore, using (36) in (34) we get

$$\epsilon_2 = \left(\frac{\lambda_p\bar{\chi}}{\bar{\lambda}_p\chi}\right)^2 . \frac{\bar{\lambda}_p\bar{\Gamma}_p}{\bar{\chi}^2}\pi_0, \quad (37)$$

$$\pi_2 = \left(\frac{\lambda_p\bar{\chi}}{\bar{\lambda}_p\chi}\right)^2 . \frac{\lambda_p\bar{\Gamma}_p}{\bar{\chi}^2}\pi_0. \quad (38)$$

From the symmetry of the incoming states in the Markov chain, one can expect that equation (36) can be generalized for any ϵ_k and π_k with $k \geq 2$, since all incoming balance equations will give the same result. Also this applies for the last two equations. Verification of this is straight forward but it is omitted due to space limits.

Therefore, we can now write the following results

- $\epsilon_0 = 0$.
- $\epsilon_1 = \frac{\lambda_p\bar{\Gamma}_p}{\chi}\pi_0$.
- $\pi_1 = \frac{\lambda_p(1-\delta\bar{\lambda}_p)}{\bar{\lambda}_p\chi}\pi_0$.

And for $k \geq 2$ we have:

- $\epsilon_k = \left(\frac{\lambda_p\bar{\chi}}{\bar{\lambda}_p\chi}\right)^k . \frac{\bar{\lambda}_p\bar{\Gamma}_p}{\bar{\chi}^2}\pi_0$.
- $\pi_k = \frac{\lambda_p}{\bar{\lambda}_p}\epsilon_k$.

Using the normalization condition $\sum_{k=0}^{\infty}(\pi_k+\epsilon_k) = 1$, one can get the value of π_0. First, we will divide the summation as follows

$$\sum_{k=0}^{\infty}(\pi_k+\epsilon_k) = \pi_0 + \underbrace{(\pi_1+\epsilon_1)}_{A} + \underbrace{\sum_{k=2}^{\infty}(\pi_k+\epsilon_k)}_{B} = 1. \quad (39)$$

Simplifying the term B, since we have

$$\pi_k + \epsilon_k = \psi^k\frac{\bar{\Gamma}_p}{\bar{\chi}^2}\pi_0, \quad \text{where} \quad \psi = \frac{\lambda_p\bar{\chi}}{\bar{\lambda}_p\chi}.$$

Hence,

$$B = \frac{\bar{\Gamma}_p\pi_0}{\bar{\chi}^2}\sum_{k=2}^{\infty}\psi^k = \left(\frac{\lambda_p\bar{\Gamma}_p}{\bar{\lambda}_p\chi}\right)\left(\frac{\lambda_p}{\chi-\lambda_p}\right)\pi_0. \quad (40)$$

The last summation only exists when $\psi < 1$, that is equivalent to $\lambda_p < \chi$, we will see later that this is the stability condition for the PU queue. Now simplifying the term A

$$A = \frac{\lambda_p\bar{\Gamma}_p}{\bar{\lambda}_p\chi}\underbrace{\left(\bar{\lambda}_p + \frac{1-\delta\bar{\lambda}_p}{\bar{\Gamma}_p}\right)}_{C}\pi_0.$$

After some manipulations, we can have $C = \frac{\chi+\bar{\Gamma}_p}{\bar{\Gamma}_p}$, therefore the term A can finally be written as:

$$A = \left(\frac{\lambda_p\bar{\Gamma}_p}{\bar{\lambda}_p\chi}\right)\left(\frac{\chi+\bar{\Gamma}_p}{\bar{\Gamma}_p}\right)\pi_0. \quad (41)$$

From (40) and (41), one can write

$$A + B = \frac{\lambda_p\bar{\Gamma}_p}{\bar{\lambda}_p\chi}\underbrace{\left(\frac{\lambda_p}{\chi-\lambda_p} + \frac{\chi+\bar{\Gamma}_p}{\bar{\Gamma}_p}\right)}_{D}. \quad (42)$$

After some involved manipulations, the term D can be rearranged and written as

$$D = \frac{\bar{\lambda}_p\chi(\bar{\delta}+\bar{\Gamma}_p)}{\bar{\Gamma}_p(\chi-\lambda_p)}.$$

Substituting for D in (42) we get

$$A + B = \frac{\lambda_p(\bar{\Gamma}_p+\bar{\delta})}{\chi-\lambda_p}\pi_0. \quad (43)$$

Using this final result of (43) in (39), we can write the value of π_0 as shown in (44) on the next page, which can be checked to satisfy the balance equation given in (29).

Appendix B

Here, we provide the convexity proof of the term π_0 in (19). We shall use the property that states that a function is convex if and only if it is convex when restricted to a line that intersects its domain [16]. Let $\mathbf{a}$ be a column vector with elements a_i, and $\mathbf{P}_1$ be a column vector with the elements p_i^1. Therefore we have $\pi_0 = 1-\lambda_p(1+\frac{M_p}{1-P_{pd}^o})+\lambda_p\big(1-\mathbf{P}_1^T\mathbf{a}\big)^{M_s}$, where the superscript T is the transpose operation. We form the function

$$g(t) = 1-\lambda_p(1+\frac{M_p}{1-P_{pd}^o})+\lambda_p\big(1-\mathbf{P}_1^T\bar{\mathbf{a}}-t\mathbf{P}_1^T\mathbf{v}\big)^{M_s}, \quad (45)$$

where t is a scalar parameter, $\bar{\mathbf{a}}$ belongs to the domain of the problem, and $\mathbf{v}$ is a vector such that $\bar{\mathbf{a}}+t\mathbf{v}$ also belongs to the domain of the problem. The domain of the problem is specified by the inequality constraints found in (17). According to the aforementioned convexity condition, if $g(t)$ is proved to be convex with respect to t, hence, π_0 will be considered convex with respect to $\mathbf{a}$. The convexity of $g(t)$ can be easily verified by differentiating it twice and examining the sign of the second derivative as follows

$$\ddot{g}(t) = \lambda_pM_s(M_s-1)\big(1-\mathbf{P}_1^T\bar{\mathbf{a}}-t\mathbf{P}_1^T\mathbf{v}\big)^{M_s-2}(\mathbf{P}_1^T\mathbf{v})^2 \geq 0, \forall t. \quad (46)$$

Therefore, the term π_0 is convex in $\mathbf{a}$.

$$\begin{aligned}\pi_0 = \frac{\chi-\lambda_p}{\bar{\delta}} &= \frac{\lambda_p(1-P_{pd}^o)\left(1-\sum_{i\in\mathcal{I}}p_i^1 a_i\right)^{M_s}+(1-\lambda_p)(1-P_{pd}^o)-M_p\lambda_p}{1-P_{pd}^o}\\ &= 1-\lambda_p\left[\left(1+\frac{M_p}{1-P_{pd}^o}\right)-\left(1-\sum_{i\in\mathcal{I}}p_i^1 a_i\right)^{M_s}\right], \end{aligned} \tag{44}$$

$$E\{Q_p\} = A + A' = \left(\frac{\lambda_p\bar{\Gamma}_p}{\bar{\lambda}_p\chi}\right)\left(\frac{\chi-\lambda_p}{\bar{\delta}}\right)\Bigg[\underbrace{\frac{\chi+\bar{\Gamma}_p}{\bar{\Gamma}_p}+\frac{(\bar{\lambda}_p\chi)^2-(\chi-\lambda_p)^2}{\bar{\chi}(\chi-\lambda_p)}}_{\mathcal{K}}\Bigg]. \tag{50}$$

APPENDIX C

Here we prove the delay formula in (28). In this case, the expected number of packets in the primary queue is given by

$$E\{Q_p\} = \underbrace{\pi_1+\epsilon_1}_{A} + \underbrace{\sum_{k=2}^{\infty} k(\pi_k+\epsilon_k)}_{A'}, \tag{47}$$

and since we already calculated the first part before (see equation (41)), we only have to calculate the term A'. As before, let $\psi = \frac{\lambda_p\bar{\chi}}{\bar{\lambda}_p\chi}$, therefore we have

$$A' = \frac{\bar{\Gamma}_p}{\bar{\chi}^2}\psi\left(\sum_{k=1}^{\infty} k\psi^{k-1}-1\right). \tag{48}$$

After some simplifications we can finally get

$$A' = \left(\frac{\lambda_p\bar{\Gamma}_p}{\bar{\lambda}_p\chi}\right)\left(\frac{1}{\bar{\delta}\bar{\chi}}\right)\left(\frac{(\bar{\lambda}_p\chi)^2-(\chi-\lambda_p)^2}{\chi-\lambda_p}\right). \tag{49}$$

Therefore, from (41) and (49), we can write $E\{N_p\}$ as (50) at the top of this page. After some simplifications, the term $\mathcal{K}$ can be simplified to

$$\mathcal{K} = \frac{\chi}{\bar{\chi}}\left(\frac{(\Gamma_p-\chi)(\chi-\lambda_p)^2+\bar{\lambda}_p^2\bar{\Gamma}_p\chi}{\bar{\Gamma}_p(\chi-\lambda_p)^2}\right). \tag{50}$$

Substituting by $\mathcal{K}$ in $E(N_p)$, we can finally write the delay formula as

$$D_p = \frac{(\Gamma_p-\chi)(\chi-\lambda_p)^2+(1-\lambda_p)^2(1-\Gamma_p)\chi}{(1-\lambda_p)(1-\chi)(1-\delta)(\chi-\lambda_p)}. \tag{51}$$

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Ahmed Arafa received his B.Sc. with highest honors in Electrical Engineering from Alexandria University, Alexandria, Egypt, in July 2010, and his M.Sc. in Wireless Communications from Nile University, Cairo, Egypt, in August 2012. His thesis work was on the analysis of feedback-based access schemes for cognitive radio networks. Since August 2012, he has been a graduate research assistant in the Electrical and Computer Engineering department at the University of Maryland, College Park, working towards his Ph.D. degree. His research focuses on information and network theoretical aspects of energy harvesting communication systems. He is a graduate student member of the IEEE.

Karim G. Seddik (S 04, M 08) is an assistant professor in the Electronics Engineering Department at the American University in Cairo (AUC), Egypt. Before joining AUC, he was an assistant professor in the Electrical Engineering Department at Alexandria University, Egypt. Dr. Seddik received the B.S. (with highest honors) and M.S. degrees in electrical engineering from Alexandria University, Alexandria, Egypt, in 2001 and 2004, respectively. He received his Ph.D. degree at the Electrical and Computer Engineering Department, University of Maryland, College Park 2008. His research interests include cooperative communications and networking, MIMO–OFDM systems, cognitive radio, layered channel coding, and distributed detection in wireless sensor networks.

Dr. Seddik is a recipient of the Certificate of Honor from the Egyptian President for being ranked first among all departments in College of Engineering, Alexandria University in 2002, the Graduate School Fellowship from the University of Maryland in 2004 and 2005 and the Future Faculty Program Fellowship from the University of Maryland in 2007.

Ahmed Sultan-Salem (Member, IEEE) received the B.S. and M.S. degrees in electrical engineering from Alexandria University, Egypt. He finished his Ph.D. from the Electrical Engineering Department at Stanford University in 2007. His thesis work was focused on the development of new models and signal analysis techniques to infer planetary surface parameters from data acquired by earth-based and spaceborne remote sensing experiments. He is an assistant professor at the Electrical Engineering Department, Alexandria University, Egypt. His current research interests are dynamic spectrum access, cognitive radio networks, OFDM for optical communications, distributed and sequential detection, cooperative communications, physical layer-based secrecy, and synthetic aperture radar.

Tamer ElBatt received the B.S. and M.S. degrees in Electronics and Communications Engineering from Cairo University, Giza, Egypt, in 1993 and 1996, respectively, and the Ph.D. degree in Electrical and Computer Engineering from the University of Maryland, College Park, MD, in 2000. From 2000 to 2009 he was with major U.S. industry R&D labs, e.g., HRL Laboratories and Lockheed Martin ATC, at various positions. In July 2009, he joined the Electronics & Communications Dept., Faculty of Engineering, Cairo University as an Assistant Professor. He also holds an appointment with the School of Communication and Information Technology and the Wireless Intelligent Networks Center (WINC) at Nile University, Cairo, Egypt. Dr. ElBatt research has been supported by DARPA, General Motors and Boeing and is currently being supported by Qatar QNRF, the Egyptian NTRA, ITIDA, EU FP7, General Motors, Microsoft and Google. He has published more than 60 papers in prestigious journals and major international conferences. Dr. ElBatt holds seven issued U.S. patents.

Dr. ElBatt is a Senior Member of the IEEE and has served on the technical program committees of numerous IEEE and ACM conferences in the areas of wireless networks and mobile computing. He is the Demos Co-Chair of ACM Mobicom 2013 and the Publications Co-Chair of IEEE Globecom 2012. Dr. ElBatt currently serves on the Editorial Board of IEEE Transactions on Mobile Computing and Wiley International Journal of Satellite Communications and Networking. Dr. ElBatt has also seved on NSF and Fulbright review panels. He has been invited to Google's EMEA Faculty Summit in Zurich, Feb. 2010. In Aug. 2010, Dr. ElBatt was a Visiting Professor at the Dept. of Electronics, Politecnico di Torino, Italy. His research has thus far collected more than 1900 citations on the Google Scholar Index and has been cited by media, such as *EE Times* and *Information Week*. Dr. ElBatt is the recipient of the 2002, 2004 HRL Achievement Award. His research interests lie in the broad areas of performance analysis and design of wireless and mobile networks with emphasis on cognitive radios and networks, cooperative networking, cross-layer optimization, MAC, sensor and vehicular networks and emerging mobile applications.

Amr A. El-Sherif (S 00, M 08) received his B.Sc. (with highest Honors) and M.Sc. degrees in electrical engineering form Alexandria University, Alexandria, Egypt in 2002 and 2005, respectively. He received his Ph.D. degree in electrical engineering from the University of Maryland, College Park, in 2009. He is currently an Assistant Professor in the Electrical Engineering Department at Alexandria University, Egypt. His research interests include cooperative communications and networking, cross-layer design for wireless networks, multiple access technologies for wireless and sensor networks, and spectrum sharing and cognitive radio systems.