UNC Charlotte 2011 Comprehensive

March 7, 2011

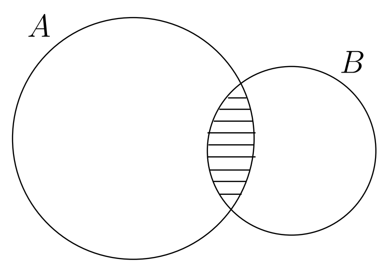
1. Three cowboys entered a saloon. The first ordered 4 sandwiches, a cup of coffee, and 10 doughnuts for \$8.45. The second ordered 3 sandwiches, a cup of coffee, and 7 doughnuts for \$6.30. How much did the third cowboy pay for a sandwich, a cup of coffee, and a doughnut?

   (A) \$2.00 **(B)** \$2.05 **(C)** \$2.10 **(D)** \$2.15 **(E)** \$2.20

2. Given that $a+b+c=5$ and $ab+bc+ac=5$, what is the value of $a^2+b^2+c^2$?

   (A) 5 **(B)** 10 **(C)** 15 **(D)** 20 **(E)** 25

3. Two circles, A and B, overlap each other as shown. The area of the common part is 2/5 of the area of Circle A, and 5/8 of the area of Circle B. What is the ratio of the radius of Circle A to that of Circle B?

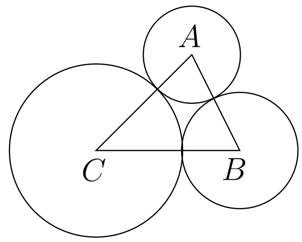


   (A) $2:1$ **(B)** $3:2$ **(C)** $4:3$ **(D)** $5:4$ **(E)** $6:5$

4. A pine tree is 14 yards high, and a bird is sitting on its top. The wind blows away a feather of the bird. The feather moves uniformly along a straight line at the speed 4 yards per second; it falls on the ground 4.5 seconds later at a distance D yards from the pine tree's base. Which of the following intervals contains the number D?

   (A) $(0,10]$ **(B)** $(10,11]$ **(C)** $(11,12]$ **(D)** $(12,13]$ **(E)** $(13,\text{inf})$

5. Three mutually tangent circles have centers A, B, C and radii a, b, and c respectively. The lengths of segments AB, BC, CA are 17, 23, and 12 respectively. Find the lengths of the radii.



(A) $a = 13, b = 9, c = 7$ **(B)** $a = 5, b = 12, c = 6$ **(C)** $a = 6, b = 9, c = 12$

(D) $a = 4, b = 8, c = 12$ **(E)** $a = 3, b = 14, c = 9$

6. A ladder is leaning against a wall with the top of the ladder 8 feet above the ground. If the bottom of the ladder is moved 2 feet farther from the wall, the top of the ladder slides all the way down the wall and rests against the foot of the wall. How long is the ladder?

   (A) 14 feet **(B)** 15 feet **(C)** 16 feet **(D)** 17 feet **(E)** 18 feet

7. Each person in a group of five makes a statement: Amy says "At least one of us is lying." Ben says "At least two of us are lying." Carrie says "At least four of us are lying." Donna says "All of us are lying." Eddie says "None of us is lying." Based on these statements, how many are telling the truth?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

8. The hour and minute hands of a clock are exactly 50 degrees apart at which of these times?

   (A) 2 : 20 **(B)** 5 : 36 **(C)** 7 : 50 **(D)** 9 : 30 **(E)** 11 : 10

9. The language of a pre-historic tribe is very simple and it contains only the 4 letters A, M, N, and O. Every word in this language has at most 4 letters. What is the largest number of words that can be created if every word must contain at least one vowel (A or O)?

   (A) 256 **(B)** 310 **(C)** 340 **(D)** 400 **(E)** 420

10. English and American spellings are 'rigour' and 'rigor', respectively. A randomly chosen man staying at a Parisian hotel writes this word, and a letter taken at random from his spelling is found to be a vowel. If 40 percent of the English-speaking men at the hotel are English and 60 percent are Americans, what is the probability that the writer is an Englishman?

    (A) 1/4 **(B)** 1/3 **(C)** 2/5 **(D)** 5/11 **(E)** 1/2

11. A rational number $3.abc$ when rounded to the nearest tenth is 3.5. If $a+b+c = 8$ and $a > b > c > 0$, what is the value of $2a + bc$?

    (A) 17 **(B)** 14 **(C)** 12 **(D)** 11 **(E)** 10

12. A farmer has 200 yards of fencing material. What is the largest rectangular area he can enclose if he wants to use a 4 yard wide gate that does not need to be covered by the fencing material?

    (A) $2,500$ square yards **(B)** $2,601$ square yards **(C)** $2,704$ square yards **(D)** $2,809$ square yards **(E)** $2,916$ square yards

13. Find the value of $b^{-\log_b(a)}$.

    (A) $\frac{b}{a}$ **(B)** $\frac{a}{b}$ **(C)** b **(D)** $\frac{1}{a}$ **(E)** $\frac{1}{b}$

14. Find the real numbers m, n such that $m + n = 3$, and $m^3 + n^3 = 117$. What is the value of $m^2 + n^2$?

    (A) 29 **(B)** 3 **(C)** 9 **(D)** 17 **(E)** 45

15. A taxicab driver charges \$3.00 for the first half mile or less and \$0.75 for each quarter mile after that. If f represents the fare in dollars, which of the following functions models the fare for a ride of x miles, where x is a positive integer?

    (A) $f(x) = 3.00 + 0.75(x - 1)$ **(B)** $f(x) = 3.00 + 0.75(x/4 - 1)$

    (C) $f(x) = 4.50 + 3(x - 1)$ **(D)** $f(x) = 3.00 + 3(x - 1)$

    (E) $f(x) = 3.00(x + 1)$

16. The cells of an infinite chess board are labeled with two integers each (the number of the column and that of the row). A child rolls a very small ball that starts from the center of the cell (8,11). There are tiny bugs at the centers of some cells. The ball rolls along a straight line; it hits a bug if and only if it rolls exactly through the center of a cell containing a bug. This does not affect the subsequent motion of the ball. It is known that the ball has hit a bug at the center of the cell (20,24). The other bugs are at the centers of the cells listed below. Which of them will be hit?

    (A) $(68, 75)$ **(B)** $(69, 76)$ **(C)** $(67, 75)$ **(D)** $(69, 77)$ **(E)** $(68, 76)$

17. Football teams score $1, 2, 3$,or 6 points at a time. They can score 1 point (point-after-touchdown) only immediately after scoring 6 points (a touchdown). A scoring sequence is a sequence of numbers $1, 2, 3, 6$, where all the 1's are immediately preceded by 6. Both $2, 6, 1, 3, 2$ and $3, 3, 3, 3, 2$ are scoring sequences with *value* 14. How many scoring sequences have value 10?

    (A) 12 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 18

18. Let $r = 11 \cdot \sqrt{10!}, s = 10 \cdot \sqrt{11!}, t = \sqrt{12!}$. Rank the numbers r, s, t from smallest to largest.

    (A) $r < t < s$ **(B)** $r < s < t$ **(C)** $t < r < s$ **(D)** $t < s < r$ **(E)** $s < t < r$

19. The integers $3, 4, 5, 6, 12$, and 13 are arranged, without repetition, in a horizontal row so that the sum of any two numbers in adjoining positions is a perfect square (the square of an integer). Find the sum of the first and last.

    (A) 9 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 17

20. You are walking across a very large field with no obstructions when you see your friend, Sandy, Y yards to your north. Sandy is walking at 1 yard per second toward the north-east. If you walk at a steady speed of 2 yards per second and Sandy does not alter her speed or direction, what is the shortest distance (measured in yards) you must walk before you catch up with her?

    (A) $2Y\left(\frac{\sqrt{3}}{2}\right)$ **(B)** $2Y\left(\frac{\sqrt{3}+\sqrt{8}}{6}\right)$ **(C)** $2Y\left(\frac{1+\sqrt{3}}{3}\right)$

    (D) $2Y\left(\frac{\sqrt{2}+\sqrt{14}}{6}\right)$ **(E)** $2Y\left(\frac{\sqrt{2}+\sqrt{15}}{6}\right)$

    This problem can also be solved using the law of cosines.

21. Let n be the product of four consecutive positive integers. How many of the numbers 4, 8, 10, 12, and 15 must be divisors of n?

    (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

22. The inequality $|x+3| \leq 2$ is equivalent to the inequality $a \leq \frac{6}{x+7} \leq b$. Find the value of a^2+b.

    (A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

    $isequivalentto$-$2 \leq x+3 \leq 2$. Adding 4 to each part of this inequality gives $2 \leq x+7 \leq 6$. Therefore, $\frac{1}{6} \leq \frac{1}{x+7} \leq \frac{1}{2}$ or $1 \leq \frac{6}{x+7} \leq 3$. Thus, $a=1$ and $b=3$.

23. What is the area of a triangle with sides of length 13, 16, and $\sqrt{41}$?

    (A) 36 **(B)** 38 **(C)** 40 **(D)** 42 **(E)** 44

24. Suppose that $f(x)=ax+b$, where a and b are real numbers. Given that $f(f(f(x)))=8x+21$, what is the value of $a+b$?

    (A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

25. For how many integers n is the value of $\frac{n}{50-n}$ the square of an integer?

    (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5