# UNC Charlotte 2011 Algebra
March 7, 2011

1. Find the product of all real solutions of $16^{x^2+x+4} = 32^{x^2+2x}$.

   (A) -18 (B) -16 (C) -15 (D) -12 (E) -10

2. If $4(9a - 13b) = 6(a - 2b)$ and $b \neq 0$, what is the ratio of a to b?

   (A) $3:4$ (B) $3:5$ (C) $1:1$ (D) $5:3$ (E) $4:3$

3. The function f satisfies the equation $f(x) + 2f(3 - x) = 4x + 5$ for all real x. What is the value of $f(1)$?

   (A) $\dfrac{17}{3}$ (B) $\dfrac{16}{3}$ (C) 5 (D) $\dfrac{14}{3}$ (E) 4

4. Each person in a group of five makes a statement: Amy says "At least one of us is lying." Ben says "At least two of us are lying." Carrie says "At least four of us are lying." Donna says "All of us are lying." Eddie says "None of us is lying." Based on these statements, how many are telling the truth?

   (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

5. A worm is slowly crawling to the top of a pole 70cm high. During the day it advances 7cm and during the night it slips down 4cm. When will it finally reach the top?

   (A) In 24 days   (B) In 23 days   (C) In 22 days

   (D) In 21 days   (E) In 20 days

6. Three cowboys entered a saloon. The first ordered 4 sandwiches, a cup of coffee, and 10 doughnuts for $8.45. The second ordered 3 sandwiches, a cup of coffee, and 7 doughnuts for $6.30. How much did the third cowboy pay for a sandwich, a cup of coffee, and a doughnut?

   (A) $2.00   (B) $2.05   (C) $2.10   (D) $2.15   (E) $2.20

7. Given that $a + b + c = 5$ and $ab + bc + ac = 5$, what is the value of $a^2 + b^2 + c^2$?

   (A) 5 (B) 10 (C) 15 (D) 20 (E) 25

8. Two circles, A and B, overlap each other as shown. The area of the common part is 2/5 of the area of Circle A, and 5/8 of the area of Circle B. What is the ratio of the radius of Circle A to that of Circle B?
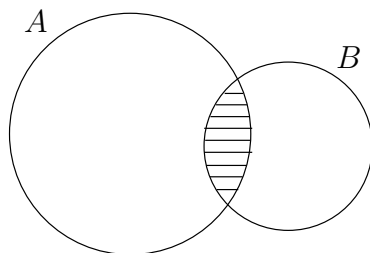
   (A) $2:1$ **(B)** $3:2$ **(C)** $4:3$ **(D)** $5:4$ **(E)** $6:5$

9. For integers a and b, the quadratic equation $x^2 + ax + 4b = 0$ has one solution at $x = 1$ and the other solution is between 2 and 5. Find the value of ab.

   (A) -5 **(B)** -3 **(C)** -1 **(D)** 4 **(E)** 6

10. A pine tree is 14 yards high, and a bird is sitting on its top. The wind blows away a feather of the bird. The feather moves uniformly along a straight line at the speed 4 yards per second; it falls on the ground 4.5 seconds later at a distance D yards from the pine tree's base. Which of the following intervals contains the number D?

   (A) $(0, 10]$ **(B)** $(10, 11]$ **(C)** $(11, 12]$ **(D)** $(12, 13]$ **(E)** $(13, infinity)$

11. If Bob can beat Jim by one-tenth of a mile in a two-mile race and Jim can beat Henry by one-fifth of a mile in a two mile race, by what distance could Bob beat Henry in a two-mile race?

   (A) 0.13 miles   **(B)** 0.20 miles   **(C)** 0.29 miles

   (D) 0.37 miles   **(E)** 0.53 miles

12. Three mutually tangent circles have centers A, B, C and radii a, b, and c respectively. The lengths of segments AB, BC, CA are 17, 23, and 12 respectively. Find the lengths of the radii.
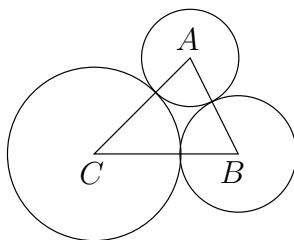
    (A) $a = 13, b = 9, c = 7$ **(B)** $a = 5, b = 12, c = 6$ **(C)** $a = 6, b = 9, c = 12$
    (D) $a = 4, b = 8, c = 12$ **(E)** $a = 3, b = 14, c = 9$

13. A ladder is leaning against a wall with the top of the ladder 8 feet above the ground. If the bottom of the ladder is moved 2 feet farther from the wall, the top of the ladder slides all the way down the wall and rests against the foot of the wall. How long is the ladder?

    (A) 14 feet   **(B)** 15 feet   **(C)** 16 feet   **(D)** 17 feet   **(E)** 18 feet

14. Find the real numbers m, n such that $m + n = 3$, and $m^3 + n^3 = 117$. What is the value of $m^2 + n^2$?

    (A) 29   **(B)** 3   **(C)** 9   **(D)** 17   **(E)** 45

15. A right triangle has legs of length 8 and 15. What is the radius of the inscribed circle?

    (A) 1   **(B)** 1.5   **(C)** 2   **(D)** 2.5   **(E)** 3

16. A farmer has 200 yards of fencing material. What is the largest rectangular area he can enclose if he wants to use a 4 yard wide gate that does not need to be covered by the fencing material?

    (A) $2,500$ square yards   **(B)** $2,601$ square yards   **(C)** $2,704$ square yards
    (D) $2,809$ square yards   **(E)** $2,916$ square yards

17. Suppose that $f(x) = ax + b$, where a and b are real numbers. Given that $f(f(f(x))) = 8x + 21$, what is the value of $a + b$?

    (A) 2   (B) 3   (C) 4   (D) 5   (E) 6

18. The points $(2, k)$ and $(5, 5)$ belong to the line perpendicular to the line $3x - 2y = 7$. Find the value of k.

    (A) 3   (B) 4   (C) 5   (D) 6   (E) 7

19. What is the product of all solutions of $||2x - 5| - 3| = 2$?

    (A) -12 (B) 0   (C) 6   (D) 12   (E) 30

20. What is the radius of the circle with equation $x^2 - 4x + y^2 + 6y = 3$?

    (A) $\sqrt{3}$ (B) 2   (C) $\sqrt{5}$ (D) 3   (E) 4

21. Suppose $2 \log x + \log 4 - \log 2x = 1$. Find the value of $x^2 + x$.

    (A) 12   (B) 20   (C) 30   (D) 42   (E) 56

22. What is the coefficient of x^5 in the polynomial $(x + 2)^8$?

    (A) 112   (B) 224   (C) 448   (D) 560   (E) 896

23. Given that $x - \frac{1}{x} = 2\sqrt{3}$ for some $x \neq 0$. find $(x^2 - \frac{1}{x^2})^2$.

    (A) 75   (B) 108   (C) 147   (D) 192   (E) 243

24. Which of the following numbers is a perfect square?

    (A) $23! \cdot 24!$ (B) $24! \cdot 25!$ (C) $25! \cdot 26!$ (D) $26! \cdot 27!$ (E) $27! \cdot 28!$

25. Find the sum of all the zeros of the function $F(x) = 18x^3 - 9x^2 - 5x + 2$.

    (A) 0.25   (B) 0.5   (C) 0.75   (D) 1.00   (E) 1.25