

UNC Charlotte 2006 Algebra

March 6, 2006

1. The name 'algebra' derives its origin from
 - (A) a middle eastern terrorist organization
 - (B) an insidious green plant
 - (C) a piece of the female wardrobe
 - (D) evidence of bad dental hygiene
 - (E) an arabic text on 'Restoring and Simplification'

2. What is the length of the interval of solutions to the inequality $1 \leq 3-4x \leq 11$?
 - (A) 1.75
 - (B) 2.00
 - (C) 2.25
 - (D) 2.50
 - (E) 3.25

3. What is the sum of the digits of the integer solution to $\sqrt{14 + \sqrt{27 - \sqrt{x - 1}}} = 4$?
 - (A) 5
 - (B) 6
 - (C) 8
 - (D) 9
 - (E) 11

4. Scott and Adam have some chickens and horses. When asked how many chickens and horses they had altogether, Adam said, "There are a total of 17 heads and 42 feet. I have three times as many chickens as horses." "Yes," replied Scott, "and we both have the same number of horses." How many chickens does Scott have?
 - (A) 2
 - (B) 4
 - (C) 6
 - (D) 7
 - (E) 13

5. In a box there are red and blue balls. If you select a handful of them with eyes closed, you have to grab at least 5 of them to make sure at least one of them is red and you have to grab at least 10 of them to make sure both colors appear among the balls selected. How many balls are there in the box?
- (A) 10 (B) 11 (C) 12 (D) 13 (E) 14
6. Some hikers start on a walk at 9 a.m. and return at 2 p.m. One quarter of the distance walked is uphill, one half is level, and one quarter is downhill. If their speed is 4 miles per hour on level land, 2 miles per hour uphill, and 6 miles per hour downhill, approximately how far did they walk?
- (A) 16.4 miles (B) 17.1 miles (C) 18.9 miles
(D) 20.0 miles (E) 21.2 miles
7. Suppose a, b, c are integers such that
1. $0 < a < b$,
 2. The polynomial $x(x - a)(x - b) - 17$ is divisible by $(x - c)$.
- What is $a + b + c$?
- (A) 14 (B) 17 (C) 21 (D) 24 (E) 27
8. Let u and v be the solutions to $2x^2 - 3x + c = 0$. If $2uv = 5$, find $u + v + c$.
- (A) -1 (B) -1.5 (C) 6.5 (D) 8.5 (E) 10
9. Let x, y be positive integers with $x > y$. If $1/(x + y) + 1/(x - y) = 1/3$, find $x^2 + y^2$.
- (A) 52 (B) 58 (C) 65 (D) 73 (E) 80
10. If x and y are integers, under what conditions is $x^2 + xy + (x - y)$ odd?
- (A) x is odd and y is odd (B) x is odd and y is even (C) x is even and y is odd
(D) x is even and y is even (E) The expression is always even

11. In five years Vic will be half as old as his dad. Twenty five years ago he was $\frac{1}{8}$ as old as his dad. How old was his dad on the day Vic was born?

(A) 20 (B) 25 (C) 30 (D) 35 (E) 40

12. A man's salary is reduced by p percent. By what percent would his salary then have to be raised to bring it back to the original amount?

(A) $\frac{2p}{100 - p}$ (B) $\frac{p - 100}{100 - 2p}$ (C) $\frac{100p}{100 - p}$

(D) $\frac{p}{p - 100}$ (E) $\frac{2p}{p - 100}$

13. Benny eats a box of cereal in 14 days. He eats the same size box of cereal with his younger brother Nathan in 10 days. How many days will it take Nathan to finish the box of cereal alone?

(A) 20 (B) 25 (C) 30 (D) 35 (E) 40

14. Given the following system of equations

$$\begin{aligned}\frac{1}{x} + \frac{1}{y} &= \frac{1}{3} \\ \frac{1}{x} + \frac{1}{z} &= \frac{1}{5} \\ \frac{1}{y} + \frac{1}{z} &= \frac{1}{7}\end{aligned}$$

What is the value of the ratio $\frac{z}{y}$?

(A) 17 (B) 23 (C) 29 (D) 31 (E) 36

15. A two-digit number N is 10 more than 3 times the sum of its digits. The units digit is 1 more than twice the 10's digit. Find the product of the digits.

(A) 24 (B) 28 (C) 30 (D) 32 (E) 36

16. Let $A, B,$ and C be digits satisfying

$$\begin{array}{r} A B \\ + A A \\ \hline C B 2 \end{array}$$

What is $A + B + C$?

- (A) 10 (B) 11 (C) 12 (D) 13 (E) 14
17. For how many integer values of n is
- $$\frac{3}{17} < \frac{n}{68} < \frac{32}{51}?$$
- (A) 28 (B) 29 (C) 30 (D) 32 (E) 34
18. A book has pages numbered starting at 1. The digit 3 is printed 237 times during page number printing. What is the greatest number of pages the book could have?
- (A) 663 (B) 664 (C) 672 (D) 673 (E) 683
19. The odd numbers from 1 to 17 can be used to build a 3×3 magic square (the rows and columns have the same sum). If the 1, 5, and 13 are as shown, what is x ?

- (A) 7 (B) 9 (C) 11 (D) 15 (E) 17

	1	
5		13
x		

20. Let a and b be two different real numbers. The equation $(x - a)(x - b - 1) + (x - a)(x - b + 1) = 0$
- (A) has three roots, one of which is $x = b$ (B) has three roots, one of which is $x = a$
- (C) has three roots, one of which is $x = b + 1$ (D) has two roots, one of which is $x = b$
- (E) has two roots, one of which is $x = b + 1$

21. Let N denote the two-digit number whose cube root is the square root of the sum of its digits. How many positive divisors does N have?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

22. Find the number of odd divisors of $7!$.

(A) 4 (B) 6 (C) 10 (D) 12 (E) 24

23. Farmer Brown's chickens are unusual. One and a half of them can lay an egg and a half in a day and a half, on average. At this rate, how many days would it take all 60 of his chickens to lay 480 eggs?

(A) 8 (B) 10 (C) 12 (D) 14 (E) 20

24. The radius of the circle given by

$$x^2 - 6x + y^2 + 4y = 12$$

is

(A) 5 (B) 6 (C) 7 (D) 8 (E) 36

25. The integer-valued numerator of a certain fraction is 10 more than the denominator. If the number that is two larger than the denominator is doubled, the result is the same as the numerator. In which interval does the value of the fraction belong?

(A) $(0, 1]$ (B) $(1, 2]$ (C) $(2, 3]$ (D) $(3, 4]$ (E) $(4, 5]$

26. John was contracted to work A days. For each of these A days that John actually worked, he received B dollars. For each of these A days that John didn't work, he had to pay a penalty of C dollars. After the A days of contracted work was over, John received a net amount of D dollars for his work. How many of the A days of contracted work did John not work?

(A) $(AB - D)/(B + C)$ (B) $(AB + D)/(B + C)$ (C) $(AB - D)/(B - C)$
(D) $(AB + D)/(B - C)$ (E) $(AC - B)/(D - C)$

27. Let N denote the 180-digit number obtained by listing the 90 two-digit numbers from 10 to 99 in order. Thus $N = 10111213 \dots 99$. What is the remainder when N is divided by 99?

- (A) 0 (B) 10 (C) 45 (D) 54 (E) 90