

UNC Charlotte 2004 Algebra

March 8, 2004

1. Let z denote the real number solution to $\sqrt{3 + \sqrt{x - 1}} = 5$. What is the sum of the digits of z ?

(A) 13 (B) 14 (C) 15 (D) 16 (E) 17

2. Which of the following is equivalent, where it is defined, to

$$\frac{1 - \frac{1-x}{1+x}}{1 + \frac{x-1}{x+1}}?$$

(A) 1 (B) $\frac{4x}{x+1}$ (C) $\frac{1}{x}$ (D) 0 (E) $\frac{2x}{2x-2}$

3. A two-inch cube ($2 \times 2 \times 2$) of silver weighs 3 pounds and is worth \$320. How much is a three-inch cube of silver worth?

(A) \$480 (B) \$600 (C) \$800 (D) \$900 (E) \$1080

4. Which of the following lines has a slope that is less than the sum of its x - and y - intercepts?

(A) $y = 2x + 1$ (B) $y = 3x/2 - 1$ (C) $y = -4x - 1$
(D) $y = 4x + 16/3$ (E) $y = 3x$

5. What is the remainder when $x^4 - x^2 + 1$ is divided by $x^2 + 1$?

(A) -3 (B) -1 (C) 0 (D) 3 (E) 4

6. An ancient Greek problem is the following—Make a crown of gold, copper, tin, and iron weighing 60 minae: gold and copper shall be $\frac{2}{3}$ of it; gold and tin $\frac{3}{4}$ of it; and gold and iron $\frac{3}{5}$ of it. How many minae of gold are in the crown?
- (A) 30.5 (B) 31.5 (C) 33.5 (D) 35.5 (E) 36.5
7. Find the area of the polygon $ABCDE$ if the vertices are located at the following coordinates: $A = (-1, 0)$, $B = (0, 1)$, $C = (1, 0)$, $D = (2, 1)$, $E = (2, -2)$.
- (A) 3 (B) 3.5 (C) 4 (D) 4.5 (E) 5
8. The mean of three numbers is ten more than the least of the numbers and fifteen less than the greatest of the three. If the median of the three numbers is 5, then the sum of the three is
- (A) 5 (B) 20 (C) 25 (D) 30 (E) 36
9. Let N denote the smallest four-digit number with all different digits that is divisible by each of its digits. What is the sum of the digits of N ?
- (A) 7 (B) 9 (C) 10 (D) 12 (E) 16
10. The polynomial $p(x) = 4x^3 - ax^2 - 41x + b$ has zeros at $x = -1/2, 3$, and $-7/2$. What is the product of a and b ?
- (A) -55 (B) 55 (C) 0 (D) 60 (E) 84
11. Suppose a and b are positive integers for which $(2a + b)^2 - (a + 2b)^2 = 9$. What is ab ?
- (A) 2 (B) 6 (C) 9 (D) 12 (E) 24
12. For which of the following values of a does the line $y = a(x - 3)$ and the circle $(x - 3)^2 + y^2 = 25$ have two points of intersection, one in the 1st quadrant and one in the 4th quadrant?
- (A) -1 (B) 0 (C) 1 (D) 2 (E) None of A,B,C, and D

13. Which of the following is not an asymptote of the function

$$R(x) = \frac{|x|(x-2)(x+3)}{x(x+2)(x-3)}?$$

- (A) $x = 0$ (B) $x = -2$ (C) $x = 3$ (D) $y = 1$ (E) $y = -1$
14. The non-zero real numbers a, b, c, d have the property that $\frac{ax+b}{cx+d} = 1$ has no solution in x . What is the value of $\frac{a^2}{a^2+c^2}$?
- (A) 0 (B) $1/2$ (C) 1 (D) 2 (E) an irrational number
15. What is the sum of the coefficients of the expanded form of $(2x - 3y + 3)^4$?
- (A) 0 (B) 16 (C) 81 (D) 625 (E) 1000
16. For what value of k do the graphs $f(x) = |x + 1|$ and $g(x) = |x/2 - 3| + k$ intersect in exactly one point?
- (A) -3.5 (B) -2.5 (C) -1.5 (D) 0.5 (E) 2.5
17. Let the function f be defined by $f(x) = x^2 + 40$. If m is a positive number such that $f(2m) = 2f(m)$ which of the following is true?
- (A) $0 < m \leq 4$ (B) $4 < m \leq 8$ (C) $8 < m \leq 12$ (D) $12 < m \leq 16$
(E) $16 < m$
18. Suppose that $\log_b 7 = 1.209062$. Which of the following values is closest to b ?
- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7
19. Suppose the number a satisfies $a \log a + \log \log a - \log(\log \log 2 - \log \log a) = 0$. What is the value of $a^{(a^{(a^a)})}$?
- (A) 1 (B) 2 (C) 4 (D) 8 (E) 16

20. Find an ordered pair (n, m) of positive integers satisfying

$$\frac{1}{n} - \frac{1}{m} + \frac{1}{mn} = \frac{2}{5}.$$

What is mn ?

- (A) 5 (B) 10 (C) 15 (D) 20 (E) 45
21. A standard deck of playing cards with 26 red and 26 black cards is split into two non-empty piles. In pile A there are four times as many black cards as red cards. In pile B, the number of red cards is an integer multiple of the number of black cards. How many red cards are in Pile B?
- (A) 16 (B) 18 (C) 20 (D) 22 (E) 24
22. The graphs of $x^2 + y^2 = 24x + 10y - 120$ and $x^2 + y^2 = k^2$ intersect when k satisfies $0 \leq a \leq k \leq b$, and for no other positive values of k . Find $b - a$.
- (A) 10 (B) 14 (C) 26 (D) 34 (E) 144
23. The product of three consecutive non-zero integers is 33 times the sum of the three integers. What is the sum of the digits of this product?
- (A) 5 (B) 6 (C) 12 (D) 16 (E) 18
24. It is possible that the difference of two cubes is a perfect square. For example, $28^2 = a^3 - b^3$ for certain positive integers, a and b . In this example, what is $a + b$?
- (A) 12 (B) 14 (C) 16 (D) 18 (E) 20
25. The three faces of a rectangular box have areas of 40, 45, and 72 square inches. What is the volume, in cubic inches, of the box?
- (A) 300 (B) 330 (C) 360 (D) 400 (E) 450
26. Two women and three girls wish to cross a river. Their small rowboat will carry the weight of only one woman or two girls. What is the minimum number of times the boat must cross the river in order to get all five females to the opposite side? At least one person must be in the boat each time it crosses the river.
- (A) 9 (B) 10 (C) 11 (D) 13 (E) 15