My Favorite Problems, 1

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This is the first of a series of columns about problems. I am soliciting problems from the readers of *M&I Quarterly*. Mathematical problems and challenges have enlivened my life over many years. In this column, I hope to share some of the 'ah ha's' I've enjoyed with my students and friends. I'd like to share your 'ah ha's' too. I'm looking for problems with solutions that don't depend on highly technical ideas. Ideal problems, like those in the lovely but now defunct *Quantum*, should be **easily understood** and accessible to bright high school students. Their solutions usually require a clever use of a well-known problem solving technique. For example, counting, using the principle of inclusion/exclusion, the pigeonhole principle, place value, Pick's theorem and other topics in plane geometry, and the Euclidean algorithm. Submitted problems need not be original. However, if the problem appeared in a contest, I want to acknowledge the contest. And, of course, if the name of the creator is available, that should be included with the problem. If you have a few problems whose solutions provoke you to say 'ah ha', please share them with *M&I* readers. Send your problems and solutions by email to me at `hbreiter@email.uncc.edu` In general, we'll list the problems in one issue and their solutions in the next issue.

1. Dinner Bill Splitting Problem.
   Years ago, my neighbors agreed to celebrate our wedding anniversary with my wife and me. We went to a lovely restaurant, enjoyed a fine dinner, and asked for the bill. When it came, we asked that it be split as equally as possible. Realizing the waiter's discomfort, we all set to work on the problem. Soon someone at the table noticed that the bill was for an odd amount, so it could not be split perfectly. However, we realized that, except for the penny problem, we could take half the bill by simply reversing the dollars and the cents. In other words, if we double t dollars and s cents, the result differs by 1 cent from s dollars and t cents. We told the waiter about this. He was astounded: "I never knew you could do it that way." Later, over another dinner with mathematical friends, the question of uniqueness came up, and pretty soon we realized that this number is the only one with this surprising splitting property. What was the amount of the original bill?

2. The 7-11 problem. A man goes into a convenience store, picks out four items, and goes to check out. The clerk tells him that her cash register is broken, and she will use her calculator. She proceeds to process the four amounts, and says, "that will be \$7.11". "Wait a minute", he protests, "you multiplied the prices together". She promptly repeats the calculation, this time adding the four amounts, and exclaims, "there, you owe \$7.11, just as I said." (There is no tax on food in this state.) There are two questions. First, what is the name of the convenience store, and what are the four prices? Challenge: try this problem with only three items. You'll have to change the \$7.11, of course. Then try the problem for just two items. There are lots of solutions. Find them all. Then try the 7-11 problem with three items and a total bill of \$8.25. Find some other total cost that could be used to solve the three item 7-11 problem.

3. (1997 American High School Math Examination, problem 29) A number is called 7-special if its decimal representation consists of only two digits, 0 and 7. For example, $7/99 = 0.\overline{07}$ and 7.707 are such numbers. It is possible to write 1 as a sum of 7-special numbers. If so, what is the fewest number of 7-special numbers whose sum is 1?