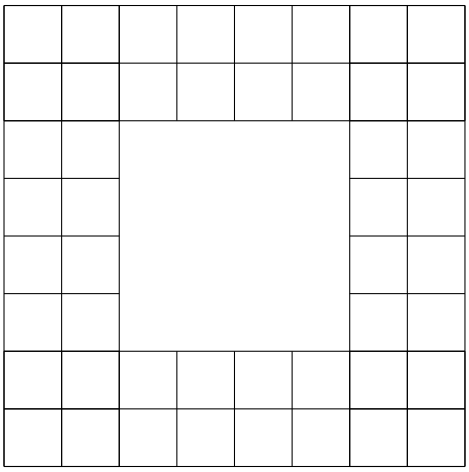
MathCounts Workshop **Iolani School** **Summer 2000**

Problem Set 1, Handbook 2000 Problems

1. How many rectangles have sides determined by the grid lines below? How many of these rectangles are squares?



2. How many ordered triples (x, y, z) satisfy the equation

$$(x^2 - 1)^2 + (y^2 - 4)^2 + (z^2 - 9)^2 = 0?$$

3. The point $(2, 3)$ is reflected about the x-axis to a point P. Then P is reflected about the line $y = x$ to a point Q. What is the x-coordinate of Q?

4. What is the probability that a randomly selected three-digit number is a multiple of 15?

5. The 'equation' $6 \div 3 = 3$ is incorrect. Replace each of the five symbols with another a symbol of the same type, one at a time, to make the equation correct. For example, $9 \div 3 = 3$, so the 6 can be replaced by 9. Is the 'equation' consisting of replacement symbols true?

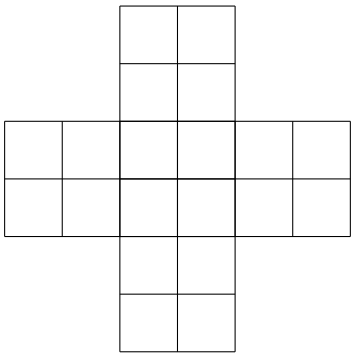
6. Two different circles and a triangle are given in the plane. What is the largest number of points that can belong to at least two of the three figures?

7. Find the area of the set S defined by

$$S = \{(x, y) \mid \lfloor x^2 \rfloor + \lfloor y^2 \rfloor = 0\}$$

where $\lfloor x \rfloor$ denotes the largest integer that is not bigger than x.

8. How many rectangles have sides determined by the grid lines below? What is the probability that a randomly picked rectangle in the diagram is a square?



9. How many distinguishable cubes can be constructed using red and green plastic squares for faces assuming that each cube must have at least one face of each color.

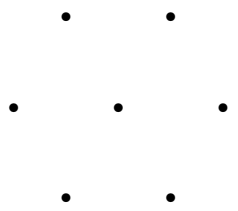
10. A regular dodecahedron is a solid with 12 pentagonal faces, 20 vertices, and 30 edges.

    (a) How many face diagonals does a regular dodecahedron have? A face diagonal is a line segment connecting two vertices that is not an edge, but which is a subset of a face.

    (b) How many space diagonals does a regular dodecahedron have?

11. Let $P(n)$ and $S(n)$ denote the product and the sum of the digits of the integer n, respectively. For example, $P(23) = 6$ and $S(23) = 5$. Find all two-digit numbers N such that $N = P(N) + S(N)$.

12. In the hexagonal lattice shown, each point is one unit from its nearest neighbors. How many circles of radius one contain at least two points of the lattice?



13. The Bernoulli-Euler(BE) triangle , like Pascal's triangle, is a triangular array of numbers each row of which is obtainable from the previous row. The BE triangle:

$$\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 0 & & & & \\
 & & & 0 & & 1 & & 1 & & & \\
 & & 2 & & 2 & & 1 & & 0 & & \\
 & 0 & & 2 & & 4 & & 5 & & 5 & \\
16 & & 16 & & 14 & & 10 & & 5 & & 0
\end{array}$$

Find the next two rows of the triangle.

14. How many squares in the plane have two or more vertices in the set

$$S = \{(0,0),(1,0),(2,0),(0,1),(1,1),(2,1)\}?$$

15. Each rectangle in the diagram is 2×1. What is the length of the longest path from A to B that does not retrace any part of itself?