1. Find the minimum value of the function $f(x) = x^2 + \dfrac{1}{x^2 + 1}$, $x \in \mathbb{R}$.

   (A) $\dfrac{1}{16}$ (B) $\dfrac{3}{16}$ (C) $\dfrac{1}{4}$ (D) $\dfrac{3}{4}$ (E) 1

2. How many solutions does the equation $\sqrt{x + 1} + 2\exp(x^3 + 1) = 2019$ have?

   (A) 0   (B) 1   (C) 2   (D) 3   (E) 4

3. What is the remainder when $x^{2019} + 2019x - 2018$ is divided by $x - 1$?

   (A) 1   (B) 2   (C) 2017   (D) 2019   (E) 2020

4. Let $n \geqslant 2$. Assume that $(x - a_1)(x - a_2)\ldots(x - a_n) = x^n + P(x)$ for all $x \in \mathbb{R}$, where $P(x)$ is a polynomial of degree $n - 2$. Find the value of the sum $a_1 + a_2 + \cdots + a_n$. (A polynomial of degree k is a function of the form $\alpha_k x^k + \alpha_{k-1}x^{k-1} + \cdots + \alpha_0$.)

   (A) 1   (B) -1 (C) n (D) $-n$ (E) 0

5. Let a be a real number. The system of equations $3x + 2y = 8$ and $ax - 8y = 9$ has no solutions (x, y). What is the value of a?

   (A) 0   (B) 1   (C) 3   (D) -8 (E) -12

6. How many real numbers x with $0 < x \leqslant 10$ are solutions to $\log_{10}(x) = \sin(x)$, where x in $\sin(x)$ is in radians and $\log_{10}(x)$ is the logarithm of x to base 10?

   (A) 0   (B) 1   (C) 2   (D) 3   (E) 4

7. Positive integer numbers a and b satisfy the equation $\sqrt{3 + 2\sqrt{2}} = a + b\sqrt{2}$. What is the value of $a + b$?

   (A) 1   (B) 2   (C) 3   (D) 4   (E) 5

8. Let $x^2 + y^2 = 10$. What is the biggest value for xy?

   (A) 10   (B) 20   (C) 5   (D) $10\sqrt{5}$ (E) 6

9. If $n! = 7!\,6!$ then what is n?

   (A) 8   (B) 9   (C) 10   (D) 13   (E) Such n does not exist

10. What is the value of $\sqrt{1 + 2 + 4 + 8 + 16 + \cdots + 2^{2019}}$, rounded up to the nearest whole number?

    (A) $2^{1010} - 1$ (B) 2^{1010} (C) $2^{1010} + 1$ (D) $2^{2019} - 1$ (E) $2^{2019} + 1$

11. The numbers x and y satisfy $2^x = 9$ and $3^y = 16$. What is the value of xy?

    (A) 7   (B) 8   (C) $\dfrac{64}{9}$ (D) $\dfrac{69}{8}$ (E) $\dfrac{25}{3}$

12. Let $f(x) = \dfrac{x-1}{x+1}$ and let $f^{(n)}(x)$ denote the n-fold composition of $f(x)$ with itself. That is, $f^{(1)}(x) = f(x)$ and $f^{(n)}(x) = f(f^{(n-1)}(x))$. Which of the following is $f^{(2019)}(x)$?

(A) $-\dfrac{x+1}{x-1}$ (B) $-\dfrac{1}{x}$ (C) $\dfrac{x-1}{x+1}$ (D) x (E) $-\dfrac{x-1}{x+1}$

13. It is known that $a + b + c = 5$ and $ab + bc + ac = 5$. What could be the value of $a^2 + b^2 + c^2$?

(A) 10   (B) 15   (C) 20   (D) 25   (E) 30

14. For which value of a does the straight line $y = 6x$ intersect the parabola $y = x^2 + a$ at exactly one point?

(A) 5   (B) 6   (C) 7   (D) 8   (E) 9

15. The solutions of the quadratic equation $x^2 + px + q = 0$ are obtained by adding 5 to each of the solutions of $x^2 - 4x + 2 = 0$. Find the value of $3p + q$.

(A) 5   (B) 6   (C) 7   (D) 8   (E) 9

16. How many solutions (a, b, c) does the following system have?

$$1 + a + b = ab,$$
$$2 + a + c = ac,$$
$$5 + b + c = bc.$$

(A) 0   (B) 1   (C) 2   (D) 3   (E) Infinitely many

17. Find the value of the product $P = \left(1 - \dfrac{1}{2^2}\right) \cdot \left(1 - \dfrac{1}{3^2}\right) \cdot \ldots \cdot \left(1 - \dfrac{1}{10^2}\right)$.

(A) 0.25   (B) 0.33   (C) 0.44   (D) 0.55   (E) 0.66

18. The sequence a_n is defined by $a_n = 1 + \sqrt{\dfrac{1}{n}} - \sqrt{\dfrac{1}{n+1}} - \sqrt{\dfrac{1}{n} - \dfrac{1}{n+1}}$. What is the value of the product $a_1 a_2 \cdots a_{99}$?

(A) $\dfrac{1}{55}$ (B) $\dfrac{1}{110}$ (C) $\dfrac{1}{99}$ (D) $\dfrac{2}{99}$ (E) $\dfrac{1}{100}$

19. The graph of the function $y = \dfrac{x-3}{x^2 - x + 6}$ is obtained from the graph of $y = \dfrac{1}{x+2}$ by deleting a single point (u, v). What is the value of $u \cdot v$?

(A) $-\dfrac{3}{5}$ (B) $-\dfrac{1}{5}$ (C) 0   (D) $\dfrac{1}{5}$ (E) $\dfrac{3}{5}$

20. Find the value of the expression $S = 1! \cdot 3 - 2! \cdot 4 + 3! \cdot 5 - 4! \cdot 6 + \ldots - 2016! \cdot 2018 + 2017!$.

(A) 1   (B) -1 (C) -2018 (D) 2018   (E) 2017