March 7, 2016

1. The value of $\frac{(2\times10^{-3})(5\times10^5)}{(0.2\times10^2)(0.5\times10^2)}$ is

   (A) 10^{-9} (B) 10   (C) 10^2 (D) 10^{-1} (E) none of these

2. The little tycoon Johnny says to his fellow capitalist Annie, "If I add 7 dollars to 3/5 of my funds, I'll have as much capital as you have." To which Annie replies, "So you have only 3 dollars more than me." How much money does Annie have?

   (A) 9   (B) 17   (C) 22   (D) 15   (E) 20

3. Four little girls, Katrina, Helen, Marie, and Naomi, sang in a concert. Each song was sung by three girls. Katrina sang 8 songs, which was more than the other girls. Helen sang 5 songs, which was fewer than the other girls. How many songs were sung in the concert?

   (A) 27   (B) 9   (C) 10   (D) 6   (E) 15

4. A state program allows people to collect empty milk bottles and exchange them for bottles full of milk. Four empty bottles may be exchanged for one full bottle. How many bottles of milk can a family drink if it has collected 24 empty bottles?

   (A) 6   (B) 7   (C) 8   (D) 9   (E) 12

5. Nick left Nicktown at 10:18 a.m. and arrived at Georgetown at 1:30 p.m., walking at a constant speed. On the same day, George left Georgetown at 9:00 a.m. and arrived at Nicktown at 11:40 a.m., walking at a constant speed along the same road. The road crosses a wide river. Nick and George arrived at the bridge simultaneously, each from his side of the river. Nick left the bridge one minute later than George. When did they arrive at the bridge?

   (A) $10:45$ (B) $11:20$ (C) $11:00$ (D) $2:10$ (E) $1:30$

6. A man is filling two tanks with water using two hoses. When the smaller tank is half full, he switches hoses. He keeps filling the tanks, and they both fill up completely at the same moment. The smaller tank is filled first from the less powerful hose. The first hose delivers water at the rate of 2.9 liters per minute, the second at a rate of 8.7 liters per minute. What is the volume of the larger tank if the volume of the smaller tank is 12.6 liters?

   (A) 19.3   (B) 18.2   (C) 25.2   (D) 30   (E) 21

7. Let a and b be the two solutions to the equation

$$2x^2 - 3x - 3 = 0.$$

Find the value $\frac{1}{a} + \frac{1}{b}$.

(A) 2   (B) 1   (C) 0   (D) -1 (E) 5

8. What is the sum of the digits of all numbers from 1 to 1000?

(A) 13501   (B) 13601   (C) 13701   (D) 13801   (E) 13901

9. How many integers n satisfy $|n^3 - 222| < 666$?

(A) 11   (B) 15   (C) 17   (D) 19   (E) 20

10. Which of the five fractions is smallest?

(A) $\frac{250388749}{250388751}$ (B) $\frac{250388748}{250388750}$ (C) $\frac{250388747}{250388749}$ (D) $\frac{250388750}{250388751}$

   (E) $\frac{250388751}{250388752}$

11. A total of n cards numbered 1 through n is divided into two stacks. What is the minimum value of n such that at least one stack will include a pair of cards whose numbers add up to an exact square?

(A) 36   (B) 15   (C) 52   (D) 48   (E) 12

12. A bug crawls along the edges of a cube. Each time it gets to a vertex, it chooses one of the three edges leaving that vertex. How many of the $3^4 = 81$ paths of length 4 lead back to the original vertex?

(A) 81   (B) 4   (C) 8   (D) 21   (E) 37

13. The outside of an $a \times b \times c$ block of unit cubes is painted, where $a < b < c$. Exactly two-thirds of the abc cubes have some paint. Which of the following could be (a, b, c)?
(A) (5,7,9)   (B) (6,8,10)   (C) (7,9,11)   (D) (6,10,12)   (E) (7,10,12)

14. What is the remainder when the product $N = 1008 \cdot 1009 \cdot 1010$ is divided by 77?

(A) 18   (B) 27   (C) 42   (D) 63   (E) 65

15. Find the sum
$$1! \cdot 1 + 2! \cdot 2 + 3! \cdot 3 + \ldots + 2016! \cdot 2016$$

(A) $2017! \cdot 2016$ (B) $2017! - 1$ (C) $2017! - 2016$ (D) $2017! + 1$ (E) $2017! + 2016$

16. If a, b, c, and d are nonzero real numbers, $a/b = c/d$, and $a/d = b/c$, then which one of the following must be true?

    (A) $a = \pm b$ (B) $a = \pm c$ (C) $a = \pm d$ (D) $b = \pm c$ (E) none of A, B, C or D

17. Suppose $ab < 0$. Which of the following points could not satisfy $y = ax + b$?

    (A) $(0, 1)$ (B) $(1, 0)$ (C) $(-1, 0)$ (D) $(0, -1)$ (E) $(1, 1)$

18. Let x and y be two real numbers satisfying $x + y = 6$ and $xy = 7$. Find the value of $x^3 + y^3$.

    (A) 55   (B) 62   (C) 78   (D) 90   (E) 216

19. Suppose a and b are positive integers neither of which is a multiple of 3. Then the remainder when $a^2 + b^2$ is divided by 3

    (A) must be 0   (B) must be 1   (C) must be 2   (D) may be 1 or 2 but not 0
    (E) may be 0, 1 or 2

20. A 3 by 3 by 3 wooden cube is painted on all 6 faces and then cut into 27 unit cubes. One unit cube is randomly selected and rolled. What is the probability that exactly two of the five visible faces are painted?

    (A) $\frac{1}{27}$ (B) $\frac{2}{27}$ (C) $\frac{12}{27}$ (D) $\frac{15}{27}$ (E) $\frac{5}{81}$

21. The solution set of $8x^6 + 7x^4 + 6x^2 + 5 < 0$ is

    (A) $\varnothing$ (B) $\{x|x < 0\}$ (C) $\{0, 1\}$ (D) $\{x|x > 0\}$ (E) none of these

22. If f is a function such that $f(3) = 2$, $f(4) = 2$ and $f(n + 4) = f(n + 3) \cdot f(n + 2)$ for all the integers $n \geq 0$, what is the value of $f(6)$?

    (A) 4   (B) 5   (C) 6   (D) 8   (E) it cannot be determined from the information given

23. From a group of three female students and two male students, a three student committee is selected. If the selection is random, what is the probability that exactly 2 females and 1 male are selected?

    (A) 0.3   (B) 0.4   (C) 0.5   (D) 0.6   (E) 0.7

24. When a missile is fired from a ship, the probability it is intercepted is $\frac{1}{3}$. The probability that the missile hits the target, given that it is not intercepted, is $\frac{3}{4}$. If three missiles are fired independently, what is the probability that all three hit their targets?

    (A) $\frac{1}{12}$ (B) $\frac{1}{8}$ (C) $\frac{9}{64}$ (D) $\frac{3}{8}$ (E) $\frac{3}{4}$

25. A cubic polynomial $p(x)$ with leading coefficient 1 has three zeros, $x = 1, x = -1$, and $x = 3$. What is the value $p(2)$?

    (A) -3 (B) -1 (C) 1   (D) 2   (E) 3