UNC Charlotte 2012 Algebra

March 5, 2012

1. In the English alphabet of capital letters, there are 15 "stick" letters which contain no curved lines, and 11 "round" letters which contain at least some curved segment. How many different 3-letter sequences can be made of two different stick letters and one curved letter?

   Stick: A E F H I K L M N T V W X Y Z

   Round: B C D G J O P Q R S U

   (A) 2310 (B) 4620 (C) 6930 (D) 13860 (E) none of these

2. The area bounded by the graph of the function $y = |x|$ and by the line $y = c$ is 5. What is c equal to?

   (A) 1 (B) 5 (C) 25 (D) $\sqrt{5}$ (E) $2\sqrt{5}$

3. Find the value of $\dfrac{266\ldots6}{66\ldots65}$, where both the numerator and the denominator have 2012 digits.

   (A) 13/35 (B) 3/8 (C) 2/5 (D) 3/7 (E) 16/35

4. A positive integer equals 11 times the sum of its digits. The units digit of this number is

   (A) 8 (B) 6 (C) 4 (D) 2 (E) 0

5. Let a and b be the two roots of the equation $x^2 + 3x - 3 = 0$. Evaluate the value $a^2 + b^2$.

   (A) 4 (B) 9 (C) 10 (D) 12 (E) 15

6. Elizabeth Joy has 30 coins consisting of nickels and dimes. If her nickels were dimes and her dimes were nickels, she would have 80 cents more than she has. How many nickels does she have?

   (A) 18 (B) 19 (C) 20 (D) 21 (E) 23

7. What is the sum of the squares of the roots of $x^4 - 7x^2 + 10 = 0$?

   (A) 4 (B) 6 (C) 9 (D) 10 (E) 14

8. How many five-digit integers of the form $(a11bc)_{10} = 10^4 a + 1100 + 10b + c$ (where $a \geq 1$) are divisible by 45?

   (A) 18 (B) 19 (C) 20 (D) 21 (E) 22.

9. Let a and b denote any of the digits from 1 to 9. How many ordered pairs (a, b) are there with the property that both $10a + b$ and $10b + a$ are primes?

   (A) 8 (B) 9 (C) 10 (D) 11 (E) 13

10. From the list of all natural numbers $2, 3, \ldots 999$, one deletes sublists of numbers nine times: At first, one deletes all even numbers, then all numbers divisible by 3, then all numbers divisible by 5, and so on, for the nine primes $2, 3, 5, 7, 11, 13, 17, 19, 23$. Find the sum of the composite numbers left in the remaining list.

    (A) 0 (B) 961 (C) 3062 (D) 2701 (E) 899

11. A number c is called a multiple zero of a polynomial $P(x)$ if $P(x)$ has a factor of the form $(x - c)^m$ where $m \geq 2$. How many numbers are multiple zeros of the polynomial $P(x) = (x^6 - 1)(x - 1) - (x^3 - 1)(x^2 - 1)$?

    (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

12. Two adjacent sides of the unit square and two sides of an equilateral triangle bisect each other. Find the area of the equilateral triangle.

    (A) $\sqrt{3}/2$ (B) 1 (C) 0.8 (D) $\sqrt{2}$ (E) $\sqrt{2}/2$

13. After a ship wreck, a surviving mouse finds himself on an uninhabited island with one kilogram of cheese and no other food. On the 1st day, he eats 1/4 of the cheese, on the 2nd, 1/9 of the remaining cheese, on the 3rd, 1/16 of the remainder, and so on. Let M be the total amount of cheese (measured in kilograms) consumed during the first 6 weeks. Which of the following statements is true?

    (A) $0.4878 < M \leq 0.4881$ (B) $0.4881 < M \leq 0.4884$
    (C) $0.4884 < M \leq 0.4887$ (D) $0.4887 < M \leq 0.4890$
    (E) $0.4890 < M \leq 0.4893$

14. Three crazy painters painted the floor in three different colors. One painted 75% of the floor in red on Monday, the second painted 70% of the floor in green on Tuesday, and the third painted 65% of the floor in blue on Wednesday. At least what percent of the floor must be painted in all the three colors?

    (A) 10% (B) 25% (C) 30% (D) 35% (E) 65%

15. After each mile on a highway from Charlotte (CLT) to the City of Mathematics Students (CMS) there is a two sided distance marker. One side of the marker shows the distance from CLT while the opposite side shows the distance to CMS. A curious student noticed that the sum of the digits from the two sides of each marker stays a constant 13. Find the distance d between the cities.

    (A) $10 \leq d < 20$ (B) $20 \leq d < 30$ (C) $30 \leq d < 40$ (D) $40 \leq d < 50$
    (E) $50 \leq d < 60$

16. When an orchestra plays a national anthem, its musicians are ordered in a square. When the orchestra plays any other song, the musicians are ordered in a rectangle such that the number of rows increases by five. What is the number m of musicians in the orchestra?

    (A) $333 \leq m < 444$ (B) $222 \leq m < 333$ (C) $111 \leq m < 222$
    (D) $50 \leq m < 111$ (E) $m < 50$

17. Find the number of solutions (m, n) to the equation $1/n - 1/m = 1/12$ where m and n are both natural numbers.

    (A) 8 (B) 7 (C) 6 (D) 5 (E) 4

18. Infinitely many empty boxes, each capable of holding six dots are lined up from right to left. Each minute a new dot appears in the rightmost box. Whenever six dots appear in the same box, they **fuse** together to form one dot in the next box to the left. How many dots are there after 2012 minutes? For example, after seven minutes we have just two dots, one in the rightmost box and one in the next box over.

    (A) 8 (B) 9 (C) 10 (D) 11 (E) 12

19. The Fibonacci numbers $1, 1, 2, 3, 5, 8, \ldots$ form a sequence where each term, after the first two, is the sum of the two previous terms. How many of the first 1000 terms are even?

    (A) 222 (B) 333 (C) 499 (D) 500 (E) 501

20. Let $\dfrac{m}{n}$ denote the fraction, in lowest terms, that is caught between $\frac{4}{7}$ and $\frac{5}{8}$ and has the smallest denominator of any such fraction. What is $m + n$?

    (A) 5 (B) 6 (C) 7 (D) 8 (E) 11

21. Find the remainder of 3^{2012} when it is divided by 7.

    (A) 0 (B) 1 (C) 2 (D) 3 (E) 5

22. How many two-digit numbers are divisible by each of their digits?

    (A) 9 (B) 10 (C) 11 (D) 13 (E) 14

23. Let C be a circle that intersects each of the circles $(x+2)^2 + y^2 = 2^2$, $(x-4)^2 + (y-2)^2 = 2^2$ and $(x-4)^2 + (y+2)^2 = 2^2$ in exactly one point and does not contain any of these circles inside it. If the radius r of C is of the form $r = p/q$ where p and q are relatively prime integers, what is $p + q$?

    (A) 5 (B) 7 (C) 9 (D) 11 (E) 13

24. What is the sum of the digits for a 5 digit number that has the property that if we put a 1 at the end of the number (making a 6 digit number ending in 1) the value is three times what we would get if we put the number 1 in front of our five digit number (this gives a six digit number starting with 1)?

    (A) 18 (B) 20 (C) 24 (D) 26 (E) 30

25. Find $\sqrt{\frac{100}{2\cdot 3}+\frac{100}{3\cdot 4}+\cdots+\frac{100}{2011\cdot 2012}}$. Round your answer to the nearest decimal digit.

(A) 3.3 (B) 5.3 (C) 7.1 (D) 8.1 (E) 9.9