UNIVERSITY OF NORTH CAROLINA CHARLOTTE
2000 HIGH SCHOOL MATHEMATICS CONTEST
March 6, 2000

1. How many six-digit multiples of 5 can be formed from the digits $1, 2, 3, 4, 5$, and 6 using each of the digits exactly once?

   (A) 21 **(B)** 32 **(C)** 36 **(D)** 64 **(E)** 120

2. Laura jogs seven blocks the first day of her training program. She increases her distance by two blocks each day. On the last day, she jogs 25 blocks. How many days was she in training?

   (A) 5 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 15

3. The line that passes through the points $(2, 5)$ and $(7, -2)$ also passes through the point $(17, y)$ for some y. What is y?

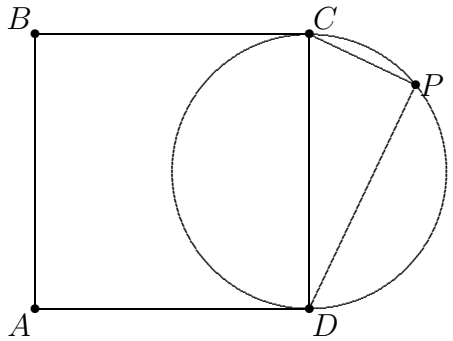
   (A) -16 **(B)** -15 **(C)** -14 **(D)** -5 **(E)** 5

4. The number $((N-2)(N-4)(N-6)(N-8)-1)/2$ is an integer if N is

   (A) 1 only **(B)** 2 only **(C)** 9 only

   (D) any odd integer **(E)** any even integer

5. In the diagram, $ABCD$ is a square and P is a point on the circle with diameter CD, $CP = 7$, and $PD = 11$. What is the area of the square?



   (A) 144 **(B)** 169 **(C)** 170 **(D)** 180 **(E)** 225

6. An equilateral triangle and a regular hexagon have the same perimeter. What is the ratio of the area of the triangle to the area of the hexagon?

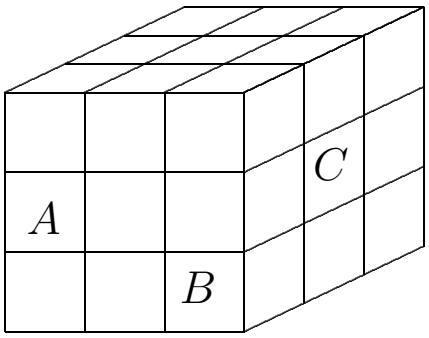
(A) $1/2$ **(B)** $2/3$ **(C)** $3/4$ **(D)** $\sqrt{2}/2$ **(E)** $\sqrt{3}/3$

7. A school has b boys and g girls, where $g < b$. How many girls must be enrolled so that 60% of the student body is female.

(A) $0.6g$ **(B)** $0.6g - 0.4b$ **(C)** $0.6b - 0.4g$ **(D)** $1.5b - g$ **(E)** $2b - g$

8. What is the surface area of the figure obtained by removing the three labeled unit cubes from the large $3 \times 3 \times 3$ cube shown?

(A) 50 **(B)** 54 **(C)** 58 **(D)** 60 **(E)** 64



9. How many ordered triples (x, y, z) satisfy the equation

$$(x^2 - 1)^2 + (y^2 - 4)^2 + (z^2 - 9)^2 = 0?$$

(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

10. Let $f(x) = (x - b)/(x - a)$ for constants a and b. If $f(2) = 0$ and $f(1)$ is undefined, what is $f(1/2)$?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

11. The slope of the line tangent to the graph of $y = x^2$ at the point $(2, 4)$ is 4. What is the y-intercept of the line?

(A) -12 **(B)** -4 **(C)** 0 **(D)** 4 **(E)** 12

12. What is the coefficient of x^7 in the polynomial $(x+3)^{10}$?

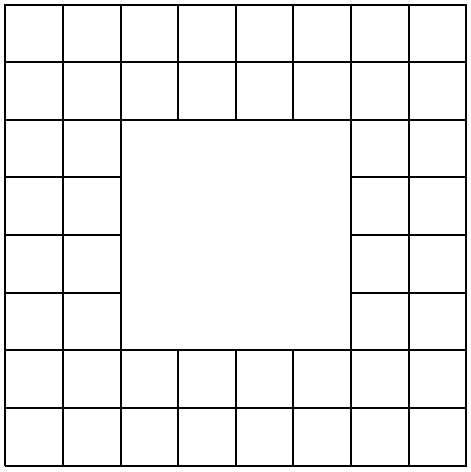
(A) 120 **(B)** 2187 **(C)** 3240 **(D)** 3402 **(E)** 5670

13. Niki just completed a 10 mile bike trip. If she had been able to ride 2 miles per hour faster, she would have completed her trip in 20 fewer minutes. Find her speed to the nearest tenth of a mile per hour.

(A) 6.2 **(B)** 6.3 **(C)** 6.5 **(D)** 6.7 **(E)** 6.8

14. How many squares of all sizes have sides determined by the grid lines below?

(A) 48 **(B)** 72 **(C)** 75 **(D)** 89 **(E)** 91



15. An ant located at a corner of a 2in. $\times$ 3in. $\times$ 5in. rectangular block of wood wants to crawl along the surface to the opposite corner of the block. What is the length of the shortest such path?

(A) $\sqrt{50}$ **(B)** $\sqrt{58}$ **(C)** 8 **(D)** $\sqrt{68}$ **(E)** 10

16. The students in Professor Einstein's class decided to reward the fine teacher with a CD player at the end of the course. A total of \$529 was collected from the students, with each student contributing the same amount, which was equal to the total number of students in the class. Only ordinary US bills were used and none of these were \$2 dollar bills. In addition, each student paid using the same five bills. How many ten-dollar bills were collected?

(A) 10 **(B)** 12 **(C)** 15 **(D)** 23 **(E)** 46

17. If the operation $\oplus$ is defined for all positive x and y by $x \oplus y = (xy)/(x+y)$, which of the following must be true for positive x, y, and z?

i. $x \oplus x = x/2$

ii. $x \oplus y = y \oplus x$

iii. $x \oplus (y \oplus z) = (x \oplus y) \oplus z$

(A) i. only **(B)** i. and ii. only **(C)** i. and iii. only

(D) ii. and iii. only **(E)** all three

18. Suppose that $f(n+1) = f(n)+f(n-1)$ for $n = 2, 3, \ldots$. Given that $f(6) = 23$ and $f(4) = 8$, what is $f(1) + f(3)$?

(A) 6 **(B)** 7 **(C)** 8 **(D)** 12 **(E)** 13

19. The point $A = (2, 3)$ is reflected about the x-axis to a point B. Then B is reflected about the line $y = x$ to a point C. What is the area of the triangle ABC?

(A) 12 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 24

20. The sides a, b, and c of a triangle satisfy $\sqrt{a}+\sqrt{b} = \sqrt{c}$. Which of the following best describes the triangle?

(A) acute **(B)** scalene **(C)** isosceles

(D) non-existent **(E)** equilateral

21. In triangle ABC, $AB = 9, BC = 10$, and $AC = 11$. Among the following, which number is closest to $\cos(\angle ABC)$?

(A) 2/9 **(B)** 1/4 **(C)** 2/7 **(D)** 3/10 **(E)** 1/3

22. If $a > 0$ and $ab = 3, bc = 5$, and $ac = 7$, what is c.?

(A) 3 **(B)** $\sqrt{3}$ **(C)** $\sqrt{\frac{35}{3}}$ **(D)** 2 **(E)** 1

23. Let N be the smallest four digit number such that the three digit number obtained by removing the leftmost digit is one ninth of the original number. What is the sum of the digits of N?

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

24. There are some elevens in a collection of numbers and the rest of the numbers are twelves. There are three more elevens than twelves. Which of the following could be the sum of the numbers in the collection?

(A) 232 **(B)** 234 **(C)** 235 **(D)** 240 **(E)** 256

25. A treasure is located at a point along a straight road with towns A, B, C, and D in that order. A map gives the following instructions for locating the treasure:

(a) Start at town A and go $1/2$ of the way to C.

(b) Then go $1/3$ of the way towards D.

(c) Then go $1/4$ of the way towards B, and dig for the treasure.

If $AB = 6$ miles, $BC = 8$ miles, and the treasure is buried midway between A and D, find the distance from C to D.

(A) 4 **(B)** 6 **(C)** 8 **(D)** 10 **(E)** None of the above

26. The area of $\triangle ABC$ is 144 square units. Point U is on $\overline{AB}$ such that the ratio AU to UB is 5 to 7. Point V is on $\overline{BC}$ such that the ratio $BV$to VC is 2 to 1. What is the area of $\triangle UVB$?

(A) 50 **(B)** 54 **(C)** 55 **(D)** 56 **(E)** 60

27. When a missile is fired from a ship, the probability it is intercepted is $1/3$. The probability that the missile hits the target, given that it is not intercepted, is $3/4$. If three missiles are fired independently from the ship, what is the probability that all three hit their targets?

(A) 1/12 **(B)** 1/8 **(C)** 9/64 **(D)** 3/8 **(E)** 3/4