UNCC 2001 Comprehensive

March 5, 2001

1. Compute the sum of the roots of $x^2 - 5x + 6 = 0$.

   (A) 3 **(B)** 7/2 **(C)** 4 **(D)** 9/2 **(E)** 5

2. Find the slope of the line connecting the two points that satisfy

   $$2x + 3y = 6 \text{ and } x^2 + y^2 = 36.$$

   (A) -1 **(B)** $-2/3$ **(C)** $-1/2$ **(D)** $-1/3$ **(E)** 0

3. Compute the sum of all the roots of $(2x + 3)(x - 4) + (2x + 3)(x - 6) = 0$.

   (A) 7/2 **(B)** 4 **(C)** 7 **(D)** 13 **(E)** none of **A**, **B**, **C** or **D**

4. The radius of the circle given by

   $$x^2 - 6x + y^2 + 4y = 36$$

   is

   (A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 36

5. What is the area of the pentagonal region $ABCDE$, where $A = (0, 0), B = (12, 0), C = (12, 2), D = (6, 7)$, and $E = (0, 5)$?

   (A) 60 **(B)** 62 **(C)** 63 **(D)** 65 **(E)** 66

6. How many positive integers can be represented as a product of two distinct members of the set $\{1, 2, 3, 4, 5, 6\}$?

   (A) 9 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 13

7. Two cards are selected randomly and simultaneously from a set of four cards numbered $2, 3, 4$, and 6. What is the probability that both cards selected are prime numbered cards? Express your answer as a fraction.

   (A) 1/6 **(B)** 1/4 **(C)** 1/3 **(D)** 1/2 **(E)** 2/3

8. How many positive integers less than one million have all digits equal and are divisible by 9?

   (A) 5 **(B)** 6 **(C)** 8 **(D)** 10 **(E)** 18

9. If $x^2+2x+n > 10$ for all real numbers x, then which of the following conditions must be true?

(A) $n > 11$ **(B)** $n < 11$ **(C)** $n = 10$ **(D)** $n = \infty$ **(E)** $n > -11$

10. The diagram shows six congruent circles with collinear centers in the plane. Each circle touches its nearest neighbor(s) at exactly one point. How many paths of length 3π along the circular arcs are there from $A = (0, 0)$ to $B = (6, 0)$?

(A) 16 **(B)** 32 **(C)** 64 **(D)** 128 **(E)** 256

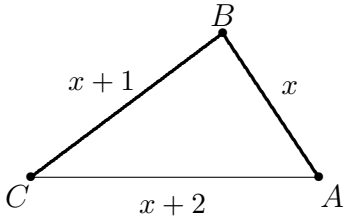
11. What is the sum of the digits of the decimal representation of

$$\frac{10^{27}+2}{3}?$$

(A) 80 **(B)** 82 **(C)** 84 **(D)** 86 **(E)** 87

12. Consider triangle ABC with $AB = x, BC = x + 1,$ and $CA = x + 2$. Which of the following statements must be true?
I. $x \geq 1$
II. $x \leq 2\sqrt{3}$
III. Angle $C < 60$ degrees.

(A) I only **(B)** II only **(C)** III only **(D)** I and II **(E)** I and III



13. If $2^{10x-1} = 1$, what is $\log x$?

(A) -1 **(B)** 0 **(C)** 1 **(D)** 2 **(E)** 3

14. How many positive divisors does 6! have?

(A) 4 **(B)** 6 **(C)** 10 **(D)** 20 **(E)** 30

15. The vertices of a triangle T are $(0,0)$, $(0,y)$, and $(x,0)$, where x and y are positive. The area of T is 30 and the perimeter is also 30. What is $x+y$?

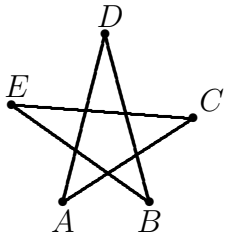
(A) 12 **(B)** 13 **(C)** 15 **(D)** 17 **(E)** 18

16. An isosceles right triangular region of area 25 is cut from a corner of a rectangular region with sides of length $5\sqrt{2}$ and $5(1+\sqrt{2})$. What is the perimeter of the resulting trapezoid?

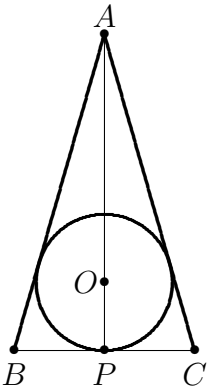
(A) 25 **(B)** 35 **(C)** $20+10\sqrt{2}$ **(D)** $10+20\sqrt{2}$ **(E)** $15+15\sqrt{2}$

17. In the following figure, what is the sum $m(\angle A)+m(\angle B)+m(\angle C)+m(\angle D)+m(\angle E)$ of the measures of the angles A, B, C, D, and E?

(A) 180° **(B)** 360° **(C)** 540° **(D)** 720° **(E)** 900°



18. In the figure below, the circle is inscribed in an isosceles triangle ABC, with segment $\overline{AP}$ passing through the center O of the circle, $AC = AB = 12$ and $BP = 4$. Find the radius r of the circle.



(A) $2\sqrt{2}$ **(B)** $3\sqrt{2}$ **(C)** $4\sqrt{2}$ **(D)** $6\sqrt{2}$ **(E)** $12+\sqrt{2}$

19. Bill and his dog walk home from the shopping center. It takes Bill 36 minutes and his dog walks twice as fast. They start together, but the dog reaches home before Bill and returns to meet Bill. After meeting Bill, the dog walks home, again at double speed, and then turns back to meet Bill again. Bill starts at noon to walk home. How many minutes later does he meet the dog for the second time?

(A) 24 **(B)** 27 **(C)** 30 **(D)** 32 **(E)** 34

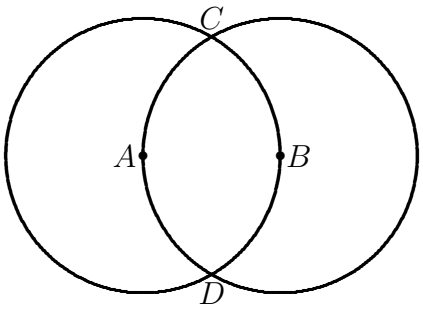
20. How many ordered triples (x, y, z) satisfy the equation

$$3x^2 + 3y^2 + z^2 - 2xy + 2yz = 0 \ ?$$

(A) 0 **(B)** 1 **(C)** 3 **(D)** 4 **(E)** infintely many

21. Let A, B be the centers of two circles with radii 1 and assume that $AB = 1$. Find the area of the region enclosed by arcs CAD and CBD.

(A) 1 **(B)** $\frac{2\pi}{3}$ **(C)** $\frac{2\pi}{3} - \frac{\sqrt{3}}{2}$ **(D)** $\frac{\sqrt{3}}{2}$ **(E)** $\sqrt{3}$



22. Suppose x satisfies $|x^2 - 2x - 3| = |x^2 - 2x + 5|$. Then x belongs to

(A) $[0, 2)$ **(B)** $[2, 4)$ **(C)** $[4, 6)$ **(D)** $[6, 8)$ **(E)** $[8, \infty)$

23. When $\sqrt{4 - 2\sqrt{3}}$ is expressed in the form $\sqrt{a} - b$, where a and b are integers, the value of $a + b$ is

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

24. The sum of the reciprocals of four different positive integers is 1.85. Which of the following could be the sum of the four integers?

(A) 15 **(B)** 16 **(C)** 17 **(D)** 18 **(E)** 19

25. What is the area of a triangle whose sides are $5, 6$, and $\sqrt{13}$?

(A) $5\sqrt{2}$ **(B)** 8 **(C)** 9 **(D)** $6\sqrt{2}$ **(E)** 10

26. Into how many regions does the solution set S of

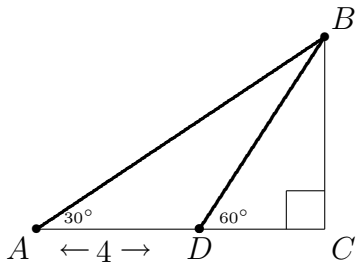
$$xy(y-x)(y+x)(x^2+y^2-1)=0$$

divide the plane? Note that some of the regions are bounded (surrounded by points of S), and some are unbounded.

(A) 4 **(B)** 8 **(C)** 16 **(D)** 18 **(E)** 22

27. In the right triangle ABC shown, D is on $\overline{AC}$, $\angle A = 30°$, $\angle BDC = 60°$, and $AD = 4$. Find BC

(A) 3 **(B)** $2\sqrt{3}$ **(C)** $\sqrt{14}$ **(D)** 4 **(E)** $3\sqrt{2}$



28. An integer between 100,000 and 199,999 becomes three times as big when we move the 1 from the leftmost position to the rightmost position. Find the sum of the digits of the number

(A) 22 **(B)** 24 **(C)** 27 **(D)** 28 **(E)** 29