Binary numbering

1.

	16	8	4	2	1		32	16	8	4	2	1
0						32						
1						33						
2						34						
3						35						
4						36						
5						37						
6						38						
7						39						
8						40						
9						41						
10						42						
11						43						
12						44						
13						45						
14						46						
15						47						
16						48						
17						49						
18						50						
19	1	0	0	1	1	51						
20						52						
21						53						
22						54						
23						55						
24						56						
25						57						
26						58						
27						59						
28						60						
29						61						
30						62						
31						63						

Representing Rational Numbers

2. Find the value of each of the binary expressions
   (a) 1010101_2
   (b) 1101101_2
   (c) 11.01101_2
3. Find the binary and ternary representations of
   (a) 1995 using both the remainder method and the subtraction method.
   (b) 1996 using both the remainder method and the subtraction method.
   (c) 0.25
   (d) 0.2.
   (e) 1/3.
4. Perform the indicated arithmetic
   (a) $1101_2 \times 10111_2$
   (b) $1001_2 \times .1_2$
   (c) $10000_2 \times 1.00101_2$
   (d) $1101_2 + 10111_2$
   (e) $1001_2 + 1011.1_2$
   (f) $1011100_2 - 100101_2$
5. Find the base -2 representation of each of the integers from 1 up to 30.
6. Find the factorial representation of each of the integers from 1 up to 30.
7. Find a Fibonacci representation of each of the integers from 1 up to 30.
8. An increasing sequence $1, 3, 4, 9, 10, 12, 13, \ldots$ consists of all those positive integers which are powers of 3 or sums of distinct powers of 3. Fine the 100^{th} term of the sequence (where 1 is the first term, 3 is the second, etc.)

Representing Rational Numbers

9. Each column in the table provides the representation of the numbers from 1 to 12 for a certain system of enumeration. Of course, the entries in column A are decimal representations. Study the pattern, and determine the method of representation. Then replace the ?'s with the appropriate representations.

A	B	C	D	E	F
1	0	1	1	1	1
2	1	10	10	2	110
3	10	100	11	10	111
4	2	101	20	11	100
5	100	1000	21	12	101
6	11	1001	100	20	11010
7	1000	1010	101	21	11011
8	3	10000	110	22	11000
9	20	10001	111	100	11001
10	101	10010	120	101	11110
11	10000	10100	121	102	11111
12	12	10101	200	110	11100
13	?	?	?	?	?
14	?	?	?	?	?
15	?	?	?	?	?
16	?	?	?	?	?
17	?	?	?	?	?
18	?	?	?	?	?

10. The floor of a real number x, denoted $\lfloor x \rfloor$ is the largest integer that is not bigger than x. The fractional part of a real number x, denoted $\langle x \rangle$, is defined by $\langle x \rangle = x - \lfloor x \rfloor$. For example, $17\frac{2}{3} = \lfloor 17\frac{2}{3} \rfloor + \langle 17\frac{2}{3} \rangle = 17 + \frac{2}{3}$ and $-2\frac{1}{3} = \lfloor -2\frac{1}{3} \rfloor + \langle -2\frac{1}{3} \rangle = -3 + \frac{2}{3}$. Solve simultaneously:

$$\begin{aligned} x + \lfloor y \rfloor + \langle z \rangle &= 1.1 \\ \langle x \rangle + y + \lfloor z \rfloor &= 2.2 \\ \lfloor x \rfloor + \langle y \rangle + z &= 3.3 \end{aligned}$$