For each of the following games, label each position with its Grundy value. That is $G(v) = \text{mex}\ \{G(x) \mid x \text{ is a successor of } v\}$. Recall that x is a successor of v if there is a move from v to x.

1. Consider the game G_1 which starts with one pile of 20 counters. The rules allow a player to take $1, 3,$ or 5 counters on each turn. The player who makes the last move wins. Denote this game by $N(20; 1, 3, 5)$. Do you want to move first? Explain why or why not.

20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

2. Consider the game G_2 which starts with one pile of 20 counters. The rules allow a player to take $1, 2,$ or 5 counters on each turn. Denote this game by $N(20; 1, 2, 5)$. Again, the player who makes the last move wins. Do you want to move first? Explain why or why not.

20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

3. Consider the game G_3 which starts with one pile of 20 counters. The rules allow a player to take $1, 2,$ or 6 counters on each turn. Denote this game by $N(20; 1, 2, 6)$. As usual, the player who makes the last move wins. Do you want to move first? Explain why or why not.

20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

4. Consider the game G_4 which starts with one pile of 20 counters. The rules allow a player to take a prime number of counters on each turn. Denote this game by $N(20; 2, 3, 5, 7, 11, 13, 17)$. The move 19 is purposely left out. As usual, the player who makes the last move wins. Do you want to move first? Explain why or why not.

20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

5. Consider the game G_5 which starts with one pile of 20 counters. The rules allow a player to take an integer power of 2 counters on each turn. Denote this game by $N(20; 1, 2, 4, 8, 16)$. As usual, the player who makes the last move wins. Do you want to move first? Explain why or why not.

20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

6. Consider the game G_6 which starts with one pile of 20 counters. The rules allow a player to take an integer power of three counters on each turn. Denote this game by $N(20; 3^0, 3^1, 3^2)$. As usual, the player who makes the last move wins. Do you want to move first? Explain why or why not.

20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0