September 27, 2005 **Name** ______________________________

On all the following questions, **show your work.** There are 144 points available on this test. Do not try to do all the problems. Try to find four or five that you you can do well.

1. (20 points) Let $f(x) = 1/x$ for all $x > 0$, and let $[a, b] = [2, 8]$.

   (a) (5) Let $n = 3$ and use left endpoints for sample points to find the approximating sum. That is, compute L_3.

   (b) (7) Find the n^{th} approximating sum, also using left endpoints. In other words, find an expression for L_n. You need not evaluate the limit as $n \to \infty$.

   (c) (8) Use the midpoint rule to approximate $\int_2^8 1/x\ dx$. Compare the two numbers M_3 and $\int_2^8 1/x\ dx$.

2. (24 points) Find the following indefinite integrals.

   (a) $\displaystyle\int \frac{(x-1)^2}{x^2+1}dx$

   (b) $\displaystyle\int \frac{1}{\sqrt{9-x^2}}\, dx$

   (c) $\displaystyle\int \frac{d}{dx}(x-3)(x^2-1)\, dx$

3. (40 points) Use the evaluation theorem as needed to find each of the definite and improper integrals below.

   (a) $\displaystyle\int_0^{\pi/2} \cos x \cos(\sin x)\, dx$

(b) $\displaystyle\int_3^4 \frac{x+1}{x^2-4}\ dx$

(c) $\displaystyle\int_e^\infty (x\ln x)^{-1}\ dx$

(d) $\displaystyle\int_0^2 xe^{x^2}\ dx$

(e) $\displaystyle\int_0^1 x^2(x-2)^8\ dx$

4. (20 points) Consider the integral $\int_{-2}^{3} 1/x \, dx$.

   (a) Explain why this integral is not defined by the usual definition of integral as a limit of Riemann sums as the number of subintervals n approaches ∞.

   (b) It is tempting to evaluate this integral by antidifferentiating $f(x) = 1/x$, getting $F(x) = \ln |x|$, and then to measuring the growth of $F(x)$ over the interval $[-2, 3]$ to get $\ln |3| - \ln | - 2| = \ln 3 - \ln 2 = \ln(3/2)$. Explain why this is wrong.

   (c) Is there are reasonable approach to this problem? What is it?

5. (25 points) Let $g(x) = \int_0^{2x^2} (t-2)(t-8)\, dt$.

   (a) Find $g'(x)$.

   (b) Find the critical points of g. IE, find the zeros of g'.

   (c) Compute $g'(-3/2), g'(-1/2), g'(1/2)$, and $g'(3/2)$.

   (d) Recall the great theorem in calculus 1 that tells you when a differentiable function is increasing: If $f'(x) > 0$ at every point of (a,b), then f is increasing over (a,b). Use this theorem and the Test Interval Technique to find the intervals over which g is increasing. Recall that $g'(x)$ must be factored completely to apply the Test Interval Technique.

6. (15 points) Use the substitution $x = \sec\theta$ to compute $\int \frac{\sqrt{x^2-1}}{x^4}\, dx$. Show the triangle you use to find $\sin\theta$, etc. You may find useful the formula $\int \sec\theta d\theta = \ln|\sec\theta + \tan\theta| + C$.