MATH 1242 SPRING 2003
COMMON FINAL EXAMINATION
PART I

This exam is divided into three parts. You will have three hours for the entire exam, but you have only one hour to complete part I. No calculators are allowed during the first hour of the exam, hence **part I must be completed without the use of any calculator.** You may start working on parts II and III as soon as you finish part I, but you cannot use a calculator until your instructor collects part I at 9.00 a.m. At that point, you may use your calculator for the remainder of the exam. A special answer sheet is provided so that your answers can be machine graded.

These pages contain Part I which consists of 12 multiple choice questions. It has to be done without using any calculators.

- You must use a pencil with a soft black led (#2 or HB) to enter your answers on the answer sheet.
- For each question choose the response which best fits the question.
- If you wish to change an answer, make sure that you completely erase your old answer and any other extraneous marks.
- There is no penalty for guessing.
- If you mark more than one answer to a question, that question will be scored as incorrect.
- You may perform your calculations on the test itself or on scratch paper, but do not make any stray marks on the answer sheet.
- **Make sure that your name appears on the answer sheet and that you fill in the circles corresponding to your name.**

At the end of the examination you MUST hand in this test booklet, your answer sheet and all scratch paper.

Name: ______________________ Student ID. ______________________

Instructor: ______________________ Section No: ______________________

1. If $\int_0^2 f(x)dx = 2$ and $\int_4^2 f(x)dx = -5$, then $\int_0^4 f(x)dx$ equals

(a) -3

(b) 7

(c) -7

(d) 3

(e) cannot be calculated from the given information.

2. $\int \left(x - \frac{2}{x}\right) dx =$

(a) $\frac{x^2}{2} - 2\ln|x| + C$

(b) $\frac{x^2}{2} + 2\ln|x| + C$

(c) $1 - 2\ln|x| + C$

(d) $\frac{x^2}{2} + 2\log x$

(e) $1 + \frac{2}{x^2}$

3. The function $f(x) = \int_0^x \sin t^2\, dt$ has the derivative

(a) $f'(x) = 2x\cos x^2$

(b) $f'(x) = \cos(x^2)$

(c) $f'(x) = \sin x^2$

(d) $f'(x) = \cos(x^2) - \sin x^2$

(e) $f'(x) = -\cos x^2$

7. The area enclosed by the curves $y = x$ and $y = x^2$ is

(a) $\frac{1}{2}$

(b) $\frac{1}{3}$

(c) $\frac{1}{4}$

(d) $\frac{1}{6}$

(e) $\frac{1}{12}$

8. $\int x(1-x^2)^{1/2}\,dx =$

(a) $-\frac{1}{3}(1-x^2)^{3/2}+C$

(b) $\frac{2}{3}(1-x^2)^{3/2}+C$

(c) $2x(1-x^2)^{-1/2}$

(d) $(1-x^2)^{3/2}+C$

(e) $x(1-x^2)^{3/2}+C$

9. Assume $1 \leq f(x) \leq 3$ on $[0,1]$ for a continuous function. Then the integral $I = \int_0^1 \frac{f(x)}{\sqrt{x}}\,dx$

(a) is divergent.

(b) is convergent and $I > 6$.

(c) is convergent and $I \leq 6$.

(d) is convergent and $I < 2$.

(e) is convergent and can have any positive value.

MATH 1242 SPRING 2003
COMMON FINAL EXAMINATION
PART II

These pages contain Part II of the exam which consists of 13 multiple choice questions. You cannot use a calculator on any part of this exam until after 9.00 a.m., when part I will be collected. After that time, calculators are permitted on parts II and III of the exam. A special answer sheet is provided so that your answers can be machine graded.

- You must use a pencil with a soft black led (#2 or HB) to enter your answers on the answer sheet.
- For each question choose the response which best fits the question.
- If you wish to change an answer, make sure that you completely erase your old answer and any other extraneous marks.
- There is no penalty for guessing.
- If you mark more than one answer to a question, that question will be scored as incorrect.
- You may perform your calculations on the test itself or on scratch paper, but do not make any stray marks on the answer sheet.
- **Make sure that your name appears on the answer sheet and that you fill in the circles corresponding to your name.**

At the end of the examination you MUST hand in this test booklet, your answer sheet and all scratch paper.

Name: ______________________ Student ID. ____________________

Instructor: ____________________ Section No: ____________________

1. If the average of $f(x) = 3x^2 - 12x + 13$ on the interval $[0, t]$ is equal to 4, then

(a) $t = 3$ or $t = 1$

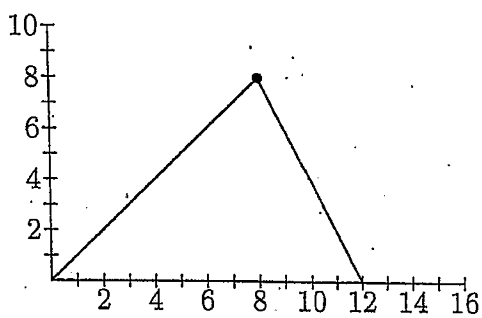
(b) $t = 3$ or $t = -3$

(c) t can have any value

(d) $t = 3$

(e) $t = 1$

2. * The function $h(x)$ is graphed below.



Calculate $\int_0^{12} h(x)\,dx$.

(a) 96

(b) 64

(c) 32

(d) 48

(e) 15

3. * The elastic force of a spring satisfies Hook's law $F(x) = kx$, with spring stiffness k. To extend the spring from its equilibrium $x = 0$ up to $x = 0.20$ m, one needs a force of 17 N. Find the work W needed to extend the spring from equilibrium to $x = 0.80$ m.

(a) $W = 5.44$ Nm

(b) $W = 85$ Nm

(c) $W = 27.2$ Nm

(d) $W = 54.4$ Nm

(e) $W = 0.32$ Nm

4. $\int_e^{e^{100}} \frac{dx}{x}$ equals

(a) 100.

(b) is divergent.

(c) $\ln 100$.

(d) 99.

(e) e^{100}.

5. The integral $\int_{-1}^{2} \frac{dx}{x}$

(a) is a common definite integral and equals $\ln 2$.

(b) is a divergent improper definite integral.

(c) is an improper definite integral and equals $\ln 2$.

(d) is a common definite integral, but can only be approximately calculated numerically.

(e) none of the above is true.

6. * The arc length of the curve $y = \sin x$ between $x = 0$ and $x = \pi$ is

(a) 4.44177

(b) 3.14317

(c) 3.82020

(d) 2.82843

(e) 4.44233

7. The definite integral $\int_0^\pi \cos x\,dx$ equals the limit

(a) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n} \cos \frac{i\pi}{n}$.

(b) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n} \cos \frac{2i\pi}{n}$.

(c) $\lim_{n\to\infty} \sum_{i=0}^{2n} \frac{\pi}{n} \cos \frac{i\pi}{n}$.

(d) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi}{n} \cos \frac{i\pi}{n}$.

(e) $\lim_{n\to\infty} \sum_{i=0}^{n-1} \frac{1}{n} \sin \frac{i\pi}{n}$.

8. Use symmetry to decide which of the following integrals equals zero for an arbitrary a.

(a) $\int_{-a}^{a} x \sin x \, dx$

(b) $\int_{-a}^{a} x^2 \sin x \, dx$

(c) $\int_{-a}^{a} (\sin x)^2 \, dx$

(d) $\int_{-a}^{a} x^4 \, dx$

(e) $\int_{0}^{a} \sin x \, dx$

9. A tank has the shape of a hemisphere with radius $5m$. It is filled with water, $5m$ deep in the middle. The integral $\int_0^5 \pi x(25 - x^2) \, dx$

(a) gives the volume of a ball with radius 5 m.

(b) when multiplied by density of water times acceleration of gravity ρg, it gives the work needed to pump all the water out the top of the tank.

(c) when multiplied by ρg, it gives the work needed to pump water out the top level of the tank, leaving water $1m$ deep in the tank.

(d) gives the volume of the water in the tank.

(e) none of the above holds true.

10. The values of A and B in the partial fraction decomposition $\frac{x+5}{x^2+x-2} = \frac{A}{x-1} + \frac{B}{x+2}$ are

(a) $A = 3, \ B = \frac{10}{3}$

(b) $A = 2, \ B = -1$

(c) $A = 6, \ B = 2$

(d) $A = 3, \ B = -1$

(e) $A = 1, \ B = \frac{10}{3}$

11. The interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$ is

(a) $-\infty < x < +\infty$

(b) $-1 \leq x < 1$

(c) $-1 < x < 1$

(d) $-2 < x < 2$

(e) $0 < x < 1$

12. The function $f(x) = \int_0^x \frac{\sin t}{t}\,dt$ has the Maclaurin series

(a) $x - \frac{x^3}{3 \cdot 3} + \frac{x^5}{5 \cdot 5} - \frac{x^7}{7 \cdot 7} + \ldots$

(b) $1 - \frac{x^2}{2 \cdot 2!} + \frac{x^4}{4 \cdot 4!} - \frac{x^6}{6 \cdot 6!} + \ldots$

(c) $1 + \frac{x^2}{2 \cdot 2!} + \frac{x^4}{4 \cdot 4!} + \frac{x^6}{6 \cdot 6!} + \ldots$

(d) $x - \frac{x^3}{3 \cdot 3!} + \frac{x^5}{5 \cdot 5!} - \frac{x^7}{7 \cdot 7!} + \ldots$

(e) $x - \frac{x^3}{(3!)^2} + \frac{x^5}{(5!)^2} - \frac{x^7}{(7!)^2} + \ldots$

13. Given that the power series $c_0 + c_1x + c_2x^2 + c_3x^3 + \ldots$ has radius of convergence $R > 0$, the corresponding termwise differentiated series $c_1 + 2c_2x + 3c_3x^2 + \ldots$

(a) can have any radius of convergence.

(b) has radius of convergence larger than R.

(c) has radius of convergence equal to R.

(d) is divergent.

(e) has radius of convergence smaller than R.

END OF PART II

MATH 1242
COMMON FINAL EXAMINATION
FREE RESPONSE SECTION
SPRING 2003

This exam is divided into three parts. These pages contain Part III which consists of 6 free response questions. An $*$ marks problems where a calculator is useful.

Please show all of your work on the problem sheet provided. We will not grade loose paper.

- If you are basing your answer on a graph on your calculator, sketch a picture of your graph on your sheet and be sure to label your window.
- Make sure that your name appears on each page.

At the end of the examination you MUST hand in this test booklet and all scratch paper.

PROBLEM	1	2	3	4	5	6
GRADE						

FREE RESPONSE SCORE:__________

Name: ______________________ Student ID. ______________________

Instructor: ______________________ Section No: ______________________

1. In the first quadrant $x \geq 0, y \geq 0$, a region $\mathcal{R}$ is bounded by the curve $y = 2x - x^2$ from above, and $y = x^3$ from below.

   (a) Find the x-coordinates of the two intersection points of the curves in the region $\mathcal{R}$. [3 points]

   (b) Set up an integral for the area of the region $\mathcal{R}$. [4 points]

   (c) * Find the area of $\mathcal{R}$. [3 points]

2. As in problem (1), in the first quadrant $x \geq 0, y \geq 0$, the two curves $y = 2x - x^2$ and $y = x^3$ bound a region $\mathcal{R}$.

(a) Set up an integral for computing the volume of the solid obtained by rotating $\mathcal{R}$ around the x-axis. (You need not evaluate the integral.) [4 points]

(b) Set up an integral for computing the volume of the solid obtained by rotating $\mathcal{R}$ around the y-axis. (You need not evaluate the integral.) [3 points]

(c) Set up an integral for computing the volume of the solid obtained by rotating $\mathcal{R}$ around the line $y = -1$. (You need not evaluate the integral.) [3 points]

3. (a) * Use Simpson's rule $S_4 = \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)]$ with $n = 4$ to estimate the integral $\int_0^2 e^{-x^2/2}dx$. [6 points]

(b) * The error estimate for Simpson's rule is given by $|E_n| \leq K\frac{(b-a)^5}{180n^4}$, where K is an upper bound for the fourth derivative of f. For the function $f(x) = e^{-x^2/2}$, this upper bound is 3. Estimate the error of the numerical integration in part (a) above. [4 points]

4. (a) Write down the first six terms of the Maclaurin series for the function e^x. [2 points]

(b) Using part (a), find the first six terms of the Maclaurin series for the function $e^{-t^2/2}$. [2 points]

(c) Using part (b), find the first six terms of the Maclaurin series for the function $f(x) = \int_0^x e^{-t^2/2}\,dt$. [4 points]

(d) * Using the fact that the series in part (c) is an alternating one, obtain lower and upper bounds for $\int_0^1 e^{-x^2/2}dx$. [2 points]

5. Determine the arc length of the curve $y = \dfrac{e^x + e^{-x}}{2}$.

   (a) Set up (but do not evaluate) the integral for the arc length on $[0, t]$. [4 points]

   (b) Using the identity $1 + \left(\dfrac{e^x - e^{-x}}{2}\right)^2 = \left(\dfrac{e^x + e^{-x}}{2}\right)^2$,

   evaluate the arc length, in part (a) above, on $[0, t]$. [6 points]

6. The integrals $\int \frac{du}{u^2+a^2} = \frac{1}{a}\arctan\frac{u}{a} + C$ and $\int \frac{u\,du}{u^2+a^2} = \frac{1}{2}\ln(u^2+a^2) + C$

allow us to calculate $\int \frac{x+3}{x^2+2x+10}\,dx$.

(a) Define u and a such that $x^2+2x+10 = u^2+a^2$ by completing the square. [4 points]

(b) Use the substitution from part (a) to evaluate the integral. [6 points]

END OF FREE RESPONSE SECTION