Series

Test	Series	Converges	Diverges	Comment
nth-Term	$\sum_{n=1}^{\infty} a_n$		$\lim_{n\to\infty} a_n \neq 0$	Only for Divergence
Geometric Series	$\sum_{n=0}^{\infty} ar^n$	$\lvert r\rvert < 1$	$\lvert r\rvert > 1$	$s = \frac{a}{1-r}$
Telescoping Series	$\sum_{n=1}^{\infty} (b_n - b_{n+1})$	$\lim_{n\to\infty} b_n = L$		$s = b_1 - L$
p-Series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	$p > 1$	$p \leq 1$	
Alternating Series	$\sum_{n=1}^{\infty} (-1)^{n-1} a_n$	$0 < a_{n+1} \leq a_n$ $\lim_{n\to\infty} a_n = 0$		Remainder: $\lvert R_N\rvert \leq a_{N+1}$
Integral	$\sum_{n=1}^{\infty} a_n$ $a_n = f(n) \geq 0$	$\int_1^{\infty} f(x)dx$ converges	$\int_1^{\infty} f(x)dx$ diverges	Remainder: $0 < R_N < \int_N^{\infty} f(x)dx$
Root	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} < 1$	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} > 1$	Inconclusive if limit=1
Ratio	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \left\lvert\frac{a_{n+1}}{a_n}\right\rvert < 1$	$\lim_{n\to\infty} \left\lvert\frac{a_{n+1}}{a_n}\right\rvert > 1$	Inconclusive if limit=1
Direct Comparison ($b_n > 0$)	$\sum_{n=1}^{\infty} a_n$	$0 < a_n \leq b_n$ $\sum_{n=1}^{\infty} b_n$ converges	$0 < a_n \leq b_n$ $\sum_{n=1}^{\infty} b_n$ diverges	
Limit Comparison ($b_n > 0$)	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \left\lvert\frac{a_n}{b_n}\right\rvert = L > 0$ $\sum_{n=1}^{\infty} b_n$ converges	$\lim_{n\to\infty} \left\lvert\frac{a_n}{b_n}\right\rvert = L > 0$ $\sum_{n=1}^{\infty} b_n$ diverges	

Summary of Convergence Tests

NAME	STATEMENT	COMMENTS
Divergence Test (11.4.1)	If $\lim_{k\to+\infty} u_k \neq 0$, then $\sum u_k$ diverges.	If $\lim_{k\to+\infty} u_k = 0$, then $\sum u_k$ may or may not converge.
Integral Test (11.4.4)	Let $\sum u_k$ be a series with positive terms, and let $f(x)$ be the function that results when k is replaced by x in the general term of the series. If f is decreasing and continuous for $x \geq 1$, then $\sum_{k=1}^{\infty} u_k$ and $\int_1^{+\infty} f(x)\,dx$ both converge or both diverge.	This test only applies to series that have positive terms. Try this test when $f(x)$ is easy to integrate.
Comparison Test (11.6.1)	Let $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$ be series with nonnegative terms such that $a_1 \leq b_1, a_2 \leq b_2, \ldots, a_k \leq b_k, \ldots$ If $\sum b_k$ converges, then $\sum a_k$ converges, and if $\sum a_k$ diverges, then $\sum b_k$ diverges.	This test only applies to series with nonnegative terms. Try this test as a last resort; other tests are often easier to apply.
Ratio Test (11.6.5)	Let $\sum u_k$ be a series with positive terms and suppose that $\rho = \lim_{k\to+\infty} \frac{u_{k+1}}{u_k}$ (a) Series converges if $\rho < 1$. (b) Series diverges if $\rho > 1$ or $\rho = +\infty$. (c) The test is inconclusive if $\rho = 1$.	Try this test when u_k involves factorials or kth powers.
Root Test (11.6.6)	Let $\sum u_k$ be a series with positive terms such that $\rho = \lim_{k\to+\infty} \sqrt[k]{u_k}$ (a) The series converges if $\rho < 1$. (b) The series diverges if $\rho > 1$ or $\rho = +\infty$. (c) The test is inconclusive if $\rho = 1$.	Try this test when u_k involves kth powers.
Limit Comparison Test (11.6.4)	Let $\sum a_k$ and $\sum b_k$ be series with positive terms such that $\rho = \lim_{k\to+\infty} \frac{a_k}{b_k}$. If $0 < \rho < +\infty$, then both series converge or both diverge.	This is easier to apply than the comparison test, but still requires some skill in choosing the series $\sum b_k$ for comparison.
Alternating Series Test (11.7.1)	If $a_k > 0$ for $k = 1, 2, 3, \ldots$, then the series $a_1 - a_2 + a_3 - a_4 + \cdots$ $-a_1 + a_2 - a_3 + a_4 - \cdots$ converge if the following conditions hold: (a) $a_1 > a_2 > a_3 > \cdots$ (b) $\lim_{k\to+\infty} a_k = 0$	This test applies only to alternating series.
Ratio Test for Absolute Convergence (11.7.5)	Let $\sum u_k$ be a series with nonzero terms such that $\rho = \lim_{k\to+\infty} \frac{\lvert u_{k+1}\rvert}{\lvert u_k\rvert}$ (a) The series converges absolutely if $\rho < 1$. (b) The series diverges absolutely if $\rho > 1$ or $\rho = +\infty$. (c) The test is inconclusive if $\rho = 1$.	The series need not have positive terms and need not be alternating to use this test.