**April 23, 2004** **Name** _____

There are 115 points available on this test. Each question is marked with its value. To get full credit for a problem, you must **show your work**. Correct answers with incorrect supporting work will receive substantially reduced credit.

1. (20 points) A car $A$ is traveling west at 40 miles per hour while car $B$ is traveling north at 50 miles per hour. At exactly noon, car $A$ is 3 miles east of an intersection $P$ and car $B$ is 4 miles south of $P$. At what speed are the cars moving toward each other?

2. (20 points) A particle is moving along the curve $y^2 = x^3 - 2xy + 3x^2 + 1$.

   (a) Show that the point $(2, 3)$ belongs to the curve.

   (b) Find the slope of the line tangent to the curve at $(2, 3)$.

   (c) If $\frac{dx}{dt} = 5$ at the point $(2, 3)$, what is $\frac{dy}{dt}$ at $(2, 3)$?

3. (20 points) Consider the function $f(x) = \frac{\cos x}{2+\sin x}$ defined over the interval $[0, 2\pi]$.

   (a) Find $f'(x)$.

   (b) Find the critical points of $f$.

   (c) Identify each critical point as a location where a max, a min, or neither occurs.

   (d) Find the absolute maximum and absolute minimum of $f$.

4. (15 points) The mean value theorem (MVT) states that if $f$ is differentiable over $[a, b]$, then there is a number $c$ in $(a, b)$ such that $f'(c)$ is the slope of the line joining $(a, f(a))$ and $(b, f(b))$.

   (a) Does the MVT apply to the function $f(x) = x \ln x$ on the interval $[1, e]$.

   (b) If not tell why. If so, find the number $c$.

5. (20 points) Suppose $f$ is a differentiable function and suppose $f''$ is given by

$$f''(x) = \frac{(x^2-4)(x+5)}{(x+2)(x+1)}.$$

Find the intervals over which $f$ is concave up. No credit for calculator solutions.

6. (20 points) Evaluate each of the following limits:

(a) $\lim_{x\to -1} \dfrac{x^2-1}{x+1}$

(b) $\lim_{x\to 0} \dfrac{x+\tan x}{\sin x}$

(c) $\lim_{x\to \infty} x^3 e^{-x^2}$

(d) $\lim_{x\to 0} (1-2x)^{\frac{1}{x}}$