March 26, 2004   **Name** _____

There are 135 points available on this test. Each question is marked with its value. To get full credit for a problem, you must **show your work**. Correct answers with incorrect supporting work will receive substantially reduced credit.

1. (15 points) Let $p(x) = x^2 - 4x + 5$.

   (a) Compute $p'(x)$

   (b) Compute $p''(x)$

   (c) Use the information in (a) to find an equation for the line tangent to the graph of p at the point $(1, 2)$.

2. (20 points) Consider the *astroid* $x^{2/3} + y^{2/3} = 4$.

   (a) Show that the point $(-3\sqrt{3}, 1)$ belongs to the graph.

   (b) Find y' as a function of x and y using implicit differentiation.

   (c) Find the slope of the line tangent to the curve at the point $(-3\sqrt{3}, 1)$.

   (d) Find an equation for the tangent line whose slope you found above.

3. (30 points) Suppose the functions f and g are given partially by the table of values shown. The next problems refer to the functions f and g given in the tables. Consider the table of values given for the functions $f, f', g,$ and g':

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	3	2	5	2
1	3	5	2	6
2	5	3	4	1
3	6	4	3	4
4	4	6	1	5
5	1	3	2	4
6	1	2	5	3

(a) Let $K(x) = f \circ g(x)$. Compute $K'(3)$

(b) Let $L(x) = f(x) \cdot g(x)$. Compute $L'(2)$.

(c) Let $U(x) = f \circ f(x)$. Compute $U'(1)$.

(d) Let $V(x) = g(x)/f(x)$. Compute $V'(4)$.

(e) Let $W(x) = (g(x))^2$. Compute $W'(5)$.

(f) Let $Z(x) = g(x^2 \cdot f(x))$. Compute $Z'(1)$.

4. (25 points)

    (a) Find $\frac{d}{dx}(\sin x)$

    (b) Write an equation involving the functions sin and $\sin^{-1}$, the composition operation, and the identity function. In other words write an equation that shows you know what $\sin^{-1} x$ is.

    (c) Differentiate both sides of the equation in (b).

    (d) Use the result in (c) to find an expression for $\frac{d}{dx}(\sin^{-1} x)$.

    (e) Let $h(x) = \sin^{-1}(x^2)$. Compute $h'(x)$.

5. (25 points) Compute the following derivatives.

   (a) $\frac{d}{dx} e^{\sin x}$

   (b) $\frac{d}{dx} \ln(\tan x)$

   (c) $\frac{d}{dx} \sqrt{x}(\ln x)$

   (d) $\frac{d}{dx} (\cos(x^2))^3$

   (e) $\frac{d}{dx} \tan^{-1}(2x)$

6. (20 points) Suppose f is defined by:

$$f(x) = \begin{cases} \ln(3x) & \text{if } x > 0 \\ \ln(-x) & \text{if } x < 0 \end{cases}$$

(a) Find $f'(3)$.

(b) Find $f'(-e)$.

(c) Find an equation for the line tangent to the graph of f at the point $(-e, f(-e))$.