1. Find the least common multiple and the greatest common divisor of each of the following pairs of integers.

   (a) $m = 2001$ and $n = 1001$.

   (b) $m = 2^3 \cdot 3^2 \cdot 5$ and $n = 3^2 \cdot 7^2$.

   (c) $m = p^3 \cdot q^2 \cdot r$ and $n = p^2 \cdot q \cdot r^3$, where p, q, and r are different primes.

2. Notice that we can find the number of positive integer divisors of an integer N in terms of its prime factorization. For example, the number $2^i 3^j 5^k$ is a divisor of $2^l 3^m 5^n$ if and only if $i \leq l, j \leq m$ and $k \leq n$. Find the number of divisors of each of the integers.

   (a) $m = 2001$.

   (b) $n = 1001$.

   (c) $m = 2^3 \cdot 3^2 \cdot 5$.

   (d) $n = 3^2 \cdot 7^2$.

   (e) $m = p^3 \cdot q^2 \cdot r$, where p, q, and r are different primes.

3. A high school with 1000 lockers and 1000 students tries the following experiment. All lockers are initially closed. Then student number 1 opens all the lockers. Then student number 2 closes the even numbered lockers. Then student number 3 changes the status of all the lockers numbered with multiples of 3. This continues with each student changing the status of all the lockers which are numbered by multiples of his or her number. Which lockers are closed after all the 1000 students have done their jobs?

4. Using the Euclidean Algorithm, evaluate $g = GCD(4144, 7596)$ and $h = LCM(4144, 7596)$. Then solve the equation $g = 4144x + 7596y$, for integers x and y.

5. Let a, b, c, d, and e be digits satisfying $4 \cdot \underline{abcde4} = \underline{4abcde}$. Find all five of the digits.

6. A 10×10 square is decomposed into exactly 75 squares of various (integer) sizes. How many 3×3 squares are in this decomposition?

7. A two-digit integer N that is not a multiple of 10 is k times the sum of its digits. The number formed by interchanging the the digits is m times the sum of the digits. What is the relationship between m and k?

8. The fido challenge: http://digicc.com/fido/

9. The crystal cabobble challenge: http://www.sinotrading.us/crystalball.htm
10. How many two-digit positive integers N have the property that the sum of N and the number obtained by reversing the order of the digits of N is a perfect square?
11. A check is written for x dollars and y cents, both x and y two-digit numbers. In error it is cashed for y dollars and x cents, the incorrect amount exceeding the correct amount by \$17.82. Find a possible value for x and y.
12. Solve the alpha-numeric problem $\underline{2abc} \times 4 = \underline{cba2}$, where a, b and c are decimal digits.
13. The rightmost digit of a six-digit number N is moved to the left end. The new number obtained is five times N. What is N?
14. Repeat the same problem with the 5 changed to a 4. That is $4(\underline{abcdef}) = \underline{fabcde}$
15. Next, consider another transformation. This time move the leftmost digit to the right end and change the 5 to a 3.
16. Consider the problem you get when you move the right TWO digits to the left end. Can this new number be exactly half the original number? Find all solutions and prove that you have them all.