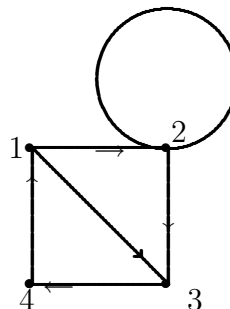**Math 1165   Homework for March 27, 2007   Relation/Matrix problem**

Your name _____

Consider the relation R defined on the set $S = \{1, 2, 3, 4\}$ as follows:

$$R = \{(1,2), (1,3), (2,2), (2,3), (3,4), (4,1)\}.$$

1. Draw the digraph of R.

   Solution:

2. Construct the matrix M_R of R.

   Solution: $M_R = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$

3. Compute the composition relation $R \circ R$ as a set of ordered pairs and construct its digraph.

   Solution: $R \circ R = \{(1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,1), (4,2), (4,3)\}.$

4. Use your graphing calculator to find the matrices M_R^2, M_R^4 M_R^8. Interpret the entry $a_{1,3}$ of the matrix M_R^8 as it relates to the digraph.

   Solution: $M_R^2 = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix}$, $M_R^4 = \begin{pmatrix} 1 & 2 & 2 & 1 \\ 1 & 2 & 2 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix}$, and

   $M_R^8 = \begin{pmatrix} 3 & 9 & 9 & 6 \\ 3 & 9 & 9 & 6 \\ 1 & 4 & 4 & 3 \\ 1 & 4 & 4 & 3 \end{pmatrix}$. The entry $a_{1,3}$ is 9, and this means that there are 9 paths in the digraph of length 8 from the vertex 1 to the vertex 3.