

Mastermind is a two-player game consisting of a codemaker and a codebreaker. The codemaker secretly selects a code consisting of an ordered sequence of four colors  $(c_1, c_2, c_3, c_4)$ , each chosen from a set  $\{A, B, C, D, E, F\}$  of six possible colors, with repetitions allowed. The codebreaker then tries to guess the code by repeatedly proposing a sequence of colors. After each guess, the codemaker tells the codebreaker two numbers: the number of correct colors in the correct positions  $b$  and the number of colors in that are part of the code but not in the correct positions  $m$ . For example, if the code is  $(A, B, C, C)$  and the codebreaker's guess is  $(B, C, D, C)$ , then the codemaker's response would be  $(1, 2)$ , since the codebreaker has guessed the second  $C$  and correctly and in the correct position, while having guessed the  $B$  and the first  $C$  correctly, but in the wrong position.

The codebreaker continues guessing until he guesses the code correctly or until he reaches a maximum allowable number of guesses without having correctly identified the secret code. See <http://mathworld.wolfram.com/Mastermind.html>

1. Find the code for the sequence given.

Guess number	Guess	$b$	$m$
1	AABB	0	0
2	CCDE	1	3
3	DCAC	0	3

**Solution:** CECD.

2. Find the code for the sequence given.

Guess number	Guess	$b$	$m$
1	AABB	0	1
2	BCDD	0	2
3	CBCE	1	2
4	EFCB	1	3

**Solution:** EBFC.

3. Consider the sequence given.

Guess number	Guess	$b$	$m$
1	AABB	0	1
2	BCDD	0	0
3	EEAE	1	2

What are the two possible codes?

**Solution:** FEEA and EFEA.

The next two problems are due to Brendan Fletcher.

4. Find the code for the sequence given.

Guess number	Guess	$b$	$m$
1	<i>AABB</i>	0	1
2	<i>BCDD</i>	0	2
3	<i>CBCE</i>	1	2
4	<i>EFCB</i>	2	2

**Solution:** We know from guess 4 that  $B, C, E,$  and  $F$  are the correct colors. We know from guess 1 that  $B$  is not in the 3rd or 4th positions, and from guess 2 that  $B$  cannot be in the first position. Therefore,  $B$  is in the 2nd position, so  $\_B\_ \_$ . Since in guess 3 ( $CBCE$ )  $b = 1$  and  $m = 2$ , the ' $b = 1$ ' corresponds to the  $B$ . Therefore, both  $C$ 's are in the wrong place. In other words,  $C$  is not in the 1st or 3rd positions. But  $B$  is in the 2nd position, so  $\_B\_C$ , which leaves only two answers:  $EBFC$  and  $FBEC$ . But guess 4 is  $EBFC$ , which is not the answer. Therefore,  $FBEC$ .

5. Find the code for the sequence given.

Guess number	Guess	$b$	$m$
1	<i>AABB</i>	0	2
2	<i>BCDD</i>	0	1
3	<i>CBCE</i>	1	0
4	<i>EFCB</i>	3	0

**Solution:** According to guess 4, three of the colors  $A, A, C,$  and  $F$  are correct. Any way you combine three of those colors, there is an  $A$ , so there is at least one  $A$  in the code. There is only one correct color in guess 3 ( $EBAE$ ), which must be  $A$ , so there are no  $B$ 's or  $E$ 's in the code. There are only two correct colors in  $AABB$ , but there are no  $B$ 's, so there must be two  $A$ 's in the code. That means  $\_ \_AA$ . Suppose  $C$  was in the code. There is only one correct color in  $BCDD$ , which would be  $C$ . Therefore, there would be no  $B$ 's or  $D$ 's, which would leave only  $F$  to put in the blank of  $C\_AA$ , since there are no  $E$ 's. But  $CFAA$  is guess 4, which is not correct. So  $C$  is not in the code, and  $\_FAA$ . There are no  $B$ 's or  $C$ 's, so according to guess 2,  $D$  is in the code. Therefore,  $DFAA$ .

6. Find the code for the sequence given.

Guess number	Guess	$b$	$m$
1	<i>BBCC</i>	1	0
2	<i>ADDB</i>	0	1
3	<i>CEFE</i>	0	2
4	<i>FBEC</i>	3	0

**Solution:** Which three colors are right in guess 4? It can't be  $B, C, E$  because of guess 1. It can't be  $B, C, F$  because of guess 1. It can't be  $C, E, F$  because of guess 2. So it must be  $B, E, F$ . What can be the fourth color? We'll try each combination  $B, E, F, x$  where  $x$  is one of the six colors. What about  $A, B, E, F$ ? No, because of guess 2; what about  $B, B, E, F$ ? No, because of guess 1; what about  $C, B, E, F$ ? No, because of guess 3; what about  $D, B, E, F$ ? No, because of guess 2; what about  $E, B, E, F$ ? No, because of guess 3. So the set must be  $F, B, E, F$ . And the order must be  $FB EF$ .