1. Write $n = 1492$ in the base $b = 2$ $b = 4$, and $b = 8$. What is the relationship among these representations? In other words, find a way to convert between these bases without translating them to and from decimal.

   Solution: First compute the base 8 representation of 1492 to get $1492 = 2724_8$. Then find the binary representation of each of the four digits 2,7,2, and 4. These are $010, 111, 010$, and 100. Now you can string these representations together to get 010111010100_2. To get the base 4 representation of 10111010100_2, interpret each pair of binary digits as a base four digit, starting at the right. Thus $00 = 0; 01 = 1; 01 = 1; 11 = 3; 01 = 1$ and $1 = 1$. Thus 113110_4 is the base four representation of 1492.

2. Find the unknown digit x from the equation $2x3_4 = 1x10_3$.

   Solution: Interpret the two numbers to get $2x3_4 = 2 \cdot 4^2 + 4x + 3$ and $1x10_3 = 1 \cdot 3^3 + 3^2x + 3$. Then solve the resulting linear equation to find that $x = 1$.

3. Construct the tables of addition and multiplication for the base $b = 6$ and evaluate $215_6 + 304_6$, $\quad 203_6 \times 405_6$.

   Solution: $(215)_6 + (304)_6 = 523_6$ and $(203)_6 \times (405)_6 = 123423_6$

4. Find the base -5 representation of all the numbers from 1 to 25.

   Solution: They are $1, 2, 3, 4, 140, 141, 142, 143, 144, 130, 131, 132, 133, 134, 120, 121, 122, 123, 124, 110, 111, 112, 113, 114, 100$.

5. Find the base 4 representation of 1/9. Prove your answer.

   Solution: $1/9 = 0.\overline{013}_4$ by the process of repeated multiplication. To prove this, let $x = 0.\overline{013}_4$, and then note that $1000_4x = 13.\overline{013}_4$ in which case we have $1000_4x - x = 333_4x = 13_4$, which is the same as the decimal equation $63x = 7$, so $x = 1/9$.

6. Find the base -4 representation of $1/3$ and then show that your answer is correct.

   Solution: $1.\overline{32}_{-4}$. To see that this is correct, let $x = 1.\overline{32}_{-4}$. Then $100x = 132.\overline{32}$, and $100_{-4} - 1_{-4} = 114_{-4}$, so $113_{-4}x = 132_{-4}$. Translating these into decimal notation, we have $15x = 1 \cdot (-4)^2 + 3 \cdot (-4) + 2 = 5$, so $x = 1/3$.

7. How many two-digit positive integers N have the property that the sum of N and the number obtained by reversing the order of the digits of N is a perfect square?

Solution: Let $N = 10x + y$. Then $10x + y + 10y + x = 11(x + y)$ must be a perfect square. Since $1 \leq x + y \leq 18$, it follows that $x + y = 11$. There are eight such numbers: 29, 38, 47, 56, 65, 74, 83, and 92.

8. A check is written for x dollars and y cents, both x and y two-digit numbers. In error it is cashed for y dollars and x cents, the incorrect amount exceeding the correct amount by \$17.82. Find a possible value for x and y.

   Solution: Note that $100y+x-(100x+y) = 99(y-x) = 1782$. It follows that $y - x = 18$, so any pair of two-digit numbers that differ by 18 will work.

9. The rightmost digit of a six-digit number N is moved to the left end. The new number obtained is five times N. What is N?

   Solution: The number is 142857. Let $N = \underline{abcdef}$ and let $x = \underline{abcde}$. Then we have $5N = 5(\underline{abcdef}) = \underline{fabcde}$. Note that we have adopted the convention of underlining the digits of an integer in decimal notation. For example $\underline{ab} = 10a + b$. Note that $5(\underline{abcdef}) = 5(\underline{abcde0} + f) = 5(10x + f) = \underline{fabcde} = 100000f+x$. This leads to $50x-x = 100000f-5f$ or $49x = 99995f$. Factoring both sides gives $7^2x = 5 \cdot 7 \cdot 2857 \cdot f$, which is equivalent to $7x = 5 \cdot 2857 \cdot f$, from which it follows that $f = 7$. Therefore $x = 14285$ and $N = 142857$.