Math 1120 Section 4 **Calculus** **Test 4**

November 22, 2004 **Name** _____

The total number of points available is 195. Throughout this test, **show your work.** For most questions, the answer along without any supporting mathematics will not be worth any points.

1. (40 points) Find each of the following derivatives.

    (a) If $f(x) = e^{x^2}$, then $f'(x) =$

    (b) If $g(x) = \ln(x^2 - 2x + 2)$, then $g'(x) =$

    (c) If $y = xe^{x^2-x}$, then $\frac{dy}{dx} =$

    (d) If $h(x) = \dfrac{\ln x}{x}$, then $h'(x) =$

    (e) If $y = (\ln e^{2x-1})^2$, then $\frac{dy}{dx} =$

2. (20 points) Consider the function $f(x) = e^{x^3 - 3x^2}$.

   (a) Compute $f'(x)$.

   (b) Find the critical points of f.

   (c) Find the relative max and min of f.

   (d) What is the maximum value of f over the interval $[-2, 4]$?

3. (10 points) Find an equation for the line tangent to the graph of $y = \ln x^2$ at the point $(1, 0)$.

4. (10 points) Find an equation for the line tangent to the graph of $y = e^{4x-2}$ at the point $(1/2, 1)$.

5. (20 points) A skull from an archeological dig has one-tenth the amount of Carbon-14 it had when the specimen was alive.

   (a) Recall that the half-life of Carbon-14 is 5770 years. Find the decay constant k.

   (b) What is the age of the specimen?

6. (20 points) How long does it take an investment of $1000 at an annual rate of 6% to triple in value if compounding takes place

   (a) takes place quarterly? Round your answer to the nearest tenth of a year.

   (b) takes place continuously? Round your answer to the nearest tenth of a year.

7. (25 points) According to Newton's Law of Cooling, the rate at which an object's temperature changes is proportional to the temperature of the medium into which it is emersed. If $F(t)$ denotes the temperature of a cup of instant coffee (initially $212°F$), then it can be proven that

$$F(t) = T + Ae^{-kt},$$

where T is the air temperature, $72°F$, A and k are constants, and t is expressed in minutes.

   (a) What is the value of A?

   (b) Suppose that after exactly 20 minutes, the temperature of the coffee is $186.6°F$. What is the value of k?

   (c) Use the information in (a) and (b) to find the number of minutes before the coffee reaches the temperature of $80°F$.

   (d) Find the rate at which the object is cooling after $t = 20$ minutes.

8. (40 points) Consider the function $g(x) = (x-3)^3(x-1)^2(x+1)^4$.

   (a) Compute $\frac{dy}{dx}$ using logarithmic differentiation.

   (b) Find the critical points of g.

   (c) Construct the sign chart for $g'(x)$.

   (d) (added after the test) Use the sign chart to find the relative extrema of g.

9. (30 points) Recall that up to this chapter in the course, there are just two types of functions whose (implied) domains are restricted. For example $y = 1/x$ has domain $(-\infty, 0) \cup (0, \infty)$ and $y = \sqrt{x}$ has domain $[0, \infty)$. We now have a third such function type. The domain of $y = \ln x$ is $(0, \infty)$.

   (a) What is the domain of $f(x) = \ln((x-1)(x+2))$?

   (b) Compute $f'(x)$ and construct its sign chart. Note that $f'(x)$ appears to have a larger domain than f. Of course that is impossible.

   (c) Over what intervals is f increasing?

   (d) Over what intervals is f concave upwards?