April 24, 1998 **Name** ____________________

In the first six problems, each part counts 7 points (total 63 points) and the final two problems count as marked.

Multiple choice section. Circle the correct choice. You do not need to show your work on these problems.

1. Let $f(x) = e^{2x+3}$. What is $f'(0)$?

   (A) 0 **(B)** 2 **(C)** e^2 **(D)** $2e^2$ **(E)** $\boxed{2e^3}$

2. Let $f(x) = \ln(x^4)$. What is $f'(e^2)$?

   (A) 0 **(B)** 2 **(C)** 4 **(D)** $\boxed{4e^{-2}}$ **(E)** $4e^2$

3. Which of the following is closest to a solution to $2e^{x^2+1} = 1998$?

   (A) 1.74 **(B)** $\boxed{2.43}$ **(C)** 6.91 **(D)** 10 **(E)** 31.59

4. For how many values of x is it true that

$$\ln[(x^2 - 1)(x^2 - 4) + e] = 1?$$

   (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** $\boxed{4}$

5. Population Growth. The population of a town increases according to the model

$$P(t) = 2200e^{0.04t}$$

   where t is measured in years with $t = 0$ corresponding to 1990. Use the model to approximate the population in 1995.

   (A) 2599 **(B)** 2608 **(C)** 2655 **(D)** 2679 **(E)** $\boxed{2687}$

6. A total of \$10,000 is invested at an annual rate of 9%. Find the balance after 5 years if it is compounded quarterly.

   (A) $\boxed{\$15,605}$ **(B)** $\$15,683$ **(C)** $\$15,720$ **(D)** $\$15,818$ **(E)** $\$15,988$

On all the following questions, **show your work.**

7. (15 points) Suppose that \$300 is deposited into an account with an annual percentage rate of 7%. What is the balance in the account after 4 years, assuming that compounding takes place

   (a) quarterly? Round your answer to the nearest penny.

   $300(1+\frac{0.07}{4})^{4\cdot 4} \approx 300 \cdot 1.3199\ldots \approx \$395.98.$

   (b) continuously? Again, round your answer to the nearest penny.

   Use the shampoo formula, $A = Pe^{rt}$ to get $A = 300e^{0.07\cdot 4} \approx 300 \cdot 1.3231 \approx \$396.94.$

8. (20 points) A radioactive substance has a half-life of 33 years.

(a) Use the exponential decay model to write an equation which shows that after 33 years, a sample with 200 grams of radioactivity has only 100 grams left.

The amount of radioactive substance A left is given by

$$A(t) = Ie^{-kt}$$

where k is a positive constant, and I is the initial amount. Then $A(33) = Ie^{-33k} = 100\text{grams}$.

(b) Use the fact that there are **initially** 200 grams of radioactivity to solve for one of the functions parameters.

Because the original amount satisfies $A(0) = Ie^{-k\cdot 0} = Ie^{0} = I$, it follows that $I = 200$.

(c) Use the information in (a) and (b) to solve for the constant k in the function.

Note that $100 = 200e^{-33k}$ is equivaelnt to saying $e^{-33k} = .5$. Solve this by taking the natural log of both sides: $k = \ln(.5)/-33 = 0.021$.

(d) How many years must elapse before the amount of radioactivity is down to 25 grams.

$25 = 200e^{-0.021t}$ which means that $e^{-0.021t} = 0.125$. Take the natural log of both sides to get $t = \frac{\ln(1/8)}{-0.021} = 99$ years.

9. (15 points) Consider the logistic growth function

$$Q(t) = \frac{A}{1 + Be^{-kt}},$$

where A, B, and k are positive constants.

(a) Find $Q'(t)$.

$$\begin{aligned} Q'(t) &= \frac{0 - A(-kBe^{-kt})}{(1 + Be^{-kt})^2} \\ &= \frac{kABe^{-kt}}{(1 + Be^{-kt})^2} \end{aligned}$$

(b) Use this information to show that $Q(t)$ is increasing in the interval $(0, \infty)$.

Both numerator and denominator of $Q'(t)$ are positive, so $Q'(t) > 0$ for $t > 0$. Thus $Q(t)$ is an increasing function.