Math 1120, Section 6 **Calculus** **Test 3**

April 10, 2008 **Name** ______________________________

The total number of points available is 141. Throughout this test, **show your work.**

1. (15 points) Consider the function $f(x) = 2x^3 - 8x^2 + 3, \quad -2 \le x \le 10$. Find the locations of the absolute maximum of $f(x)$ and the absolute minimum of $f(x)$ and the value of f at these points.

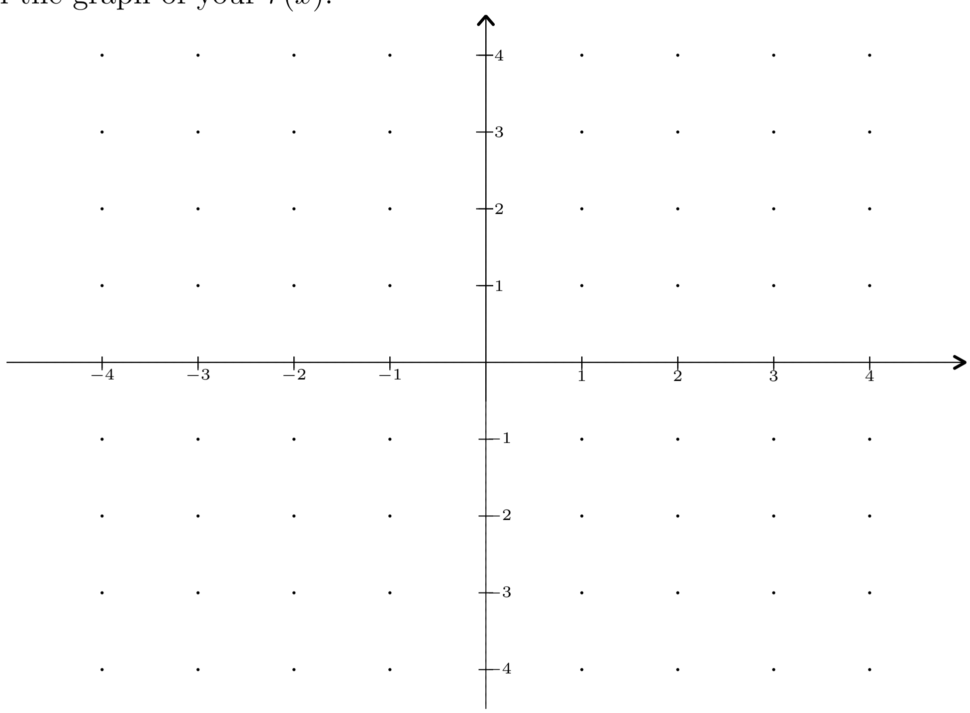
2. (20 points) Find a rational function $r(x)$ that has all the following properties:

   (a) It has exactly two zeros, $x = -3$ and $x = 3$.

   (b) It has two vertical asymptotes, $x = -2$ and $x = 2$.

   (c) It has $y = -2$ as a horizontal asymptote.

   (a) Sketch the graph of your $r(x)$.



   (b) Find a symbolic representation of r.

3. (30 points) Consider the function $f(x) = (2x+3)^2(x-4)^2$.

   (a) Find $f'(x)$ and $f''(x)$.

   (b) Find all the critical points of f.

   (c) Apply the Test Interval Technique to find the sign chart for f' and use the information in the sign chart to classify the critical points of f. In other words, tell whether each one is the location of (a) a relative maximum, (b) a relative minimum, or (c) neither a relative max or min.

   (d) List the intervals over which f is increasing.

   (e) Discuss the concavity to f and find all the inflection points on the graph of f.

4. (15 points) Find an equation for the line tangent to (the graph of) $f(x) = xe^{2x}$ at the point $(2, 2e^4)$.

5. (20 points) Certain radioactive material decays in such a way that the mass remaining after t years is given by the function

$$m(t) = 165e^{-0.02t}$$

where $m(t)$ is measured in grams.

(a) Find the mass at time $t = 0$.

(b) How much of the mass remains after 15 years?

(c) What is the half-life of the material?

(d) Find the rate of loss at $t = 1$ year.

6. (10 points) The population of the world in 1990 was 5 billion and the relative growth rate was estimated at 1.5 percent per year. Assuming that the world population follows an exponential growth model, find the projected world population in 2010.

7. (20 points) Consider the function $f(x) = x\ln(x^2 + 2)$.

   (a) Find $f'(x)$.

   (b) Find $f''(x)$.

   (c) Find the sign chart for $f''(x)$.

   (d) Find the intervals over which f is concave upwards.

8. (12 points) A rancher wants to fence in an area of 12 square miles in a rectangular field and then divide it in half with a fence down the middle parallel to one side. What is the shortest length of fence that the rancher can use?