November 21, 2006 **Name** ______________________________

The total number of points available is 139. Throughout this test, **show your work.**

1. (10 points) Find all vertical and horizontal asymptotes, if any, for $r(x) = \frac{6x^2+3x}{x^2+5x-6}$.

2. (12 points) Consider the function $f(x) = 4x^3 + 15x^2 - 18x, \quad -4 \le x \le 4.$ Find the locations of the absolute maximum of $f(x)$ and the absolute minimum of $f(x)$ and the value of f at these points.

3. (12 points) Sketch the graph of a function f that satisfies the following conditions.

   (a) The domain of f is $(-\infty, 1) \cup (1, \infty)$

   (b) $x = 1$ is a vertical asymptote.

   (c) $\lim_{x \to \infty} f(x) = 2, \lim_{x \to -\infty} f(x) = 0.$

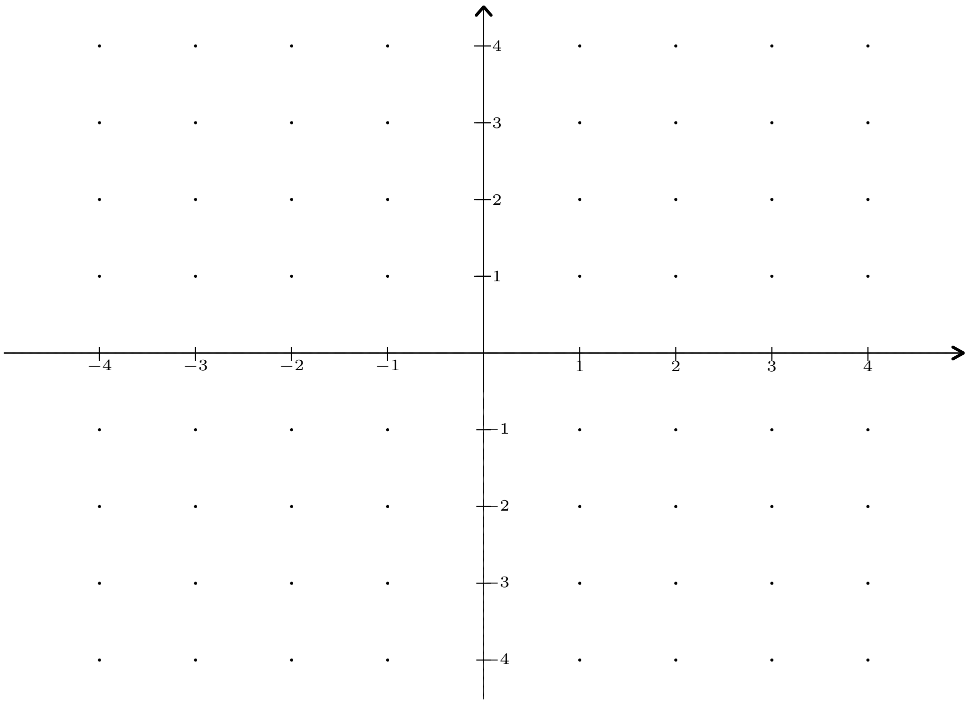
   (d) $f(-1) = 1,\ f(0) = 0$

   (e) $f'(x) > 0$ on $(-\infty, -1) \cup (0, 1)$

   (f) $f'(x) < 0$ on $(-1, 0) \cup (1, \infty)$

   (g) $f''(x) > 0$ on $(-\infty, -2) \cup \left(-\frac{1}{2}, 1\right) \cup (1, \infty)$

   (h) $f''(x) < 0$ on $\left(-2, -\frac{1}{2}\right)$



4. (20 points) For each function listed below, find all the critical points. Recall that the derivative of the exponential function $e^{f(x)}$ is given by $\frac{d}{dx}e^{f(x)} = f'(x)e^{f(x)}$. It is also useful to know that the equation $e^x = 0$ has no solutions. Tell whether each critical point gives rise to a local maximum, a local minimum, or neither.

   (a) $f(x) = (x^2 - 4)^3$

   (b) $g(x) = (x + 2)^{2/3}$

   (c) $h(x) = e^{x^3 - 3x}$

   (d) $k(x) = \dfrac{x^2 - 1}{2x + 3}$

   (e) $f(x) = \ln(x^2 + 4)$

5. (25 points) Consider the function $f(x) = (2x-5)^2(x^2-3)$.

   (a) Find the places where $f(x)$ changes signs.

   (b) Find the places where $f(x)$ has a horizontal tangent line.

   (c) Find the places where $f(x)$ changes concavity.

   (d) For the points in part (b), tell whether $f(x)$ has (a) a relative maximum, (b) a relative minimum, or (c) neither a relative max or min.

6. (10 points) Find a rational function $r(x)$ that has the following properties.
   (a) It has exactly two zeros, $x = -4$ and $x = 2$.
   (b) It has two vertical asymptotes, $x = 0$ and $x = -1$.
   (c) It has $y = 2$ as a horizontal asymptote.

7. (15 points) Find the point on the line $3x + 2y = 6$ that is closest to the point $(-4, -4)$.

8. (15 points) A baseball team plays in he stadium that holds 60000 spectators. With the ticket price at 12 dollars the average attendance has been 25000. When the price dropped to 10 dollars, the average attendance rose to 40000.

   (a) Find the demand function $p(x)$, where x is the number of the spectators and $p(x)$ is measured in dollars, assuming it is linear. In other words, if the relationship between the price and number of tickets sold is linear, find the price when x tickets are sold.

   (b) How should the ticket price be set to maximize revenue?

9. (20 points) According to Newton's Law of Cooling, the rate at which an object's temperature changes is proportional to the temperature of the medium into which it is emersed. If $F(t)$ denotes the temperature of a cup of instant coffee (initially $212°F$), then it can be proven that

$$F(t) = T + Ae^{-kt},$$

where T is the air temperature, $72°F$, A and k are constants, and t is expressed in minutes.

(a) What is the value of A?

(b) Suppose that after exactly 10 minutes, the temperature of the coffee is $186.6°F$. What is the value of k?

(c) Use the information in (a) and (b) to find the number of minutes before the coffee reaches the temperature of $80°F$.