August 2, 2005 **Name** _____

The total number of points available is 160. Throughout this test, **show your work.**

1. (10 points) Consider the function $f(x) = 5x^2 - 8x + 3, \quad 0 \leq x \leq 7$. Find the locations of the absolute maximum of $f(x)$ and the absolute minimum of $f(x)$ and the value of f at these points.

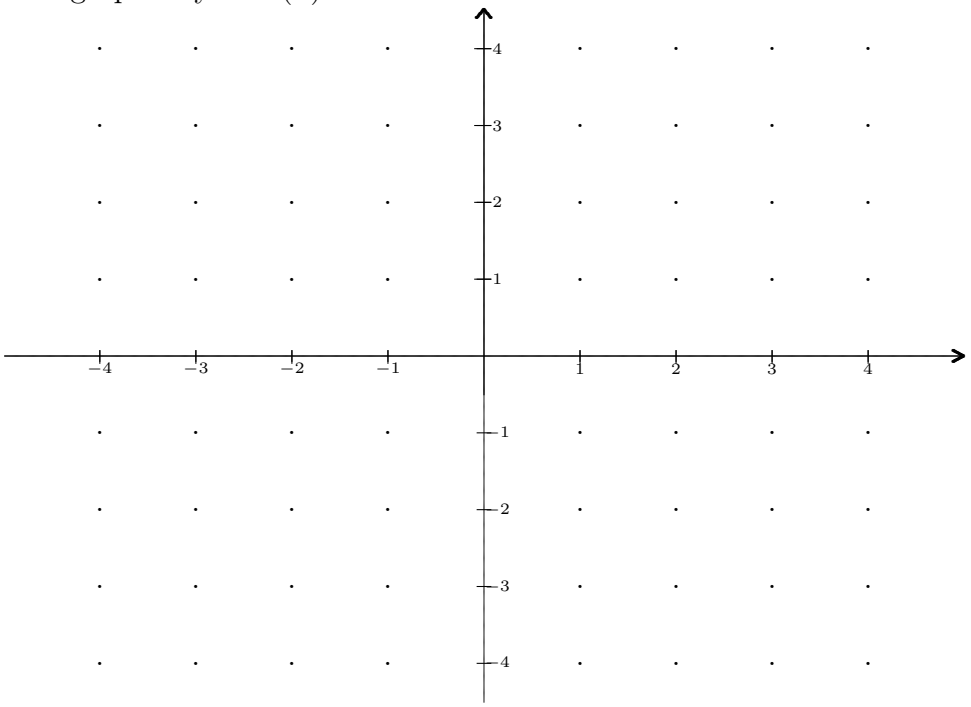
2. (20 points) Find a rational function $r(x)$ that has all the following properties:

   (a) It has exactly two zeros, $x = -2$ and $x = 3$.

   (b) It has two vertical asymptotes, $x = 0$ and $x = -3$.

   (c) It has $y = 2$ as a horizontal asymptote.

   (a) Sketch the graph of your $r(x)$.



   (b) Find a symbolic representation of r.

3. (50 points) Consider the function $f(x) = (2x-1)^2(x+3)^2$.

   (a) Find $f'(x)$ and $f''(x)$.

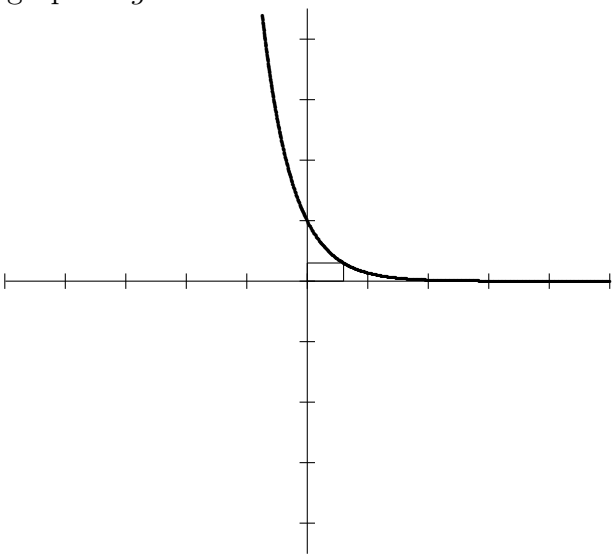
   (b) Find the three critical points of f.

   (c) Apply the Test Interval Technique to find the sign chart for f' and use the information in the sign chart to classify the critical points of f. In other words, tell whether each one is the location of (a) a relative maximum, (b) a relative minimum, or (c) neither a relative max or min.

   (d) List the intervals over which f is increasing.

   (e) Discuss the concavity to f and find all the inflection points on the graph of f.

4. (20 points) Consider the function $g(x) = e^{-2x}$. A rectangle R with sides parallel to the x- and y-axes has its lower left vertex at the origin and its upper right vertex on the graph of g as shown below.



(a) Note that the area of R depends only on the choice of x. Find the area $R(x)$. For example, $R(2) = 2 \cdot e^{-4}$.

(b) Find the value of x that maximizes the area of the rectangle. What is it about the sign chart of $R'(x)$ that convinces you the you have found a relative maximum.

5. (20 points) Consider the function $f(x) = \ln(x^2 + 1)$.

   (a) Find $f'(x)$.

   (b) Find $f''(x)$.

   (c) Find the sign chart for $f''(x)$.

   (d) Find the intervals over which f is concave upwards.

6. (20 points) A rancher wants to fence in an area of 10 square miles in a rectangular field and then divide it in half with a fence down the middle parallel to one side. What is the shortest length of fence that the rancher can use?

7. (20 points) A baseball team plays in he stadium that holds 60000 spectators. With the ticket price at 12 dollars the average attendance has been 25000. When the price dropped to 10 dollars, the average attendance rose to 40000.

   (a) Find the demand function $p(x)$, where x is the number of the spectators and $p(x)$ is measured in dollars, assuming it is linear. In other words, if the relationship between the price and number of tickets sold is linear, find the price when x tickets are sold.

   (b) How should the ticket price be set to maximize revenue?