

November 23, 2015

Name _____

The problems count as marked. The total number of points available is 130. Throughout this test, **show your work**. Use of calculator to circumvent ideas discussed in class will generally result in no credit.

1. (50 points) You must show your work.
 - (a) (10 points) Let $F(x) = (x^2 - 3x + 1)e^{-x}$. Find the intervals where F is increasing.

 - (b) (10 points) In the year 2000 the population of the earth was estimated to be 6.5 billion people. Today the estimate is 7.3 billion. What is the annual population growth? Round your answer to the nearest hundredth of a percent.

 - (c) (10 points) Let $G(x) = x \ln(x^2 - 3)$. Find an equation for the line tangent to G at the point $(2, G(2))$.

 - (d) (10 points) Let $K(x) = x^2 e^{-2x}$. Discuss the concavity of K .

 - (e) (10 points) Let $L(x) = (e^{-2x} + 1)^3$. Find an equation for the line tangent to L at the point $(0, 8)$.

2. (20 points) Let's assume that American male height can be modeled using a learning curve. Such a person is about 1 foot long at birth and about 6 feet tall at age 60, and they do not get any taller.

(a) Solve for two of the three parameters.

(b) If a child is 18 inches long at age 1, how many years will it take until he is 5 feet tall?

(c) How much growth does the person exhibit during the two year period from age 9 to age 11?

(d) Find $Q'(10)$ and compare this with your result in part (c). Explain in English what you think?

3. (30 points) Let

$$f'(x) = \begin{cases} -x - 5 & \text{if } x < 0 \\ -(x - 3)(x - 5) & \text{if } 1 \leq x \end{cases},$$

(a) What is the domain of f' ? Express your answer in interval notation.

(b) Find the intervals over which f' is increasing.

(c) Find the intervals over which the function f is increasing.

(d) Suppose $f(2) = 5$. Find an equation for the line tangent to the graph of f at the point $(2, 5)$.

4. (15 points) Let $H(x) = \ln((x^2 - 4)(x^2 - 16))$.
- (a) Recall that $\ln(x)$ is defined precisely when $x > 0$. What is the domain of H (in interval notation)?
- (b) Find the slope of the line tangent to H at the point $x = 1$.

5. (15 points) Build a function that satisfies the logistic curve model

$$Q(t) = \frac{A}{1 + Be^{-kt}}$$

that satisfies

- $Q(0) = 100$
- $\lim_{t \rightarrow \infty} Q(t) = 150$
- $Q'(0) = 20$

Then find a value of t for which $Q(t) = 125$.

6. (10 points) Find a cubic polynomial with a max at $x = -3$, a min at $x = 2$ and has value 2 at $x = 0$.
7. (10 points) Find an equation for the line tangent to the graph of $g(x) = e^{e^x}$ at the point $x = \ln(2)$.
8. (10 points) The number a satisfies $2^a = 7$. Estimate 9^a correct to three significant places.
9. (10 points) What is the y -intercept of the line tangent to the graph of $f(x) = (1 + e^{-2x+4})^2$ at the point $(2, f(2))$.