April 8, 2015 **Name** ______________________________

The total number of points available is 168. Throughout this test, **show your work. Throughout this test, you are expected to use calculus to solve problems. Graphing calculator solutions will generally be worth substantially less credit.**

1. (12 points) Find an equation for the line tangent to the graph of $f(x) = xe^{-2x+4}$ at the point $(2, f(2))$.

2. (12 points) In 1985, the tuition at Yale University was \$10000 per year. In 2015 it was about \$44000 per year. Estimate the annual percent growth. Write a sentence to justify your answer.

3. (12 points) Find an equation for the line tangent to the graph of $f(x) = x^2 \ln(x)$ at the point $(1, f(1))$.

4. (30 points) There is a function g whose derivative is given below:

$$g'(x) = \begin{cases} x^2 - 2x - 3 & \text{if } -4 \le x \le 4 \\ 9 - x & \text{if } 4 < x \le 12 \end{cases}$$

(a) What is the domain of g'. Use interval notation.

(b) Find the critical points of g'.

(c) Find the intervals over which g' is decreasing.

(d) Find the intervals over which the function g is decreasing.

(e) Find the critical points of g.

(f) Find the absolute maximum and absolute minimum of g'. You must show all your work.

5. (20 points) Consider the function $f(x) = x^4$. In this problem we are looking for the point on the graph of f that is closest to the point $(0, 16)$. We'll prove that the point exists as follows. Note that the points belonging to the graph of f are of the form $(x, y) = (x, x^4)$. Build the distance function $d(x)$ that measures the distance from $(0, 16)$ to (x, x^4). For example $d(2)$ is the distance between $(0, 16)$ and $(2, 2^4)$, which is just 2.

   (a) Let $D(x)$ be the square of $d(x)$. In other words, D is the inside part of d, but without the radical. Compute $D'(x)$.

   (b) Notice that $x = 0$ is a critical point. Is it the location of a relative maximum, a relative minimum, or an imposter? Write a sentence supporting your answer.

   (c) What is $D'(1)$? What is $D'(2)$? Since D' is continuous, you can apply the Intermediate Value Theorem. Is this critical point a relative max or min, or neither. Is $D(x)$ increasing or decreasing at $x = 2$?

6. (15 points) For each function f listed below, find the slope of the line tangent to its graph at the point $(0, f(0))$.

   (a) $f(x) = e^{e^x}$.

   (b) $f(x) = (x-1)^2 \cdot \ln(2x+1)$.

   (c) $f(x) = (1 + \ln(2x+1))^3$.

7. (10 points) For each function listed below, find a critical point.

   (a) $g(x) = 4\sqrt{x^2+1} - 2x + 20$.

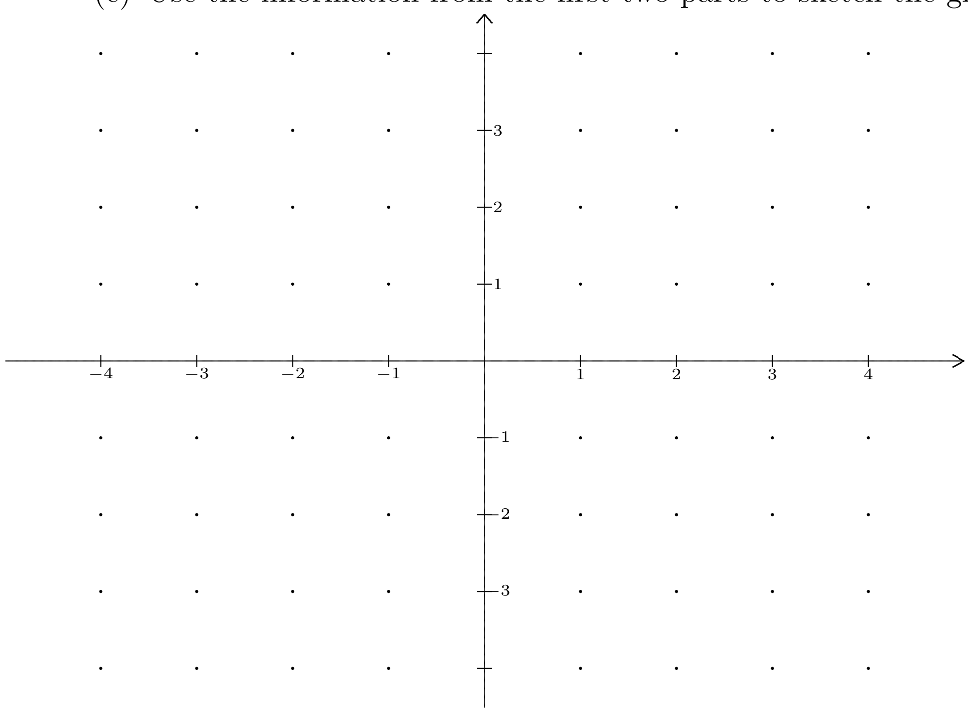
   (b) $h(x) = (2x-3)e^{4x}$.

8. (20 points) Consider the function $f(x) = \frac{(2x+3)(x-3)}{x(x-1)}$.

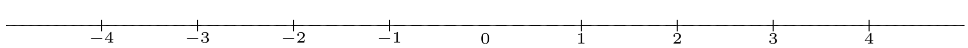
   (a) Build the sign chart for f

   (b) Find the vertical and horizontal asymptotes and the zeros, being careful not to mix them up.

   (c) Use the information from the first two parts to sketch the graph of f.



   (d) From the graph, you can speculate on the existence of critical points if there are any. Write a sentence about where you expect to find these critical points or why you think there are none. Estimate the sign chart for $r'(x)$



9. (12 points) Compound Interest.

(a) Consider the equation $2000(1 + 0.03)^{4t} = 6000$. Find the value of t and interpret your answer in the language of compound interest.

(b) Consider the equation $P(1 + 0.04)^{4 \cdot 10} = 5000$. Solve for P and interpret your answer in the language of compound interest.

(c) Consider the equation $Pe^{10r} = 2P$. Solve for r and interpret your answer in the language of compound interest.

10. (25 points) Consider the function $f(x) = \ln(3x^2 + 1)$.

    (a) Find $f'(x)$.

    (b) Find an equation for the line tangent to the graph of f at the point $(3, f(3))$.

    (c) Find $f''(x)$.

    (d) Find the sign chart for $f''(x)$.

    (e) Find the intervals over which f is concave upwards.