**Math 1120, Section 6**          **Calculus**          **Test 3**

**November 25, 2014**      **Name**

The total number of points available is 145. Throughout this test, **show your work.**

1. (10 points) Find an equation for the line tangent to the graph of $y = \ln(2x+1)$ at the point $(1, \ln(3))$.

2. (10 points) Find an equation for the line tangent to the graph of $y = e^{6x-2}$ at the point $(1/3, 1)$.

3. (55 points) Consider the function $f$ given below. The domain of $f$ is $[-4, 5]$. The function is linear on each of the three intervals $[-4, -2], [-2, 2]$ and $[2, 3]$. Over $[3, 5]$, it is a quadratic with vertex at $(3, 2)$.
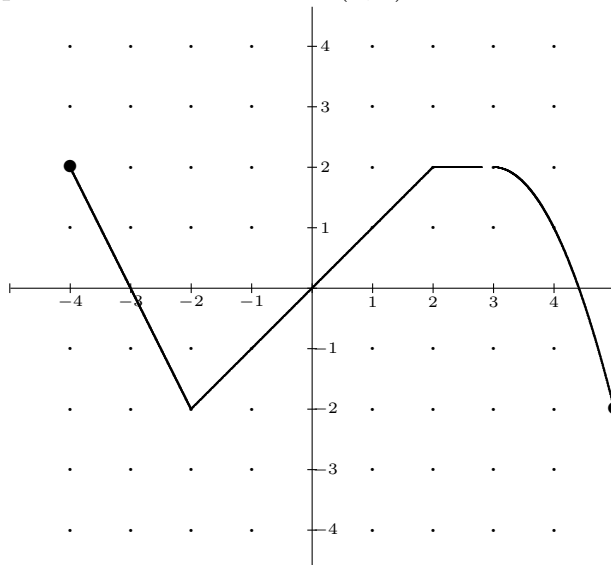


(a) (20 points) Find a symbolic representation for $f$ by filling in each clause in the function definition below:

$$f(x) = \begin{cases} & \text{if } -4 \le x < -2 \\ & \text{if } -2 \le x < 2 \\ & \text{if } \quad 2 \le x < 3 \\ & \text{if } \quad 3 \le x \le 5 \end{cases}$$

(b) (15 points) On the axes provided, sketch the graph of $f'$.

(c) Find three whole number values of $x$ where the function $f$ is increasing.

(d) Find two whole number values of $x$ where $f'$ does not exist.

(e) Find three whole number values of $x$ where $f'$ is positive.

(f) Find three whole number values of $x$ where $f''$ has the value 0.

4. (10 points) A skull from an archeological dig has one-twelfth the amount of Carbon-14 it had when the specimen was alive.

    (a) Recall that the half-life of Carbon-14 is 5770 years. Find the decay constant $k$.

    (b) What is the age of the specimen? Round off your answer to the nearest multiple of one hundred years.

5. (10 points) How long does it take an investment of $\$P$ at an annual rate of 8% to triple in value if compounding

    (a) takes place quarterly? Round your answer to the nearest tenth of a year.

    (b) takes place continuously? Round your answer to the nearest tenth of a year. As usual, no work shown, no credit!

6. (20 points) A botanist conjectures that the height of a certain type of pine tree can be modeled by a learning curve. To test his conjecture, he plants a 2 foot tall tree. He knows that eventually the tree will grow to 40 feet tall, its maximum height. Suppose that after one year, the tree is 4 feet tall.

   (a) What does the model predict for the height of the tree after two years.

   (b) How many inches does the tree grow during the fourth year?

   (c) What is the instantaneous rate of growth at $t = 3.5$ years.

   (d) Describe the connection between the two answers (b) and (c).

A. (15 points) Consider the function $f(x) = (x^2 - 4x + 4)e^{2x}$.

    (a) Use the product rule to find $f'(x)$.

    (b) List the critical points of $f$.

    (c) Construct the sign chart for $f'(x)$.

    (d) Write in interval notation the interval(s) over which $f$ is increasing.

B. (15 points) Consider the function $f(x) = \ln[(2x^2 + 3)(7x - 2)(x^2 - 4)]$.

    (a) Recalling that $\ln(x)$ is defined precisely when $x > 0$, find the domain of $f$.

    (b) Let $g(x) = (2x^2 + 3)(7x - 2)^2(x^2 - 4)^3$. Use logarithmic differentiation to find $g'$. You need not simplify your answer.

    (c) Find an equation for the line tangent to $g$ at the point $(-1, g(-1))$.