August 6, 1999 Your name ____________________

leave for solutions On all the following questions, **show your work.**

1. Let $f(x)$ be a function whose derivative $f'(x)$ is given by

$$f'(x) = \frac{(x+3)^2(x+1)(x-2)}{(x-1)^2}$$

Find the intervals over which f is increasing.

The critical points of f are the of two types. The singular point is the value of x for which $f'(x)$ does not exist, $x = 1$, and the stationary points of f are $-3, -1$, and 2. Use the test interval method to see how the sign of f' changes over the five intervals determined by the four critical points. Note that f' is positive over $(-\infty, -3)$ and $(-3, -1)$, which means that f *could* be increasing over $(-\infty, -1)$. Note that the critical point -3 is a stationary point, so that is indeed the case. Then f' changes sign at -1 and then again at 2. It is positive on $(2, \infty)$. So, f is increasing over both the iintervals $(-\infty, -1)$ and $(2, \infty)$.

2. True or false.

(a) If f is increasing on (a, b), then $f'(x) > 0$ for each x in (a, b).

False, because f' could be zero at a point of (a, b).

(b) If $f'(c) = 0$, then f has a relative maximum or a relative minimum at $x = c$.

False. The function $f(x) = x^3$ at $x = 0$ is a counter example.

(c) If f has a relative maximum or a relative min. at $x = c$, then $f'(c) = 0$.

False. The function $f$could have a relative max or min at a singular point.

(d) If $f'(c) = 0$ and $f''(c) < 0$, then f has a relative maximum at $x = c$.

True. This is the second derivative test.

(e) If $f'(x) > 0$ for each x in the interval $(-1, 1)$, then f is increasing on $(-1, 1)$.

True. This follows from the theorem at the beginning of chapter 4.

(f) If $f(a) < 0, f(b) > 0$, and $f'(x) > 0$ for each x in (a, b), then there is one and only one number c in (a, b) such that $f(c) = 0$.

True. This follows from the Intermediate value theorem together with the fact that a continuous function must have a critical point between any two of its zeros.