March 27, 1998 **Name** ________________

In the first six problems, each part counts 7 points (total 63 points) and the final two problems count as marked.

Multiple choice section. Circle the correct choice. You do not need to show your work on these problems.

1. Over which of the intervals is the function defined by $f(x) = x^3+3x^2-24x+18$ increasing?

   (A) $[-5,-2]$ **(B)** $[-3,-1]$ **(C)** $[-1,2]$ **(D)** $[1,4]$ **(E)** $\boxed{[3,7]}$

2. Suppose the functions f and g are differentiable and their values at certain points are given in the table. The next three problems refer to these functions f and g. Recall that, for example, the entry 1 in the first row and third column means that $f'(0) = 1$.

x	$f(x)$	$f'(x)$
0	2	1
1	7	3
2	5	4
3	1	2
4	3	3
5	6	4
6	0	5
7	4	1

x	$g(x)$	$g'(x)$
0	5	5
1	7	3
2	4	4
3	2	6
4	6	10
5	3	4
6	1	2
7	0	1

   (a) The function h is defined by $h(x) = f(g(x))$. Use the chain rule to find $h'(2)$.

   (A) 1 **(B)** 4 **(C)** 6 **(D)** 10 **(E)** $\boxed{12}$

   (b) The function k is defined by $k(x) = f(x) \cdot g(x)$. Use the product rule to find $k'(3)$.

   (A) 1 **(B)** 4 **(C)** 6 **(D)** $\boxed{10}$ **(E)** 12

   (c) The function H is defined by $H(x) = f(x)/g(x)$. Use the quotient rule to find $H'(4)$.

   (A) -12 **(B)** $\boxed{-1/3}$ **(C)** $-1/2$ **(D)** $3/10$ **(E)** 1

3. Which of the following is the set of all critical points of the function defined by $g(x) = \sqrt{x^3 - 12x + 40}$ on the interval $-4 \le x < \infty$?

   (A) $\{-2\}$ **(B)** $\{-2, 0\}$ **(C)** $\{-2, 0, 2\}$ **(D)** $\boxed{\{-2, 2\}}$ **(E)** $\{0, 2\}$

4. Referring again to the function g defined in 3., estimate the value of $g'(1)$.

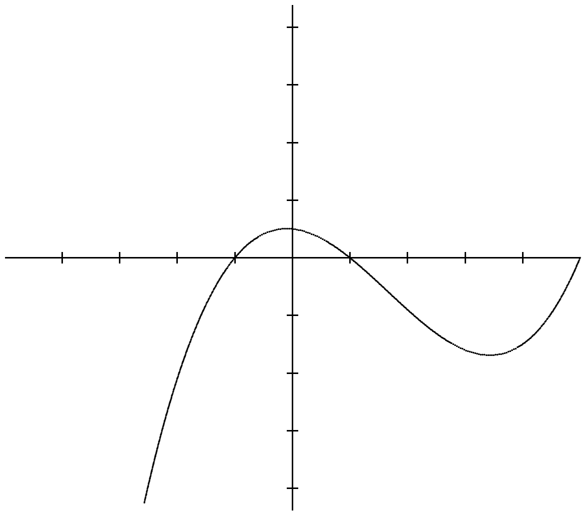
   (A) -1.32 **(B)** $\boxed{-0.84}$ **(C)** -0.11 **(D)** 0.27 **(E)** 1.42

5. There is one horizontal asymptote $y = a$ and two vertical asymptotes $x = b$ and $x = c$ of the graph of

$$y = \frac{3x^2 - 2x - 3}{x^2 + 4x}.$$

   Compute $a + b + c$.

   (A) $\boxed{-1}$ **(B)** 0 **(C)** 1 **(D)** 4 **(E)** 7



6. The x-coordinate of a point of **inflection** of the curve above is approximately

   (A) -2 **(B)** -1 **(C)** 0 **(D)** $\boxed{1.5}$ **(E)** 3.5

7. Referring again to the function whose graph appears above, at what point x does the derivative have a value very close to 1.

   (A) -2 **(B)** $\boxed{-1}$ **(C)** 0 **(D)** 1 **(E)** 3.5

On all the following questions, **show your work.**

8. (25 points) The quantity demanded per month, x, of a certain brand of electric shavers is related to the price, p, per shaver by the equation

$$p = -0.005x + 80 \qquad (0 \le x \le 16,000),$$

where p is measured in dollars. The total monthly cost for manufacturing the shavers is given by

$$C(x) = 0.0002x^3 - 0.3x^2 + 20,000 \qquad (0 \le x \le 20,000).$$

(a) Find the revenue function $R(x)$.

$\boxed{R(x) = xp = x(-0.005x + 80).}$

(b) Find the profit function $P(x)$.

$\boxed{P(x) = -0.005x^2 + 80x - (0.0002x^3 - 0.3x^2 + 20000).}$

(c) Find the critical points of the profit function.

Since $P'(x) = -0.01x + 80 - 0.0006x^2 + 0.6x$, we find the critical points of P by setting $P'(x) = 0$. The quadratic formula yields $x = -120.7$ and $x = 1104$. Only the later makes sense.

(d) How many shavers should be produced per month in order to maximize the company's profit?

$\boxed{1104}$

(e) What is the maximum profit?

$\boxed{P(1104) = 158,756}$

9. (20 points) Optimal Charter Flight Fare. If exactly 200 people sign up for a charter flight, the agency charges \$300. However, if more than 200 sign up, the agency reduces the fare by \$0.50 for each additional person.

   (a) Let x denote the number of passengers beyond 200. Construct the revenue function $R(x)$.

   $\boxed{R(x) = (300 - 0.5x)(200 + x), \text{ where } x \text{ represents the number of additional passengers.}}$

   (b) Find all the critical points of your revenue function.

   $\boxed{R'(x) = 300 - 0.5x + (200 + x)(-0.5) = 200 - x.}$

   (c) What number of passengers results in the maximum revenue?

   $\boxed{x = 200, \text{ so maximum profits are attained when there are 400 passengers.}}$

   (d) What is the maximum revenue? $\boxed{R(200) = 80,000.}$