March 3, 2011 Name _____

The problems count as marked. The total number of points available is 128. Throughout this test, **show your work.**

1. (12 points) Let $f(x) = x^2 - 2x - 3$.

   (a) Compute $f'(x)$

   (b) What is $f'(2)$?

   (c) Use the information in (b) to find an equation for the line tangent to the graph of f at the point $(2, f(2))$.

2. (12 points) Consider the function f defined by:

$$f(x) = \begin{cases} x + x^3 & \text{if } x < 1 \\ 2 & \text{if } x = 1 \\ 2x^{1/2} & \text{if } x > 1 \end{cases}$$

   (a) Find an equation for the line tangent to the graph of f at the point $(4, 4)$.

   (b) Find an equation for the line tangent to the graph of f at the point $(-2, -10)$.

3. (12 points) The function f satisfies $f(2) = 5$ and its graph has a tangent line L at the point $(2, 5)$. The line L has a y-intercept of 4. What is $f'(2)$? Note, a correct answer without supporting work is worth only 1 point.

4. (15 points) If a ball is thrown vertically upward from the roof of 212 foot building with a velocity of 48 ft/sec, its height after t seconds is $s(t) = 212 + 48t - 16t^2$.

   (a) What is the height the ball at time $t = 0$?

   (b) What is the velocity of the ball at the time it reaches its maximum height?

   (c) At what time is the velocity zero?

   (d) What is the maximum height the ball reaches?

   (e) What is the velocity of the ball when it hits the ground (height 0)?

5. (10 points) Find a point on the graph of f at which the slope of the tangent line is 3, where

$$f(x) = x - \frac{1}{x}.$$

Are there any other points where the slope is 3?

6. (10 points) The cost of producing x units of stuffed alligator toys is $C(x) = -0.003x^2 + 6x + 6000$ for $0 \le x \le 1000$.

   (a) Find the marginal cost at the production level of 500 units.

   (b) Find the (incremental) cost of producing the 501st toy.

7. (32 points) Consider the table of values given for the functions $f, f', g,$ and g':

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	2	1	3	2
1	4	6	2	5
2	6	4	3	4
3	1	2	5	3
4	3	5	2	6
5	5	3	4	1
6	0	3	2	4

(a) Let $L(x) = (f(x) + g(x))^3$. Compute $L'(2)$.

(b) Let $U(x) = g \circ g \circ g(x)$. Compute $U'(1)$.

(c) Let $K(x) = \frac{g(x) + f(x)}{g(x)f(x)}$. Compute $K'(2)$

(d) Let $Z(x) = x^2 - \frac{f(x)}{x}$. Compute $Z'(3)$.

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8. (25 points) Let $H(x) = (x^2 - 1)^2(5x + 7) + (x^2 - 1)(5x + 7)^2$.

    (a) Build the sign chart for $H(x)$.

    (b) Use the information from (a) to find the domain of the function $G(x) = \sqrt{(x^2 - 1)^2(5x + 7) + (x^2 - 1)(5x + 7)^2}$. Express your answer in interval notation.