November 5, 2008 **Name** _____

The total number of points available is 139. Throughout this test, **show your work.**

1. (15 points) Consider the function $f(x) = (2x+3)^2(x-1)^2$.

   (a) Use the product rule to find $f'(x)$.

   (b) List the critical points of f.

   (c) Construct the sign chart for $f'(x)$.

   (d) Write in interval notation the interval(s) over which f is increasing.

2. (15 points) Consider the function $f(x) = \frac{(2x+3)}{(x-1)^2}$.

   (a) Use the quotient rule to find both $f'(x)$ and $f''(x)$.

   (b) Construct the sign chart for $f''(x)$.

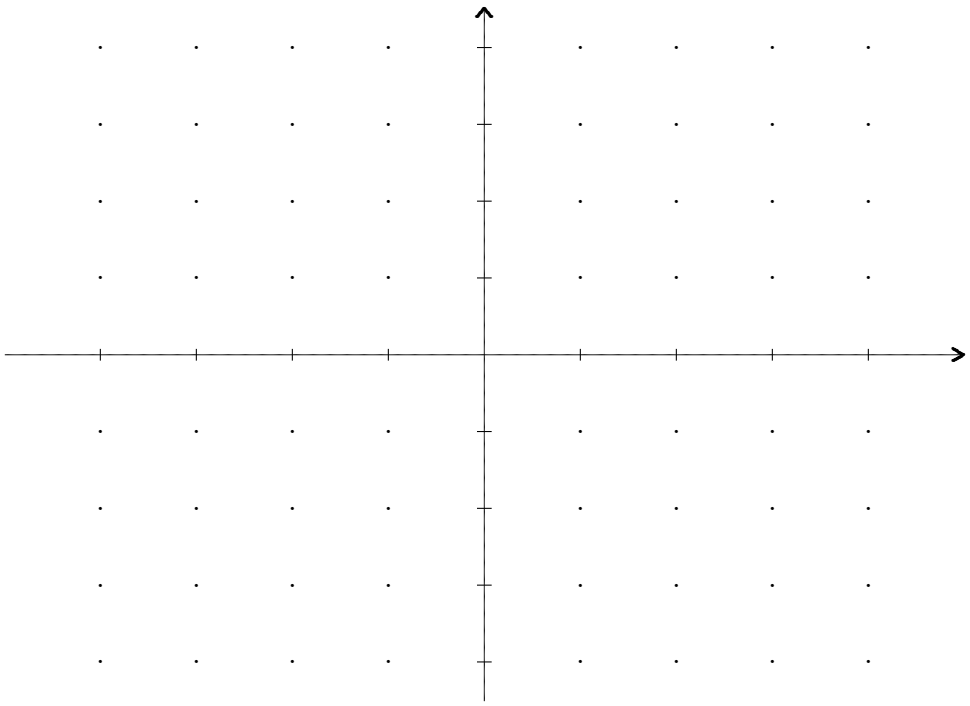
   (c) Write in interval notation the interval(s) over which f is concave upwards.

3. (15 points) Consider the function $f(x) = \frac{(2x+3)(x-3)}{x(x-1)}$.

   (a) Build the sign chart for f

   (b) Find the vertical and horizontal asymptotes.

   (c) Use the information from the first two parts to sketch the graph of f.



4. (10 points) If 1400 square centimeters of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

5. (12 points) A baseball team plays in he stadium that holds 56000 spectators. With the ticket price at 9 the average attendance has been 23000. When the price dropped to 8, the average attendance rose to 28000. If $p(x)$ represents the price which will attract x spectators,

   (a) Find the demand function $p(x)$, where x is the number of the spectators. Assume $p(x)$ is linear.

   (b) How should be set a ticket price to maximize revenue?

6. (6 points) The line $y = 3x - 5$ is tangent to the graph of the function f at the point $(2, 1)$. What is $f'(2)$?

7. (12 points) For what values of x is the tangent line of the graph of

$$f(x) = 2x^3 - 15x^2 - 72x + 12$$

parallel to the line $y = 12x - 17$?

8. (12 points) Consider the function $f(x) = x^3 - 5.5x^2 - 4x + 7, \quad -5 \leq x \leq 5$. Find the locations of the absolute maximum of $f(x)$ and the absolute minimum of $f(x)$ and the value of f at these points.

9. (12 points) For each function listed below, find all the critical points. Tell whether each critical point gives rise to a local maximum, a local minimum, or neither.

   (a) $f(x) = (x^3 - 8)^2$

   (b) $g(x) = (x - 1)^{2/3}$

10. (15 points) Let $L(x) = 3x - 4$. Of course L is a linear function. For each real number x, the point $(x, y) = (x, 3x - 4)$ belongs to the line. The point $(1, 1)$ does not belong to the line.

    (a) Let x denote the number of letters in your first name. Find the distance between $(1, 1)$ and $(x, L(x)$.

    (b) Let x denote the number of letters in your family name. If this is the same number as in (a), add one to it. Find the distance between $(1, 1)$ and $(x, L(x)$.

    (c) Find the distance function $D(x)$ that measure the distance from $(1, 1)$ to $(x, L(x)$, where x is arbitrary. The first two parts are samples of function values.

    (d) Find the derivative $D'(x)$.

    (e) Differentiate the square of $D(x)$. This should be much easier to work with.

    (f) Find a critical point of the square of D. Its the same as we would get for D itself.

    (g) Find the point on the line that is closest to $(1, 1)$.

11. (10 points) Build a (symbolic representation of a) function f satisfying
    (a) f has zeros at $x = 3$ and $x = -1$.
    (b) f has vertical asymptotes at $x = -4$ and $x = 0$.
    (c) f has $y = 2$ as a horizontal asymptote.