October 13, 2004 **Name** _____

The total number of points available is 135. Throughout this test, **show your work.**

1. (15 points) Let $f(x) = \sqrt{x^3 + 1}$.

   (a) Compute $f'(x)$

   (b) What is $f'(2)$?

   (c) Use the information in b. to find an equation for the line tangent to the graph of f at the point $(2, f(2))$.

2. (12 points) Consider the function f defined by:

$$f(x) = \begin{cases} 3x - x^3 & \text{if } x < 1 \\ 2 & \text{if } x = 1 \\ 2x^{2/3} & \text{if } x > 1 \end{cases}$$

   (a) Is f continuous at $x = 1$?

   (b) What is the slope of the line tangent to the graph of f at the point $(8, 8)$?

   (c) Find $f'(-3)$

3. (15 points) KAM Industries makes ovens. The daily cost in dollars of producing x ovens is given by

$$C(x) = -0.06x^2 + 120x + 5000,$$

for x in the range 0 to 2000.

(a) What is the actual cost of manufacturing the 101^{st} oven? ...the 201^{st} oven?

(b) Find the marginal cost function $C'(x)$. What are $C'(100)$ and $C'(200)$?

(c) Find the average cost function $\overline{C}(x)$.

(d) Find the marginal average cost function $\overline{C}'(x)$.

4. (36 points) Consider the table of values given for the functions f, f', g, and g':

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	2	1	6	2
1	4	6	2	5
2	6	4	3	4
3	1	2	5	3
4	3	5	2	6
5	5	3	4	1
6	0	3	2	4

(a) Let $K(x) = f(x) \cdot g(x)$. Compute $K'(3)$

(b) Let $L(x) = f(x)/g(x)$. Compute $L'(2)$.

(c) Let $U(x) = f \circ g(x)$. Compute $U'(1)$.

(d) Let $V(x) = g \circ f(x)$. Compute $V'(5)$.

(e) Let $W(x) = f(x^2 - g(x))$. Compute $W'(2)$.

(f) Let $Z(x) = g(x - f(x))$. Compute $Z'(3)$.

5. (30 points) Compute the following derivatives.

   (a) Let $f(x) = x^{-2} + \sqrt{x}$. Find $\frac{d}{dx} f(x)$.

   (b) Let $g(x) = \sqrt{x^4 - x^2}$. What is $g'(x)$?

   (c) Find $\frac{d}{dx}((4x+1)^2 \cdot (2x^3 - 1))$.

   (d) Find $\frac{d}{dx} \frac{2x^2+1}{x-2}$.

   (e) Find $\frac{d}{dt}(t^2 + 1/t)^4$.

6. (10 points) Let $f(x) = \sqrt{2x-3}$. The chain rule can be applied to find that $f'(x) = \frac{1}{2}(2x-3)^{-1/2} \cdot 2 = 1/\sqrt{2x-3}$. Use the limit definition of derivative to verify this fact.

7. (12 points) Intermediate Value Theorem. Recall that the IVT asserts the following: If f is a continuous function on the interval $[a, b]$ and M is a number between $f(a)$ and $f(b)$, then there exists a number c satisfying $a \leq c \leq b$ and $f(c) = M$. For this problem let $f(x) = \sqrt{2x-5}$ and let $[a, b] = [3, 15]$. Finally, suppose $M = 4$. Find the number c whose existence is guaranteed by IVT.