March 5, 2004   Name _____

The total number of points available is 120. Throughout this test, **show your work.**

1. (15 points) Let $p(x) = x^2 - 4x + 5$.

   (a) Compute $p'(x)$

   (b) Compute $p''(x)$

   (c) Use the information in a. to find an equation for the line tangent to the graph of p at the point $(1, 2)$.

2. (15 points) Intermediate Value Theorem. Recall that the IVT asserts the following: If f is a continuous function on the interval $[a, b]$ and M is a number between $f(a)$ and $f(b)$, then there exists a number c satisfying $a \leq c \leq b$ and $f(c) = M$. For this problem let $f(x) = \sqrt{4x - 3}$ and let $[a, b] = [1, 7]$. Finally, suppose $M = 2$. Find the number c whose existence is guaranteed by IVT.

3. (15 points) The total weekly cost in dollars incurred by the Lincoln Record Company in pressing x playing records is given by $C(x) = 2000 + 3x - 0.01x^2$ for x in the range 0 to 6000.

(a) Find the marginal cost function $C'(x)$.

(b) Find the average cost function $\overline{C}(x)$.

(c) Find the marginal average cost function $\overline{C}'(x)$.

(d) Interpret your results in (c). Is the average cost growing or falling as the company produces more units?

4. (30 points) Consider the table of values given for the functions f, f', g, and g':

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	2	1	6	2
1	4	6	1	5
2	6	4	3	4
3	1	2	5	3
4	3	5	2	6
5	5	3	4	1
6	0	3	2	4

(a) Let $K(x) = f(x) \cdot g(x)$. Compute $K'(2)$

(b) Let $L(x) = f(x)/g(x)$. Compute $L'(1)$.

(c) Let $U(x) = f \circ g(x)$. Compute $U'(4)$.

(d) Let $V(x) = g \circ f(x)$. Compute $V'(5)$.

(e) Let $W(x) = f(x^2)$. Compute $W'(2)$.

(f) Let $Z(x) = g(xf(x))$. Compute $Z'(3)$.

5. (25 points) Compute the following derivatives.

   (a) Let $f(x) = x^2 + x^{-\frac{1}{2}}$. Find $\frac{d}{dx} f(x)$.

   (b) Let $g(x) = \sqrt{x^4 + 4}$. What is $g'(x)$?

   (c) Find $\frac{d}{dx}((3x + 1)^2 \cdot (4x^3 - 1))$.

   (d) Find $\frac{d}{dx} \frac{2x^2 + 1}{x - 2}$.

   (e) Find $\frac{d}{dt}(t^2 + 1/t)^3$.

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6. (10 points) Let $f(x) = 2/x$. Use the limit definition of derivative to find $f'(x)$.

7. (10 points) Consider the function f defined by:

$$f(x) = \begin{cases} 2x^2 - 3x & \text{if } x < 1 \\ 4 & \text{if } x = 1 \\ \sqrt{x-1} & \text{if } x > 1 \end{cases}$$

   (a) What is the slope of the line tangent to the graph of f at the point $(5, 2)$?

   (b) Find $f'(-3)$