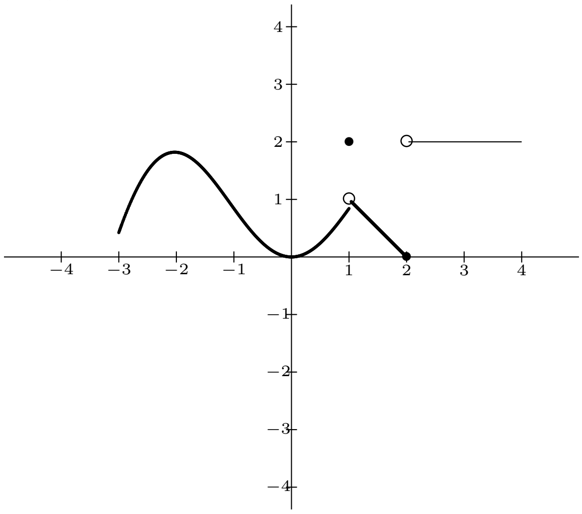
March 7, 2003 **Name** ______________________________

The first 6 problems count 5 points each and the final 4 count as marked. The total number of points available is 128.

Multiple choice section. Circle the correct choice. You do not need to show your work on these problems.

1. Questions (a) through (e) refer to the graph of the function f given below.



(a) $\lim_{x \to 1} f(x) =$

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** does not exist

Solution: B. The limit is 1 by the blotter test.

(b) $\lim_{x \to 2^+} f(x) =$

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** does not exist

Solution: C. Looking just at the part of the graph that lies to the right of 2, we see that the right limit as x approaches 2 is 2.

(c) A good estimate of $f'(-1)$ is

(A) -1 **(B)** 0 **(C)** 1 **(D)** 2 **(E)** there is no good estimate

Solution: A. The slope of the tangent line is certainly negative, and -1 is the only negative option.

(d) A good estimate of $f'(-2)$ is

(A) -1 **(B)** 0 **(C)** 1 **(D)** 2 **(E)** there is no good estimate

Solution: B. The slope of the tangent line is close to zero, so 0 is a reasonable estimate.

(e) A good estimate of $f'(3)$ is

(A) -1 **(B)** 0 **(C)** 1 **(D)** 2 **(E)** there is no good estimate

Solution: B. The function is horizontal at 3 so the slope of the tangent line is zero.

2. The line tangent to the graph of a function f at the point $(2, -3)$ on the graph also goes through the point $(-1, 6)$. What is $f'(2)$?

(A) -3 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 3

Solution: A. The derivative is the slope of the tangent line, so it is $(6 - (-3)) \div (-1 - 2) = -3$

3. True-false questions. These count 2 points each.

(a) True or false. If f and g are differentiable and a and b are constants, then $\frac{d}{dx}[af(x) + bg(x)] = af'(x) + bg'(x)$.
Solution: True. This is just the rule that talks about the derivative of the sum and of a constant times a function.

(b) True or false. If $f'(x) > 0$ for each x in the interval $(-1, 1)$, then f is increasing on $(-1, 1)$.
Solution: True.

(c) True or false. If $f(a) < 0, f(b) > 0$, and $f(x)$ is continuous for each x in $[a, b]$, then there is at least one number c in (a, b) such that $f(c) = 0$.
Solution: True. The Intermediate Value Theorem guarantees that there is at least one c in (a, b).

(d) True or false. The graph of a function cannot touch or intersect a horizontal asymptote to the graph of f.
Solution: False. There is nothing in the definition of horizontal asymptote that implies this.

(e) True or false. If f and g are differentiable, then $\frac{d}{dx}[f(x)g(x)] = f'(x)g'(x)$.
Solution: False. Look up the product rule.

(f) True or false. If f and g are differentiable, then $\frac{d}{dx}[\frac{f(x)}{g(x)}] = \frac{f'(x)}{g'(x)}$.
Solution: False. Look up the quotient rule.

(g) True or false. If f and g are differentiable and $h(x) = f \circ g$, then $h'(x) = f(g(x))g'(x)$.
Solution: False. Look up the chain rule.

4. (40 points) Suppose the functions f and g have derivatives at all their domain points and their values at certain points are given in the table. The next four problems refer to these functions f and g. Recall that, for example, the entry 1 in the first row and third column means that $f'(0) = 1$. In each case, a function $H(x)$ is given in terms of $f(x)$ and $g(x)$. You are asked to find $H'(x)$ at the value of x provided.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	2	1	5	4
1	7	3	6	2
2	5	4	1	7
3	1	2	6	8
4	3	3	2	5
5	6	4	1	4
6	0	5	4	6
7	4	1	5	1

(a) The function H is defined by $H(x) = f(x^2)$. Find $H'(1)$.

(A) 6 **(B)** 12 **(C)** 18 **(D)** 24 **(E)** 44

Solution: A. $H'(x) = f'(x^2) \cdot 2x$, so $H'(1) = f'(1) \cdot 2 = 6$.

(b) The function J is defined by $J(x) = f(g(f(x)))$. Use the chain rule to find $J'(2)$.

(A) 6 **(B)** 9 **(C)** 12 **(D)** 21 **(E)** 48

Solution: E. $J'(x) = f'(g(f(x))) \cdot g'(f(x)) \cdot f'(x)$, so $J'(2) = f'(g(f(2))) \cdot g'(f(2)) \cdot f'(2) = f'(1) \cdot g'(5) \cdot f'(2) = 48$.

(c) The function K is defined by $K(x) = g(x)/x^2$. Use the quotient and chain rules to find $K'(3)$.

(A) $-1/9$ **(B)** $1/3$ **(C)** $2/3$ **(D)** $4/9$ **(E)** $7/9$

Solution: D. Use the quotient rule to get 4/9.

(d) The function L is defined by $L(x) = (x + f(x))^{10}$. Use the chain and power rules to find $L'(0)$.

(A) 0 **(B)** $10 \cdot 2^9$ **(C)** $5 \cdot 2^9$ **(D)** $10 \cdot 2^{10}$ **(E)** 2^{11}

Solution: D. $L'(x) = 10(x + f(x))^9 \cdot (1 + f'(x)) = 10 \cdot 2^9 \cdot 2$.

(e) Use the information in the chart to find the y-intercept of the line tangent to the graph of f at the point $(2, 5)$.

(A) -3 **(B)** 0 **(C)** 2 **(D)** 3 **(E)** 5

Solution: A. The slope of the line tangent to the graph of f at the point $(2, 5)$ is given in the table as 4, so the line is $y - 5 = 4(x - 2)$ which has y-intercept -3.

On all the following questions, **show your work.**

5. (20 points) Let $k(x) = 2x^2$.

   (a) Using the definition of derivative, find $k'(x)$
   Solution: The difference quotient is $(k(x + h) - k(x)) \div h = (2(x + h)^2 - 2x^2) \div h = 2x^2 + 4xh + 2h^2 - 2x^2) \div h = (4xh + h^2) \div h$, so $k'(x) = lim_{h \to 0} 4xh \div h = 4x$.

   (b) Evaluate the function found above at $x = 1$ to find $k'(1)$.
   Solution: From above, $k'(1) = 4$.

   (c) Use the information above to find an equation for the line tangent to the graph of k at the point $(1, k(1))$.
   Solution: $y - k(1) = y - 2 = 4(x - 1)$.

6. (20 points) A division of Moreken Industries manufactures microwave ovens. The daily cost (in dollars) of producing x ovens is given by $C(x) = -0.03x^2 + 120x + 15000$

   (a) What is the actual cost of producing the 201st microwave oven?
   Solution: $C(201) - C(200) = -0.03(201^2 - 200^2) + 120(201 - 200) + 5000 - 5000 = -12.03 + 120 = \$107.97$.

   (b) Find the marginal cost function $C'(x)$.
   Solution: $C'(x) = -0.03 \cdot 2x + 120$

   (c) Find $C'(200)$.
   Solution: $C'(200) = -12 + 120 = \$108.00$

   (d) Find the average cost function $\overline{C}(x)$.
   Solution: $\overline{C}(x) = \frac{-0.03x^2+120x+15000}{x}$.