March 25, 2019 **Name** _____

The problems count as marked. The total number of points available is 156. Throughout this test, **SHOW YOUR WORK.**

1. (24 points) Demonstrate your understanding of the product, quotient and chain rules by differentiating each of the given functions. No need to simplify. You must show your work.

   (a) Let $F(x) = (x^2 - 3x + 1)(x^3 - 2x + 5)$

   (b) $G(x) = \dfrac{2x^4 - 3x + 1}{x^2 - x + 3}$

   (c) $K(x) = (x^2 - 3)^{17}$

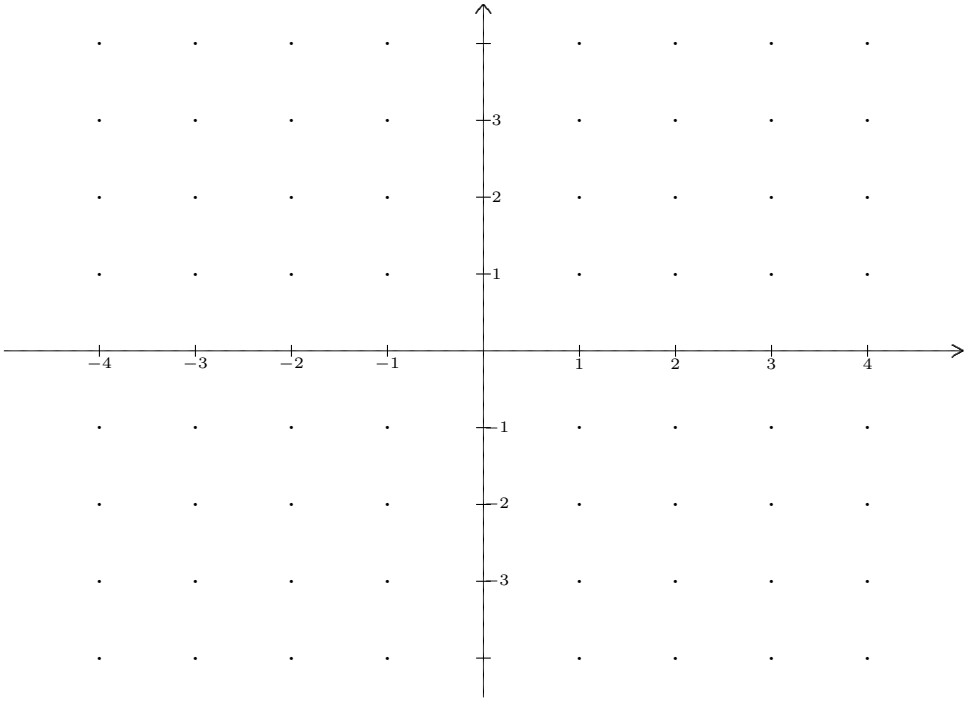
   (d) $H(x) = \sqrt{(3x + 1)^4 - 7}$.

2. (30 points) Consider the function $f(x) = \frac{(2x+3)(x-3)}{x(x-1)}$.

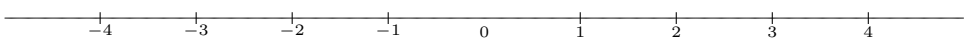
   (a) Build the sign chart for f

   (b) Find the vertical and horizontal asymptotes and the zeros, being careful not to mix them up.

   (c) Use the information from the first two parts to sketch the graph of f.



   (d) From the graph, you can speculate on the existence of critical points if there are any. Write a sentence about where you expect to find these critical points or why you think there are none. Estimate the sign chart for $r'(x)$



3. (20 points) Consider the line L given by $y = 2x$, the point $P = (-4, 2)$, and the circle whose equation is $x^2 - 8x + y^2 - 4y = -16$.

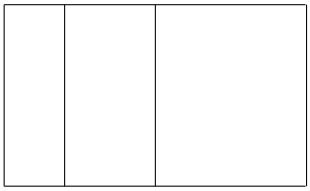
(a) Find the point on the line that is closest to the point P.

(b) Find the point on the circle that is closest to the point P.

(c) Find the point on the line that is closest to the circle.

4. (10 points) Find a quadratic polynomial $f(x)$ which is decreasing on $(-\infty, 2)$ and increasing on $(2, \infty)$.

5. (15 points) A farmer has 20000 feet of fencing to build a rectangular pasture. But he must separate the goats, horses and cows into different parts of the pasture using two vertical straight sections of fence as shown.



What is the area of the largest pasture the farmer can build?

6. (35 points) Consider the table of values given for the functions f, f', g, and g':

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	2	1	3	2
1	4	6	2	5
2	6	4	3	4
3	1	2	5	3
4	3	5	2	6
5	5	3	4	1
6	0	3	1	4

(a) $Q(x) = f(x)/g(x)$. Find $Q(5)$ and $Q'(5)$.

(b) Let $H(x) = f(x) \cdot (g(x) + 1)$. Compute $H(4)$ and $H'(4)$.

(c) Let $W(x) = f(g(x) + 1)$. Compute both $W(5)$ and $W'(5)$.

(d) Let $L(x) = g(\frac{1}{x} + 1)$. Find $L(1)$ and $L'(1)$.

(e) Let $U(x) = \sqrt{g(2x)}$. Compute $U(3)$ and $U'(3)$.

(f) Let $Z(x) = g(2x - f(x))$. Compute $Z(4)$ and $Z'(4)$.

7. (30 points) Let $g(x) = (x^2 - 4)^2(2x + 1)^2$.

   (a) Find $g'(x)$.

   (b) Find all the x-intercepts (the zeros) of $g'(x)$. That is, find the critical points of g.

   (c) Build the sigh chart for $g'(x)$.

   (d) Use the sign chart for $g'(x)$ to classify each critical point of g found in part (a) as the location of (i) a local minimum, (ii) a local maximum, or (iii) an imposter.