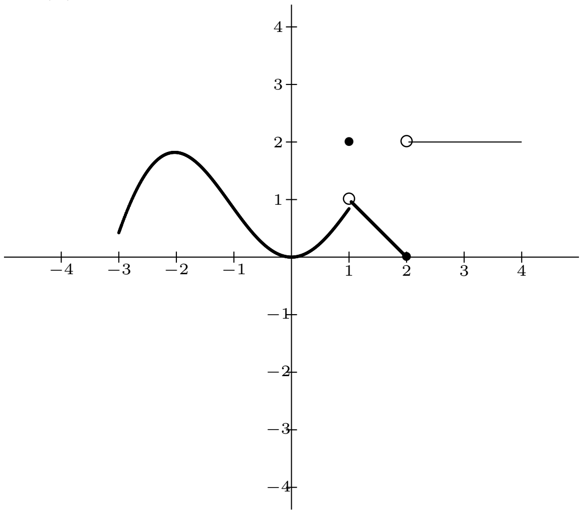
February 25, 2002 Your name ______________________

The first 9 problems count 6 points each and the final ones counts as marked. Problems 1 through 4 are multiple choice. In the multiple choice section, circle the correct choice (or choices). You do not need to show your work on problems 1 through 4, but you must show your work on the other problems. The total number of points available is 106.

1. Questions (a) through (c) refer to the graph of the function f given below.



(a) A good estimate of $f'(0)$ is

(A) -1 **(B)** 0 **(C)** 1 **(D)** 2 **(E)** there is no good estimate

Solution: B. The tangent line seems to be horizontal, therefore the best estimate of its slope is 0.

(b) A good estimate of $f'(-1)$ is

(A) -1 **(B)** 0 **(C)** 1 **(D)** 2 **(E)** there is no good estimate

Solution: A. The tangent line has a negative slope, therefore the best estimate of its slope is -1.

(c) A good estimate of $f'(1/2)$ is

(A) -2 **(B)** -1 **(C)** 0 **(D)** 1.5 **(E)** there is no good estimate

Solution: D. The tangent line has positive slope, so the only reasonable estimate of $f'(1/2)$ is 1.5.

2. The line tangent to the graph of a function f at the point $(2, -1)$ on the graph also goes through the point $(-2, 7)$. What is $f'(2)$?

   (A) -2 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 2

   Solution: A. The slope of the tangent line is $\frac{-1-7}{2-(-2)} = -2$, so $f'(2) = -2$.

3. What is the slope of the tangent line to the graph of $f(x) = 3x^2 - 4x$ at the point $(2, 4)$?

   (A) 2 **(B)** 4 **(C)** 8 **(D)** -8 **(E)** -4

   Solution: C. $f'(x) = 6x - 4$, which at $x = 2$ has the value 8.

4. Suppose the functions f and g have derivatives at all their domain points and their values at certain points are given in the table. The next four problems refer to these functions f and g. Recall that, for example, the entry 1 in the first row and third column means that $f'(0) = 1$. In each case, a function $H(x)$ is given in terms of $f(x)$ and $g(x)$. You are asked to find $H'(x)$ at the value of x provided.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	2	1	5	4
1	7	3	6	2
2	5	4	1	7
3	1	2	6	8
4	3	3	2	5
5	6	4	1	4
6	0	5	4	6
7	4	1	5	1

   (a) The function H is defined by $H(x) = f(g(x))$. Use the chain rule to find $H'(2)$.

   (A) 3 **(B)** 6 **(C)** 9 **(D)** 12 **(E)** 21

   Solution: E. By the chain rule, $H'(2) = f'(g(2))g'(2) = f'(1) \cdot 7 = 3 \cdot 7 = 21$.

   (b) The function H is defined by $H(x) = (f(x)) \cdot (g(x))$. Use the product rule to find $H'(3)$.

   (A) 10 **(B)** 12 **(C)** 16 **(D)** 20 **(E)** 24

   Solution: D. By the product rule, $H'(3) = f'(3)g(3) + g'(3)f(3) = 2 \cdot 6 + 8 \cdot 1 = 20$.

(c) The function H is defined by $H(x) = (x + f(x))/g(x)$. Use the quotient rule to find $H'(4)$.

(A) $-35/4$ **(B)** $-27/4$ **(C)** -6 **(D)** -5 **(E)** 0

Solution: B. By the quotient rule, $H'(x) = \frac{(1+f'(x)g(x)-g'(x)(x+f(x)}{g(x)^2}$. So $H'(4) = \frac{(1+f'(4))g(4)-g'(4)(4+f(4))}{g(4)^2} = \frac{(1+3)2-5(4+3)}{4} = -27/4$.

(d) The function H is defined by $H(x) = (f(x))^2 + (g(x))^2$. Find $H'(1)$.

(A) 12 **(B)** 18 **(C)** 24 **(D)** 44 **(E)** 66

Solution: E. Note that $H'(x) = 2f'(x)f(x) + 2g'(x)g(x)$, so $H'(1) = 2f'(1)f(1) + 2g'(1)g(1) = 2(7 \cdot 3 + 2 \cdot 6) = 66$.

On all the following questions, **show your work.**

5. (20 points) A division of Ditton Industries manufactures microwave ovens. The daily cost (in dollars) of producing x ovens is given by $C(x) = 0.0002x^3 - 0.03x^2 + 120x + 5000$

(a) What is the actual cost of producing the 201st, and 301st microwave oven?

Solution: $C(201) - C(200) = 0.0002(201^3 - 200^3) - 0.03(201^2 - 200^2) + 120(201 - 200) + 5000 - 5000 = 24.12 - 12.03 + 120 = \$132.09$. and $C(301) - C(300) = 0.0002(301^3 - 300^3) - 0.03(301^2 - 300^2) + 120(301 - 300) + 5000 - 5000 = 54.18 - 18.03 + 120 = \$165.15$.

(b) Find the marginal cost function $C'(x)$.

Solution: $C'(x) = 0.0002 \cdot 3x^2 - 0.03 \cdot 2x + 120$

(c) Find $C'(200)$ and $C'(300)$

Solution: $C'(200) = 24 - 12 + 120 = \$132.00$ and $C'(300) = 54 - 18 + 120 = \$156.00$.

(d) Find the average cost function $\overline{C}(x)$.

Solution: $\overline{C}(x) = \frac{0.0002x^3 - 0.03x^2 + 120x + 5000}{x}$.

6. (20 points) Compute the following derivatives.

   (a) Let $f(x) = x^{-1} + x^{-2}$. Find $\frac{d}{dx} f(x)$.
   Solution: $\frac{d}{dx} f(x) = -x^{-2} - 2x^{-3}$.

   (b) Let $g(x) = (x^2 + 4)^{2/3}$. What is $g'(x)$?
   Solution: $g'(x) = \frac{2}{3}(x^2 + 4)^{-1/3} \cdot 2x = 4x(x^2 + 4)^{-1/3}/3$.

   (c) Find $\frac{d}{dx}((3x + 1)^2 \cdot (x^2 - 1)^2)$
   Solution: $\frac{d}{dx} = 2(3x + 1) \cdot 3(x^2 - 1)^2 + 2(x^2 - 1) \cdot 2x(3x + 1)^2$
   $= 2(3x + 1)(x^2 - 1)(3(x^2 - 1) + 2x(3x + 1))$
   $= 2(3x + 1)(x^2 - 1)(9x^2 + 2x - 3)$.

   (d) Find $\frac{d}{dx} \frac{2x^3+1}{x+1}$
   Solution: Use the quotient rule to get $\frac{d}{dx} \frac{2x^3+1}{x+1} = \frac{6x^2(x+1)-(2x^3+1)}{(x+1)^2} = \frac{4x^3+6x^2-1}{(x+1)^2}$.

7. (12 points) Recall that two important functions, denoted $e(x)$ and $l(x)$, we'll study later in the course satisfy (a), $e'(x) = e(x), l'(x) = 1/x,$ and $l \circ e(x) = x$ for all x.

   (a) Compute $\frac{d}{dx} e(x^2)$
   Solution: By the chain rule, $\frac{d}{dx} e(x^2) = e'(x^2) \cdot 2x = e(x^2) \cdot 2x$.

   (b) Compute $\frac{d}{dx}(l(x)/e(x))$
   Solution: $\frac{d}{dx}(l(x)/e(x)) = \frac{l'(x)e(x)-e'(x)l(x)}{e(x)^2} = \frac{e(x)/x-e(x)l(x)}{e(x)^2}$.