March 13, 2015 **Name** ______________________________

The problems count as marked. The total number of points available is 146. Throughout this test, **show your work.** Using a calculator to circumvent ideas discussed in class will generally result in no credit. Note please that this test is a composite of the tests for sections 1 and 2.

1. (24 points) Let
$$H(x) = 3x^4 + 4x^3 - 72x^2.$$
   (a) Find $H'(x)$ and $H''(x)$.

   (b) Find all the critical points of H.

   (c) Find the intervals over which $H(x)$ is increasing.

   (d) Discuss the concavity of H.

2. (12 points) Consider the function f defined by:
$$f(x) = \begin{cases} 3x - x^3 & \text{if } -4 < x < 1 \\ 2x^{2/3} & \text{if } 1 < x < 10 \end{cases}$$
   (a) What is the domain of f? Write your answer in interval notation.

   (b) What is the slope of the line tangent to the graph of f at the point $x = 8$?

   (c) Find an equation for th line tangent to the graph of f at $(-3, f(-3))$.

3. (15 points) Let $f(x) = \sqrt{x^4 - 3x + 11}$.

   (a) Compute $f'(x)$

   (b) What is $f'(1)$?

   (c) Use the information in (b) to find an equation for the line tangent to the graph of f and at the same time perpendicular to the line $y - 3 = -6(x + 4)$.

4. (20 points)

   (a) Use the product rule for two functions (that you learned in class) to build a product rule for three functions. In other words, suppose $H(x) = f(x) \cdot g(x) \cdot h(x)$. Build a formula for $H'(x)$ in terms of the derivatives of f, g and h.

   (b) Use your rule to find $\frac{d}{dx}(2x-3)(3x+1)(5x-3)$. You must show that you used your 'new' product rule to get credit in this problem. Of course you can differentiate the cubic function to check your answer.

   (c) How many critical points does H have? Discuss your reasoning. No points for just a numerical answer.

5. (20 points) Let

$$H(x) = (2x + 7)^2(x^2 - 9).$$

(a) Find $H'(x)$ using the product and chain rules.

(b) Find all the critical points of H. Its fine to leave these is radical form.

(c) Find the intervals over which $H(x)$ is increasing.

6. (20 points) Let $f(x) = x^4/4 - x^2 - x$. Follow the instructions below to prove that f has exactly one critical point in the interval $[1, 2]$. You may assume that all polynomials are continuous for all real numbers. Thus the hypothesis of IVT is satisfied.

   (a) Use the Intermediate Value Theorem to prove that f has at least one critical point in $[1, 2]$.

   (b) Use the Big Theorem we discussed in class (about finding intervals where a function increases) to prove that f can have at most 1.

7. (20 points) Build a cubic polynomial $P(X)$ that has a relative maximum at $X = -2$ and a relative minimum at $x = 3$.

8. (35 points) Consider the table of values given for the functions f, f', g, and g':

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	2	1	3	2
1	4	6	2	5
2	6	4	3	4
3	1	2	5	3
4	3	5	2	6
5	5	3	4	1
6	0	3	2	4

(a) Let $L(x) = \sqrt{x} + f(x) + g(x)$. Compute $L(4)$ and $L'(4)$.

(b) Let $U(x) = x^2 f(x)$. Compute $U(3)$ and $U'(3)$.

(c) Let $K(x) = f(x^2)/x$. Compute $K(2)$ and $K'(2)$

(d) Let $Z(x) = (2x + g(x))^3$. Compute $Z(1)$ and $Z'(1)$.

(e) Let $Q(x) = g(x^2 + f(2x))$. Compute $Q(0)$ and $Q'(0)$.

9. (24 points) Consider the polynomial $g(x) = 2x^3 + 3x^2 - 36x + 10$.

   (a) Find the two critical points of g.

   (b) Build the sign chart for the function $g'(x)$.

   (c) Classify the critical points of g as (a) relative maxima, (b) relative minima, or (c) imposters.

   (d) Find an interval where g is concave upwards.

   (e) Find a point of the graph of g where the tangent line is parallel to the line $y = -36x - 4$.