June 13, 2001 **Name** _____

The total number of points possible is 130. **SHOW YOUR WORK**

1. (20 points) Use the definition of derivative to find $f'(a)$ for the function $f(x) = 4x - x^3$. Use this information to find the slope of the line tangent to the graph of f at the point $(-1, -3)$.

2. (10 points) Find the derivative of $f(x) = (2x^2 - \sqrt{x})^2$.

3. (10 points) Find $\frac{dy}{dx}$ when $y = (x^2 - 7x + 1)(3x - 1/x)$

4. (10 points) Find an equation for the line tangent to the graph of $h(x) = \dfrac{3x - 2}{x^2 - 1}$ at the point $(0, 2)$.

5. (10 points) The total weekly cost in dollars incurred by the Lincoln Record Company in pressing x playing records is given by $C(x) = 3000 + 3x - 0.001x^2,\ 0 \leq x \leq 6000$.

   (a) Find the average cost function $\overline{C}$.

   (b) Find the marginal average cost function $\overline{C'}$.

6. (10 points) Does the function $f(x) = \sqrt{x + 3}$ satisfy the hypothesis of Intermediate Value Theorem over the interval $[-2, 6]$. If so, find an INTEGER (ie, a whole number) M between $f(-2)$ and $f(6)$, and then find a number c in the interval $(-2, 6)$ such that $f(c) = M$.

7. (10 points) Suppose that $f'(3) = 2$ and $f(3) = 1$. What is the y-intercept of the line tangent to the graph of f at the point $(3, 1)$?

8. (30 points) Suppose the functions f and g are differentiable. Some of the values of f, f', g, and g' are given in the table. The next six problems refer to these functions f and g. Recall that, for example, the entry 10 in the fifth row and sixth column means that $g'(4) = 10$.

x	$f(x)$	$f'(x)$
0	2	1
1	7	3
2	5	4
3	1	2
4	3	3
5	6	4
6	0	5
7	4	1

x	$g(x)$	$g'(x)$
0	5	5
1	7	3
2	4	4
3	2	6
4	6	10
5	3	4
6	1	2
7	0	1

   (a) The function h is defined by $h(x) = f(g(x))$. Use the chain rule to find $h'(3)$.

   (b) The function R is defined by $R(x) = g(f(x))$. Use the chain rule to find $R'(2)$.

   (c) The function k is defined by $k(x) = f(x) \cdot g(x)$. Use the product rule to find $k'(5)$.

   (d) The function H is defined by $H(x) = f(x)/g(x)$. Use the quotient rule to find $H'(4)$.

   (e) The function K is defined by $K(x) = (f(x) + g(x))^2$. Find $K'(6)$.

   (f) The function M is defined by $M(x) = f(f(x))$. Use the chain rule to find $M'(0)$.

9. (20 points) The altitude of a rocket t seconds into flight is given

$$s = f(t) = -2t^3 + 114t^2 + 480t + 1 \qquad (t \geq 0),$$

   where s is measured in feet.

   (a) Find an expression v for the rockets velocity at any time t.

   (b) Compute the rockets velocity when $t = 10, 40, 50$, and 70. Interpret your results.

   (c) Using the results from part b., find the maximum height of the rocket. Hint: at its maximum height, the velocity of the rocket is zero.