

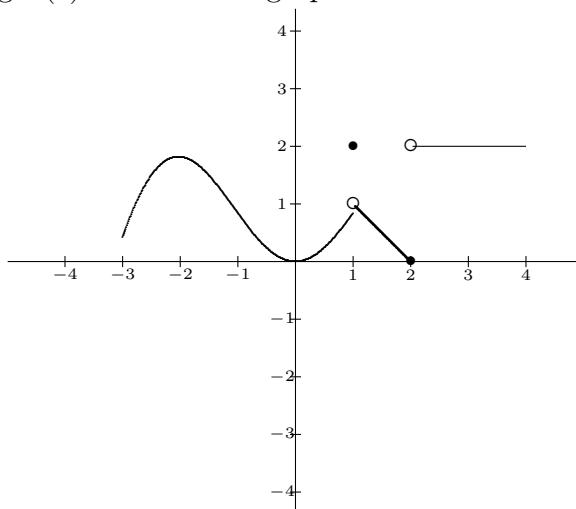
February 23, 2001

Name \_\_\_\_\_

In the first 3 problems, each part counts 6 points (total 42 points) and the final 4 problems count as marked. The total number of points available is 112.

Multiple choice section. Circle the correct choice. You do not need to show your work on these problems.

1. Questions (a) through (e) refer to the graph of the function  $f$  given below.



- (a)  $\lim_{x \rightarrow 1} f(x) =$   
(A) 0    (B) 1    (C) 2    (D) 4    (E) does not exist
- (b)  $\lim_{x \rightarrow 2^-} f(x) =$   
(A) 0    (B) 1    (C) 2    (D) 4    (E) does not exist
- (c) A good estimate of  $f'(0)$  is  
(A) -1    (B) 0    (C) 1    (D) 2    (E) there is no good estimate
- (d) A good estimate of  $f'(-1)$  is  
(A) -1    (B) 0    (C) 1    (D) 2    (E) there is no good estimate
- (e) A good estimate of  $f'(2)$  is  
(A) -1    (B) 0    (C) 1    (D) 2    (E) there is no good estimate

2. The line tangent to the graph of a function  $f$  at the point  $(2, 3)$  on the graph also goes through the point  $(-2, 7)$ . What is  $f'(2)$ ?
- (A)  $-2$    (B)  $-1$    (C)  $0$    (D)  $1$    (E)  $2$
3. What is the slope of the tangent line to the graph of  $f(x) = 2x^{-2}$  at the point  $(2, 1/2)$ ?
- (A)  $-1/2$    (B)  $-1/4$    (C)  $-1/8$    (D)  $-1/16$    (E)  $-1/512$

On all the following questions, **show your work.**

4. (15 points) Intermediate Value Theorem. Recall that the IVT asserts the following: If  $f$  is a continuous function on the interval  $[a, b]$  and  $M$  is a number between  $f(a)$  and  $f(b)$ , then there exists a number  $c$  satisfying  $a \leq c \leq b$  and  $f(c) = M$ . For this problem let  $f(x) = \sqrt{2x - 1}$  and let  $[a, b] = [1, 5]$ . Finally, suppose  $M = 2$ . Find the number  $c$  whose existence is guaranteed by IVT.

5. (15 points) The total weekly cost in dollars incurred by the Lincoln Record Company in pressing  $x$  playing records is given by  $C(x) = 2000 + 3x - 0.01x^2$  for  $x$  in the range 0 to 6000.
- (a) Find the marginal cost function  $C'(x)$ .
- (b) Find the average cost function  $\bar{C}(x)$ .
- (c) Find the marginal average cost function  $\bar{C}'(x)$ .
- (d) Interpret your results in (c). Is the average cost growing or falling as the company produces more units?
6. (15 points) Let  $f(x) = 4/x$ .
- (a) Construct  $\frac{f(3+h)-f(3)}{h}$
- (b) Simplify and take the limit of the expression in (a) as  $h$  approaches 0 to find  $f'(3)$ .
- (c) Use the information found in (b) to find an equation for the line tangent to the graph of  $f$  at the point  $(3, 4/3)$ .
7. (25 points) Compute the following derivatives.
- (a) Let  $f(x) = x^3 + x^{-\frac{1}{2}}$ . Find  $\frac{d}{dx}f(x)$ .
- (b) Let  $g(x) = \sqrt{x^2 + 4}$ . What is  $g'(x)$ ?
- (c) Find  $\frac{d}{dx}((3x + 1)^2 \cdot (4x^4 - 1))$

(d) Find  $\frac{d}{dx} \frac{2x^2+1}{x+2}$

(e) Find  $\frac{d}{dt} (t^2 + 1/t)^2$ .