October 28, 2014 **Name** _____

The problems count as marked. The total number of points available is 145. Throughout this test, **show your work.**

1. (30 points) Let $f(x) = 3x^4 + 4x^3 - 72x^2$.

   (a) Find the critical points of f.

   (b) Build the sign chart for $f'(x)$.

   (c) Use this information in part (b) to find the intervals over which f is increasing.

   (d) Discuss the concavity of f.

   (e) Find $f(1)$ and $f'(1)$. Use this information to find the line tangent to f at $(1, f(1))$ in slope-intercept form.

2. (35 points) Consider the table of values given for the functions f, f', g, and g':

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	2	1	6	2
1	4	6	2	5
2	6	4	3	4
3	1	2	5	3
4	3	5	2	6
5	5	3	4	1
6	0	3	2	4

(a) Let $L(x) = 2f(x) \cdot g(x)$. Compute $L'(5)$.

(b) Let $U(x) = f(3x) \div g(2x)$. Compute $U(2)$ and $U'(2)$.

(c) Let $K(x) = g(x + f(x))$. Compute $K(3)$ and $K'(3)$.

(d) Let $V(x) = f(g(f(x)))$. Compute $V'(3)$.

(e) Let $W(x) = g(x^2 - 1)$. Compute $W'(2)$.

3. (20 points) Recall that $\frac{d}{dx}e^{g(x)} = e^{g(x)} \cdot g'(x)$. Find the intervals over which the function $f(x) = x^2e^{2x}$ is increasing. Write your answer in interval notation.

4. (15 points) Two positive numbers x and y are related by $2x + 3y = 16$. What is the largest possible product xy could be, and what pair (x, y) achieves that product? Note that if $y = 2$, then $x = 5$ and the product $xy = 10$. If $y = 4$, then $x = 2$ and the product is 8.

5. (15 points) Two positive numbers x and y are related by $xy = 10$. What is the smallest possible value $6x + 3y$ could have?

6. (30 points) Consider the function

$$r(x) = \frac{(x^2-4)(6x)}{(3x-6)(x+1)(x-3)}.$$

Use the Test Interval Technique to find the sign chart of $r(x)$. Find the zeros and the horizontal and vertical asymptotes, and sketch the graph of r. Your graph must be consistent with the information you find in the sign chart.