March 14, 2014 **Name** ______________________________

The problems count as marked. The total number of points available is 174. Throughout this test, **show your work.** Using a calculator to circumvent ideas discussed in class will generally result in no credit.

1. (24 points) Demonstrate your understanding of the product, quotient and chain rules by differentiating each of the given functions. Find the critical points for each function and the intervals over which the function is increasing. You must show your work.

   (a) Let $F(x) = (2x - 8)(3x + 6)$.

   (b) Let $G(x) = (x^2 - 4)^3$.

   (c) Let $H(x) = \frac{2x-3}{x-4}$.

2. (24 points)

   (a) Build a cubic polynomial $p(x)$ with zeros at $x = -2$, $x = 0$ and $x = 4$.

   (b) Find the critical points of p.

   (c) Find the interval(s) over which the polynomial p is decreasing.

3. (12 points) Mike thinks of a positive number. He adds the square of his number and the reciprocal of his number. What is the smallest possible sum he could obtain? A calculator solution will get not credit. Estimate the answer to the nearest 0.01.

4. (30 points) Consider the table of values given for the functions f, f', g, and g':

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	2	1	3	2
1	4	6	2	5
2	6	4	3	4
3	1	2	5	3
4	3	5	2	6
5	5	3	4	1
6	0	3	1	4

(a) $Q(x) = g(x)/f(x)$. Find $Q(5)$ and $Q'(5)$.

(b) Let $H(x) = f(x^2 + 1)$. Compute $H(1)$ and $H'(1)$.

(c) Let $W(x) = [g(x)]^3 + f(x)$. Compute $W(4)$ and $W'(4)$.

(d) Let $L(x) = (x^2 + 1) \cdot f(x)$. Compute $L(2)$ and $L'(2)$.

(e) Let $U(x) = f \circ g \circ f(x)$. Compute $U(6)$ and $U'(6)$.

(f) Let $Z(x) = g(2x + 1) \cdot f(3x - 1)$. Compute $Z(1)$ and $Z'(1)$.

5. (20 points) Consider the line L given by $y = 2x$, the point $P = (-4, 2)$, and the circle whose equation is $x^2 - 8x + y^2 - 4y = -16$.

   (a) Find the point on the line that is closest to the point P.

   (b) Find the point on the circle that is closest to the point P.

   (c) Find the point on the line that is closest to the circle.

6. (24 points) Let

$$f(x) = \begin{cases} |x+2| & \text{if } -\infty < x \leq 0 \\ \sqrt{2x+2} & \text{if } 0 < x \leq 4 \\ x^{2/3} & \text{if } 4 < x < \infty \end{cases},$$

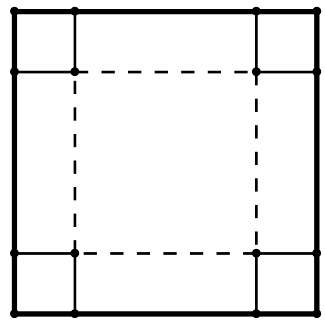
(a) Find an equation for the line tangent to the graph of f at the point $(-3, 1)$.

(b) Find an equation for the line tangent to the graph of f at the point $(1, 2)$.

(c) Find an equation for the line tangent to the graph of f at the point $(8, 4)$.

(d) Discuss the continuity of f at the point $x = 0$.

7. (20 points) We are given a 8in $\times$ 8in sheet of paper as shown. Squares of size $x \times x$ are cut from each corner and then the sides are folded upward along the dashed lines. Let $V(x)$ denote the volume of the topless box obtained. (This is the same problem as you had in Set 5).



(a) What is the natural domain of V? Calculate $V(1)$ and $V(2)$.

(b) Sketch the graph of $V(x)$ over its natural domain.

(c) Find the maximum value of $V(x)$.

8. (20 points) If a ball is thrown vertically upward from the roof of 256 foot building with a velocity of 64 ft/sec, its height after t seconds is $s(t) = 256 + 64t - 16t^2$.

   (a) What is the height the ball at time $t = 1$?

   (b) What is the velocity of the ball at the time it reaches its maximum height?

   (c) At what time t does the ball reach its maximum height?

   (d) What is the maximum height the ball reaches?

   (e) After how many seconds is the ball exactly 176 feet above the ground?

   (f) How fast is the ball going when it reaches the height 176? Write the answer with the correct units.

   (g) How fast is the ball going when it hits the ground?

   (h) The second derivative $s''(t)$ of the position function, also called the *acceleration* function, is denoted $a(t)$. Compute $a(t)$. Explain why this function is negative for all values of t.